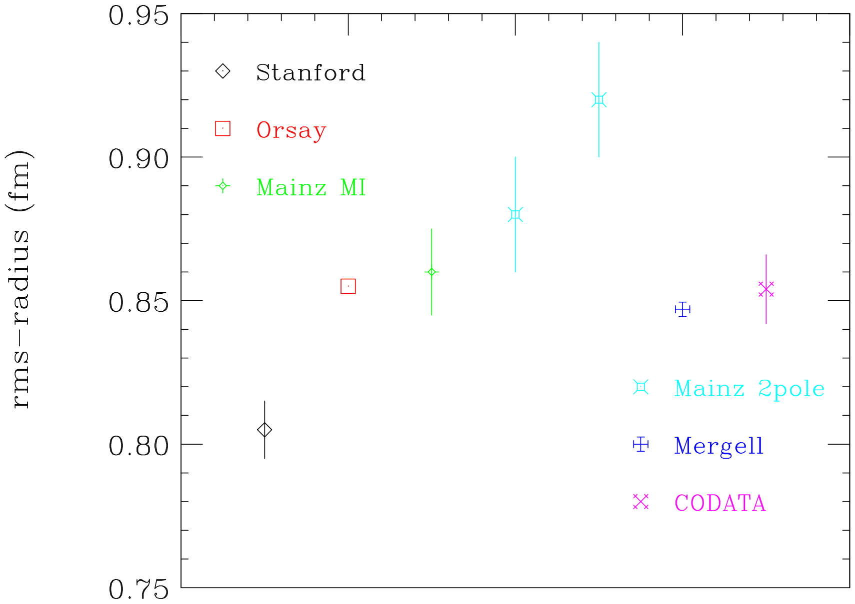
Problems with the proton rms-radius

Ingo Sick

Charge-rms radius of proton: fundamental quantity

needed in many applications

History of radius from (e,e): rather checkered



Reanalysis: IS, PLB 576 (03) 62

removed several deficiencies of previous studies

finds $r_{rms} = 0.895 \pm 0.018$fm, significantly larger than previous results

understand reasons for change

nonconvergence q^{2n}, poor fit VDM, Coulomb, fit G_e instead of σ

Unsatisfactory: size error bar 0.018fm

for A>1 error bar smaller, despite poorer data base

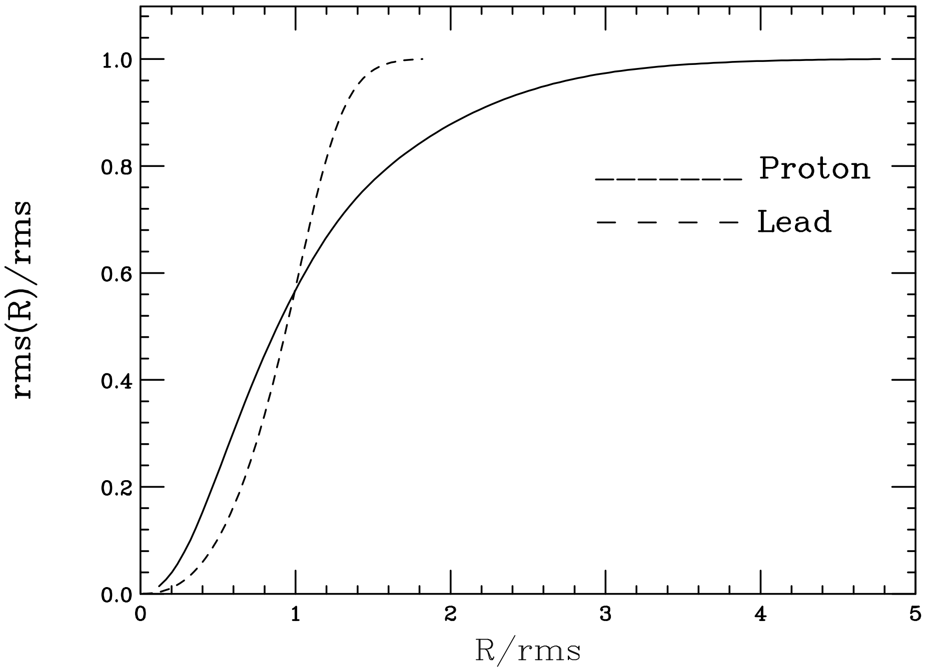
for atomic physics would want more accurate radius

Reason

for proton shape $\rho(r) \sim$ exponential

$\rightarrow$ important role of large-r tail, see $[\int_0^R \rho(r) r^4 dr / \int_0^\infty \rho(r) r^4 dr]^{1/2}$

there ρ small, poorly determined



for 1% need to integrate to 3.6·rms!

Idea: constrain *shape* of large-r tail

add physics, get more accurate *rms*-radius

Tail of nucleon charge density

Simple-most model for large r

least-bound Fock state: p = n + π^+, n = p + π^-
dominates $\rho(r)$ completely at large-enough r (>0.8fm in cloudy bag model)
will use as constraint
need to think about relation G_e(q) $\leftrightarrow$ ρ(r)

Interlude: $\rho(r)_{exp}$ from (e,e)

non-relativistic: $\rho(r)$ = Fourier-transform of $G_e(q)$

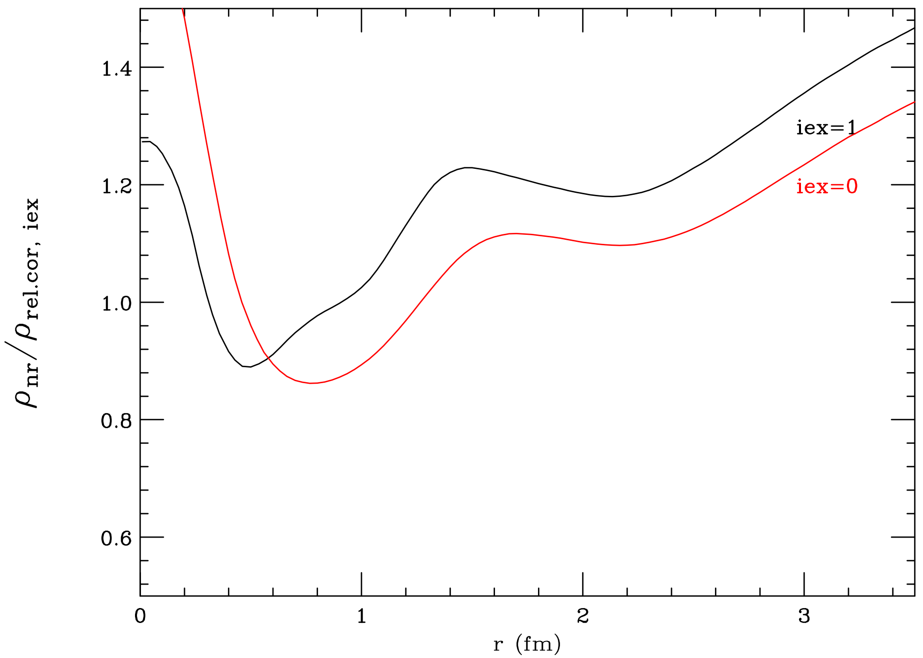
But: q very large, need to consider relativistic effects

1. determine $\rho(r)$ in Breit-frame, + Lorentz contraction
   use as momentum transfer $\kappa^2 = Q^2/(1.+\tau)$, $\quad \tau = Q^2/4M^2$

2. for composite systems boost operator depends on structure
   various prescriptions (Licht, Mitra, Ji, Holzwarth,...), all of form
   $G_e(q) \rightarrow G_e(q)(1.+\tau)^\lambda$, λ=0 or 1
   de facto λ=0 or 1 makes little difference for ρ(large r)

Test:

calculate ρ(r) from given G_e(q) with/without relativistic corr.
take ratio



find: ambiguity in relativistic effects important for ρ at small r
unimportant for large-r $\equiv$ low momenta

λ affects only normalization of large-r density, not shape
normalization *not* used in constraint

desirable side-effect: ρ(r=0) flat after application of relativistic corrections

Density at very large r

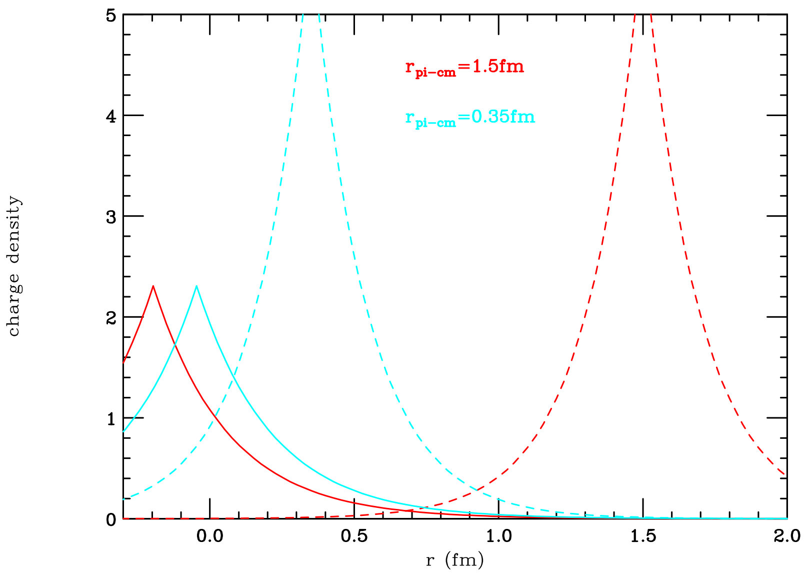
a priori use asymptotic form: Whittaker function $W_{-\eta,3/2}(2\kappa r)/r$

use physical masses m_N, m_π, l=1

use separation energy = m_π, include CM-correction

makes sense only at *large* n-π relative distance: rms$_p$=0.89fm, rms$_\pi$=0.66fm

only at large r overlap n, π small (see red curves)



potential difficulty

need to fold with charge distribution of n, π

could get into trouble with r=0 divergence of W/r

in practice

calculate w.f. in square well potential, V(r>R)=0 (courtesy D.Trautmann)

radius R=0.8fm (not important) , depth adjusted to separation energy

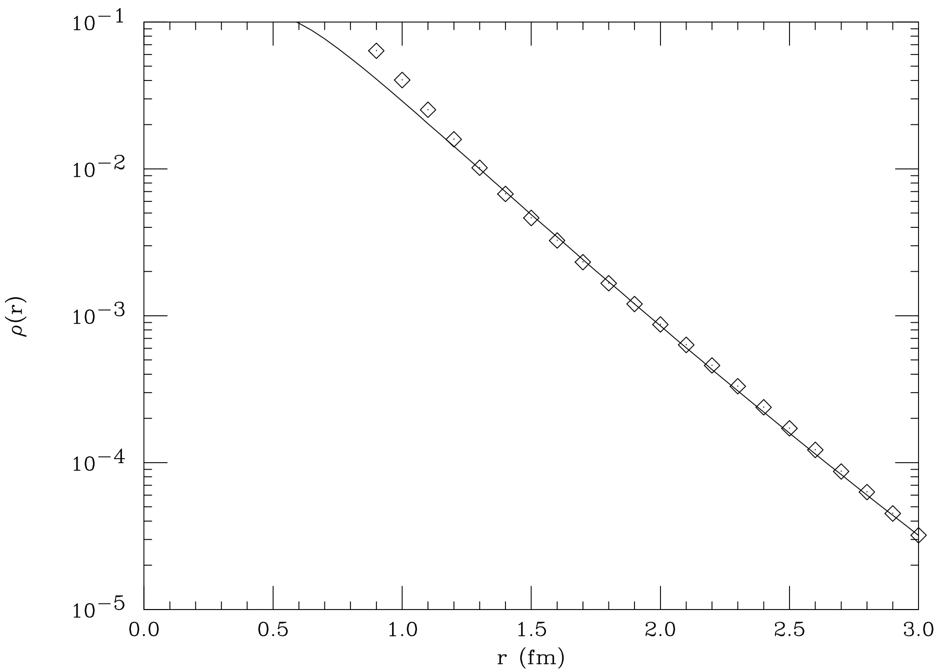
for r>R shape $\rho(r) \equiv$ Whittaker function

can easily fold

according to DT small difference Schrödinger-KleinGordon

Result

excellent agreement with shape of ρ_{exp}(r) (norm fit to ρ_{exp})



"Refinements" of model

allow also for $\Delta + \pi$ contribution

coefficients of various terms from Dziembowski,...,Speth

'Pionic contribution to nucleon EM properties in light-front approach'

for p,n get contributions from $\pi^+ n$, $\pi^- p$, $\pi^- \Delta^{++}$, $\pi^+ \Delta^0$, $\pi^- \Delta^+$, $\pi^+ \Delta^-$

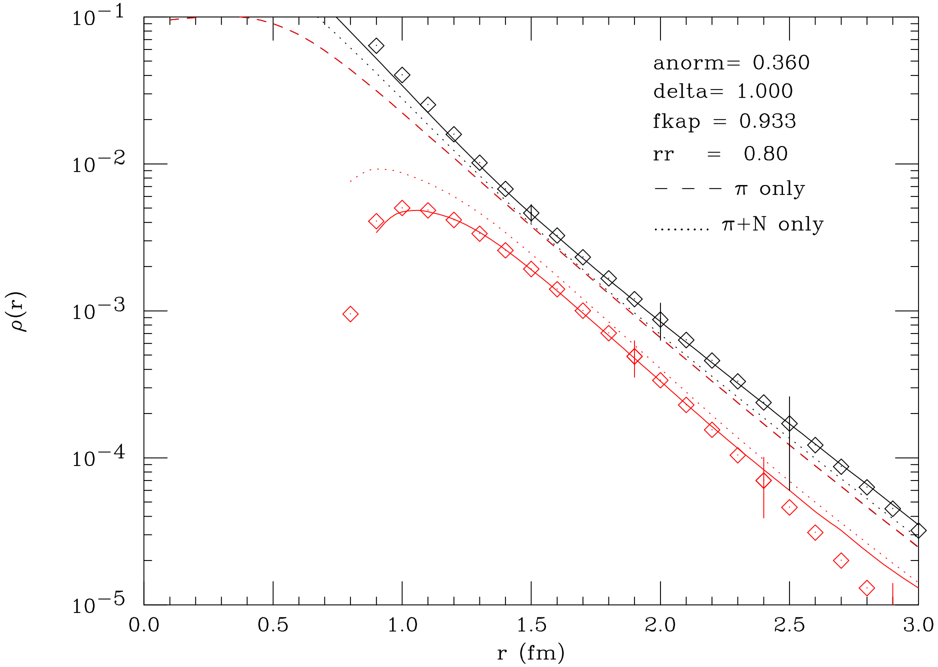
calculate similarly

effect on p-tail: small, improves a bit towards smaller r

effect on n-tail: larger, gets close to data with *same* normalization factor

not really relevant as will ignore n, components $\neq \pi^- p$ too important

$\diamond \rho_p(r)$, $\diamond - \rho_n(r)$, ——— shape tail



Plausibility checks

fraction of norm in π-tail

experimental charge distribution

$\int_{1.}^{\infty} = 0.17$ $\int_{1.3}^{\infty} = 0.08$

Myhrer+Thomas, cloudy bag model ($\sim$ tail)

important to reduce spin sum rule, from value for relativistic quarks, 0.65 by factor 0.7-0.8 down to exp. value of 0.33$\pm$0.06

$P_{n\pi} = 0.2 - 0.25,$ $P_{\Delta\pi} = 0.05 - 0.1$

Bunyathyan+Povh, Deep inelastic scattering

reaction p + e $\rightarrow$ n(forward) + e' + X (only integral information)

$P_{n\pi} = 0.24 - 0.39$

Nikolaev *et al.* Drell-Yan (integral)

$P_{\pi n} = 0.21 - 0.28$

Hammer et al, VDM

$\int_{1.}^{\infty} = 0.03$ $\int_{1.3}^{\infty} = 0.017$

....continue with fit p-data

Data used in fit

world (e,e) data up to 12 fm^{-1}

both cross sections and polarization data

two-photon exchange corrections (Arrington *et al.*)

makes G_{ep} from σ and P to agree

(relative) tail density for r >1.3fm

Parameterization

r-space parameterization to implement constraint

use Sum-Of-Gaussians (SOG) parameterization for G_{ep} and G_{mp}

Detail

placed every ∼0.3fm, for r<3.3 fm

amplitudes fit to σ, P, constraint

include relativistic corrections (unimportant for large r)

24 parameters

Results

605 data points for $q_{max} = 12$ fm^{-1}

20 values for constraint on shape, for r > 1.3fm, *i.e.* for $\rho(r) < 0.01\ \rho(0)$

χ^2=518 (812) when floating (or not) data

excellent fit of tail-constraint

Find: $\mathbf{r}_{rms} = 0.894 \pm 0.008$ **fm**

error bar includes statistics+systematics
important reduction of uncertainty!

Added benefit of tail-constraint:

floating changes $\mathbf{r}_{rms}$ **by 0.0014 fm only**
fit without constraint: floating changes .02 fm, bigger than error bar!
exemplifies dangers of floating *without* large-r constraint

constraint suppresses unphysical wiggles in $G_e(q)$ **at very low q**

Radius from spectroscopy of atomic Hydrogen

spectacular progress of experiments
transition energies measured to 13 digits
1s Lamb shift measured to 5 significant digits
most of higher-order QED-terms now calculated
for summary see RMP 80 (2008) 633

find rms-radius = 0.877±0.007 fm

agreement with (e,e) satisfactory
considering tiny effect of rms in Lamb-shift

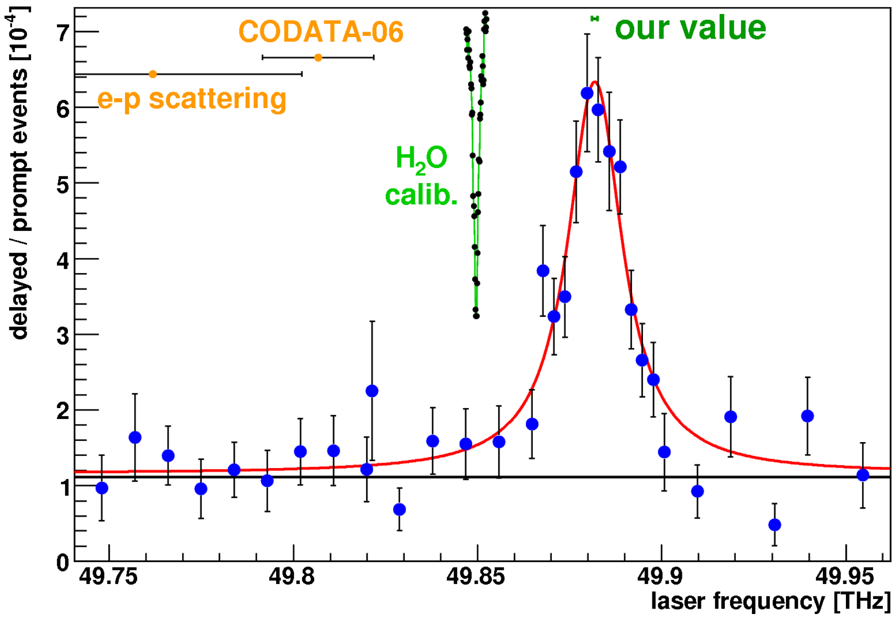
Big problem

recent data on muonic Hydrogen

Pohl *et al.*, PSI-experiment
subm. to Nature

find rms-radius = 0.842±0.001 fm

Convincing data

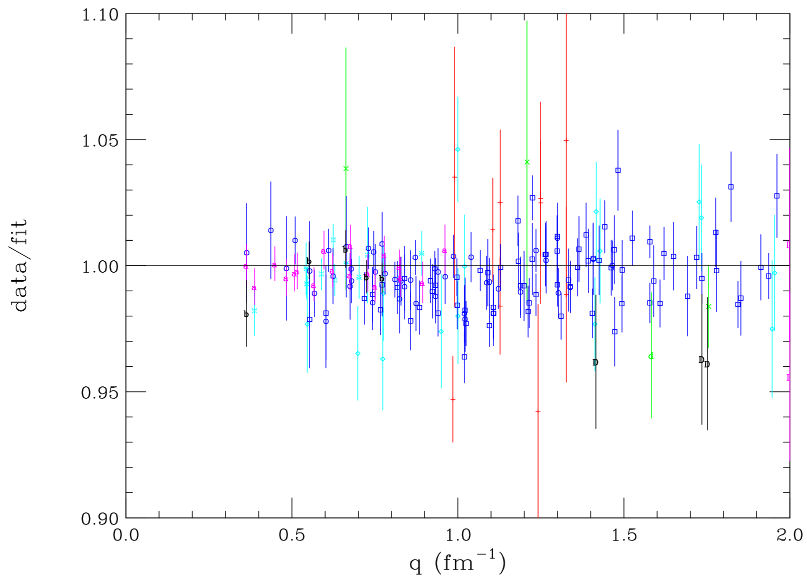


Can (e,e) and μX be made compatible?

analyze world (e,e)-data with constraint on rms-radius

data	tailconstraint	χ^2	rms
(e,e) not floated	no	822	0.897
(e,e), floated	no	422	0.881
(e,e)+μX, not floated	no	926	0.842
(e,e)+μX, floated	no	574	0.843
(e,e), floated	yes	518	0.893
(e,e)+μX, floated	yes	715	0.845

Find large increase in χ^2: 422 $\rightarrow$ 574
for fit with tailconstraint $\rightarrow$ 715
ratio data/fit show systematic trend



my conclusion:

serious discrepancy (e,e)$\leftrightarrow$ μX

Explanations??

missing QED terms??

Zemach-term $(Z\alpha)^5$ apparently still in doubt

polarization of proton??

problems common to all (e,e)-data, *e.g.* rad. corrections??

defect of present (e,e) data set??

new MAMI-experiment finds $0.880\pm0.004\pm0.004$ fm

2-photon effects larger than calculated??