Microscopic theories of strongly interacting many-body systems

Pisa Research Unit : **Ignazio Bombaci**

Proposed research activity

1. **Nuclear Matter Equation of State: role of three-body forces**

   in collaboration with Alejandro Kievsky (Pisa),
   Domenico Logoteta and Isaac Vidaña (Coimbra)

2. **Quark deconfinement phase transition in neutron stars.**

   Hybrid stars with the Field Correlator Model (FCM)

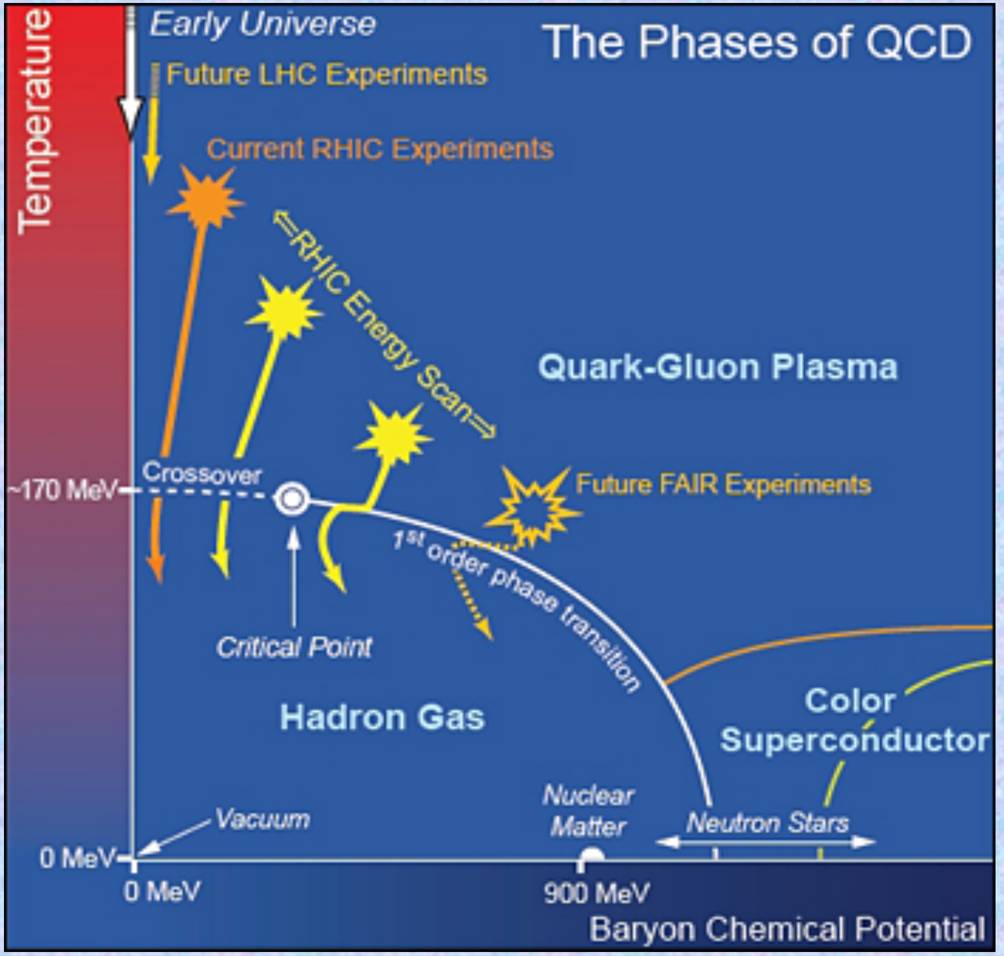
   in collaboration with D. Logoteta (Coimbra)

Microscopic theories of strongly interacting many-body systems
Otranto, May 31, 2013

A link between lattice QCD and measured neutron star masses

Ignazio Bombaci (Università di Pisa e INFN Sez di Pisa)
and
Domenico Logoteta (University of Coimbra, Portugal)

Monthly Notices of the Royal Astronomical Society, 433 (2013) L79



The Phases of QCD
Temperature
Early Universe
Future LHC Experiments
Current RHIC Experiments
⇐RHIC Energy Scan⇒
Quark-Gluon Plasma
~170 MeV
Crossover
Future FAIR Experiments
1st order phase transition
Critical Point
Hadron Gas
Color Superconductor
Vacuum
Nuclear Matter
Neutron Stars
0 MeV
0 MeV
900 MeV
Baryon Chemical Potential

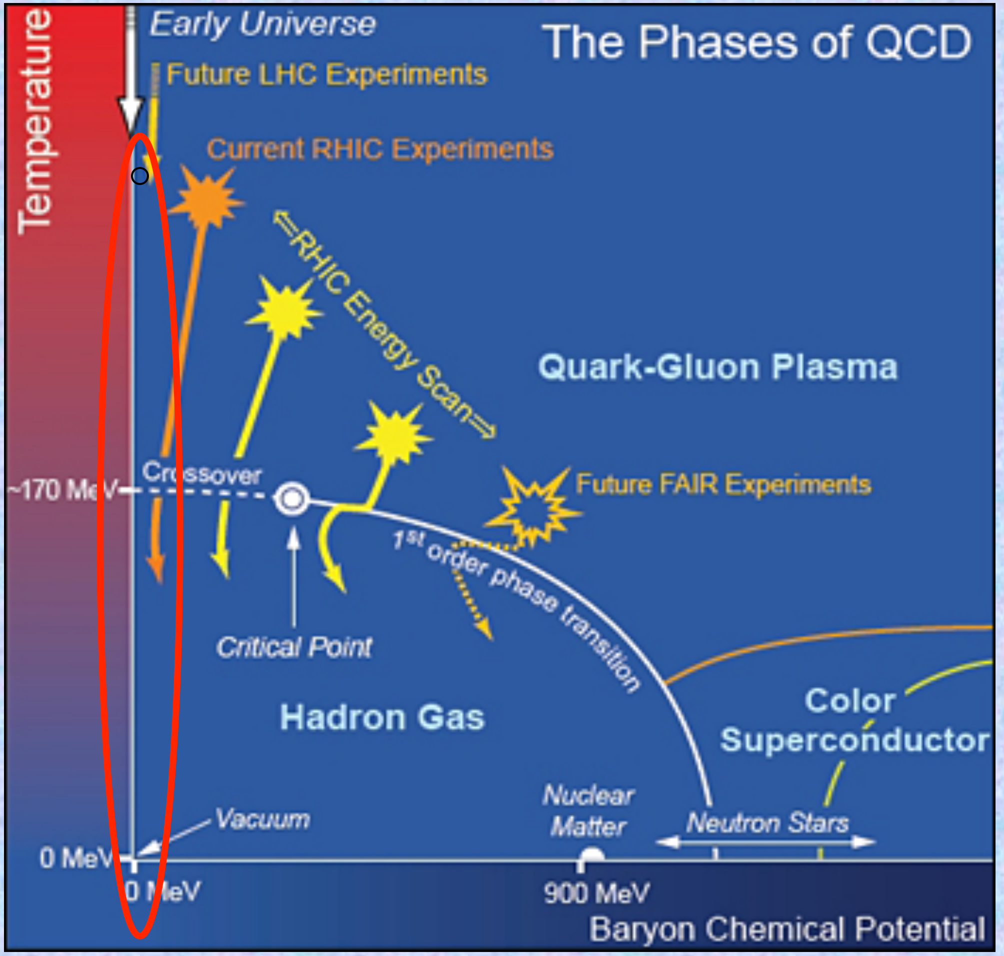
Lattice QCD at μ_b=0 and finite T

- The transition to Quark Gluon Plasma is a crossover
  Aoki et ,al., Nature, 443 (2006) 675

- Deconfinement transition temperature T_c

HotQCD Collaboration
T_c= 154 ± 9 MeV
Bazarov et al., Phys.Rev. D85 (2012) 054503

Wuppertal-Budapest Collab.
T_c= 147 ± 5 MeV
Borsanyi et al., J.H.E.P. 09 (2010) 073



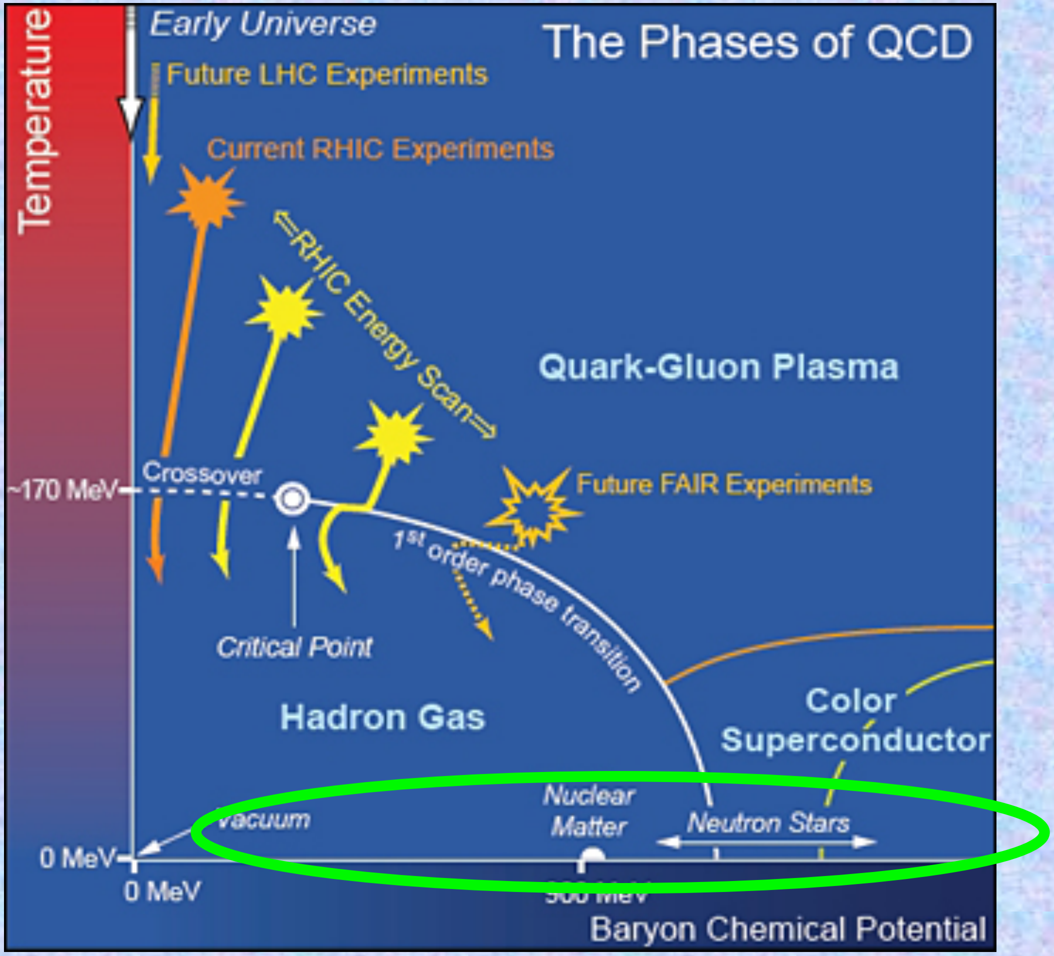
Neutron Star Matter high μ_b and low T

► **Quark deconfinement transition expected of the first order** Z. Fodor, S.D. Katz, Prog. Theor Suppl. 153 (2004) 86

► **Lattice QCD calculations are presently not possible**

Deconfined phase:

QCD inspired models



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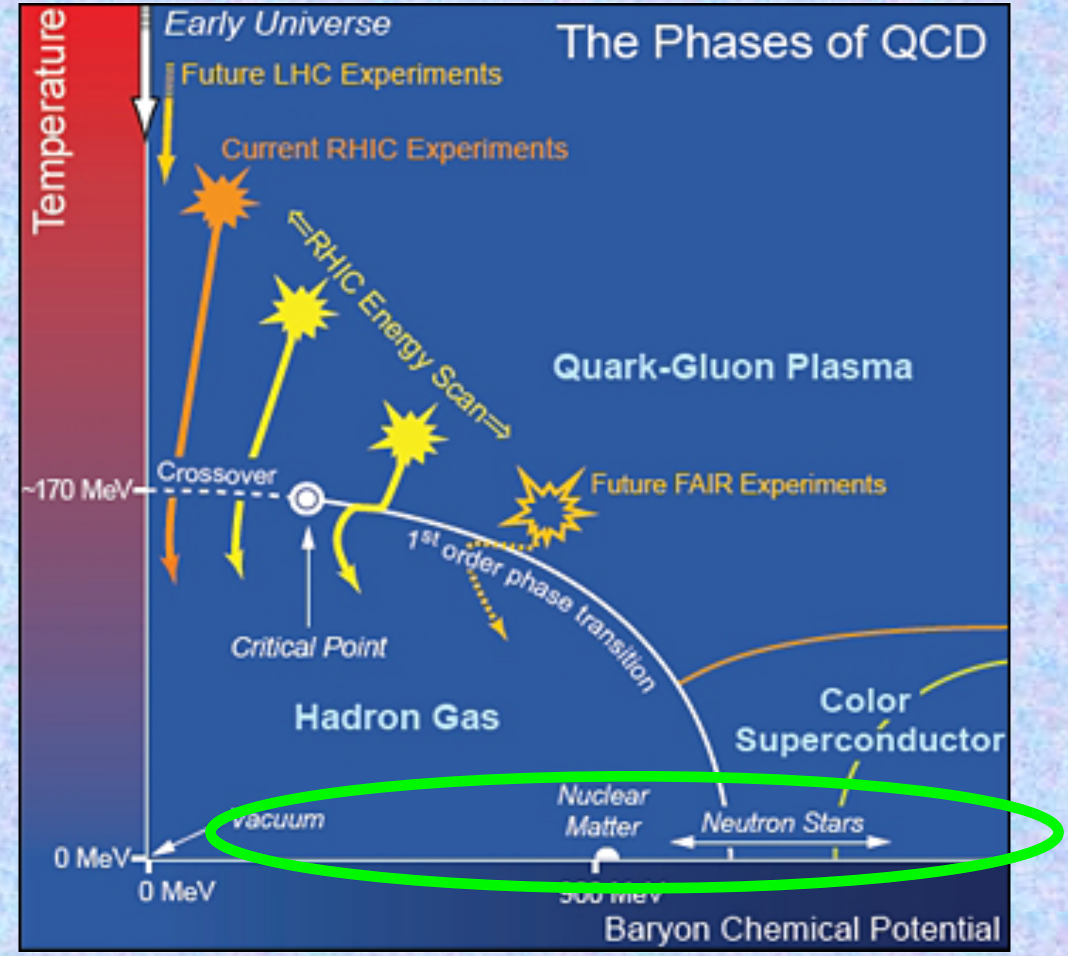
Deconfined phase:

QCD inspired models



► Unreliable as $\rho \rightarrow \rho_{deconf}$

► can not make predictions at high T and μ_b= 0 can not be tested using Lattice QCD calculations

Field Correlator Method (FCM)

The FCM is a nonperturbative approach to QCD which includes from first principles the dynamics of confinement in terms of color electric (CE) $D^E(x)$, $D^E_1(x)$, and color magnetic (CM) $D^H(x)$, $D^H_1(x)$, correlators

H.G. Dosch, Phys.Lett. B 190, 177 (1987); H.G. Dosch, Yu. Simonov, Phys.Lett. B 2005, 339 (1988); Yu. Simonov, Nucl.Phys. B 307, 512 (1988) A. Di Giacomo et al., Phys.Rep. 372, 319 (2002).

Recently the FCM has been extended to $\mu_b \neq 0$

Yu. Simonov, M.A. Trusov, JEPT Lett. 85, 598 (2007) Yu. Simonov, M.A. Trusov, Phys.Lett. B 650, 36 (2007)

The FCM can describe the whole QCD phase diagram as it can span from high temperature and low baryon chemical potential, to low T and high μ_b limit

Two-points correlator of the gluonic field strength tensor ("Field Correlator")

$$\Delta^{(2)}_{\mu_1\nu_1\mu_2\nu_2} = \frac{1}{N_c} Tr \left\langle F_{\mu_1\nu_1}(x)\Phi(x,y)F_{\mu_2\nu_2}(y)\Phi(y,x)\right\rangle$$

$$\Phi(x,y) = P\exp\left[i\int_x^y A_\mu dz_\mu\right]$$

Parallel transporter

$$\frac{1}{N_c} Tr \left\langle E_i(x)\Phi(x,y)E_j(y)\Phi(y,x)\right\rangle =$$

$$\delta_{ij}\left[D^E(z) + D_1^E(z) + z_4^2\frac{dD_1^E(z)}{dz_4^2}\right] + z_i z_j \frac{dD_1^E(z)}{d\vec{z}^{\,2}}$$

$$z = x - y$$

Similar expression for the CM correlators

FCM Equation of State of the Quark Phase

Quark pressure (q = u, d, s)

$$P_q \Big/ T^4 = \frac{1}{\pi^2}\left[\phi_\nu\left(\frac{\mu_q - V_1/2}{T}\right) + \phi_\nu\left(-\frac{\mu_q - V_1/2}{T}\right)\right]$$

$$\phi_\nu(a) = \int_0^\infty du \frac{u^4}{\sqrt{u^2+\nu^2}} \frac{1}{\exp(\sqrt{u^2+\nu^2} - a) + 1}$$

$$\nu = \frac{m_q}{T}$$

Yu. Simonov, M.A. Trusov, JEPT Lett. 85, 598 (2007); Phys.Lett. B 650, 36 (2007)

Large distance static $q\bar{q}$ potential

$$V_1(T) = \int_0^{1/T} d\tau \left(1 - \tau T\right) \int_0^{\infty} d\xi \; \xi \; D_1^E\left(\sqrt{\xi^2 + \tau^2}\right)$$

$$D_1^E(x) = D_1^E(0) \exp\left(-|x|/\lambda\right)$$

$\lambda \approx 0.34 \;\; \mathrm{fm}$ Vacuum correlation lenght (D'Elia et al.(1997))

Large distance static $q\bar{q}$ potential

$$V_1(T) = \int_0^{1/T} d\tau \left(1 - \tau T\right) \int_0^{\infty} d\xi \; \xi \; D_1^E\left(\sqrt{\xi^2 + \tau^2}\right)$$

$$D_1^E(x) = D_1^E(0) \exp\left(-|x|/\lambda\right)$$

$\lambda \approx 0.34 \;\; \text{fm}$ **Vacuum correlation lenght (D'Elia et al.(1997))**

$V_1(T)$ is independent on the baryon chemical potential

supported by lattice QCD similations at small μ_b (Doring et al. 2006)

We take [as in Ref.: Baldo, et al. Phys. Rev. D78, 063009 (2008)]

$V_1 \equiv V_1(T=0)$ **as a model parameter**

Gluon pressure

$$P_g \Big/ T^4 = \frac{8}{3\pi^2}\int_0^\infty d\chi\ \chi^3 \left[\exp\left(\chi + \frac{9V_1}{8T}\right) - 1\right]^{-1}$$

Yu. Simonov, M.A. Trusov, JEPT Lett. 85, 598 (2007)

Yu. Simonov, M.A. Trusov, Phys.Lett. B 650, 36 (2007)

A.V. Nefediev, Yu.S Simonov, M.A.Trusov, Int. J. Mod. Phys. E1 8, 549 (2009)

Total pressure of the deconfined phase

$$P_{QP} = P_g + \sum_{u,d,s} P_q + \Delta\varepsilon_{vac}$$

$$\Delta\varepsilon_{vac} \approx -\frac{11-\frac{2}{3}N_f}{32}\frac{G_2}{2}$$

$$N_f = 3 \Rightarrow \Delta\varepsilon_{vac} = -\frac{9}{64}G_2$$

G_2 **gluon condensate** Shifman, Vainshtein, Zakharov, Nucl.Phys. B 147, 385 (1979)

$G_2 = 0.012 \pm 0.06$ GeV^4 (QCD sum-rules)

G_2 model parameter

Previous **Hybrid Stars calculations with the FCM**

M. Baldo, *et al.* Phys. Rev. D78, 063009 (2008)

Hadronic phase: BHF with 3NF
(nucleonic or hyperonic matter)

Quark phase: FCM

Deconf. Phase trans.: first order, Maxwell construction

Equation of State of the Hadronic Phase

Nonlinear relativistic mean field model:
Glendennig – Moszkowski GM1 parametriztion

K = 300 MeV, $M^* = 0.7$ M

Pure nucleonic matter (N)

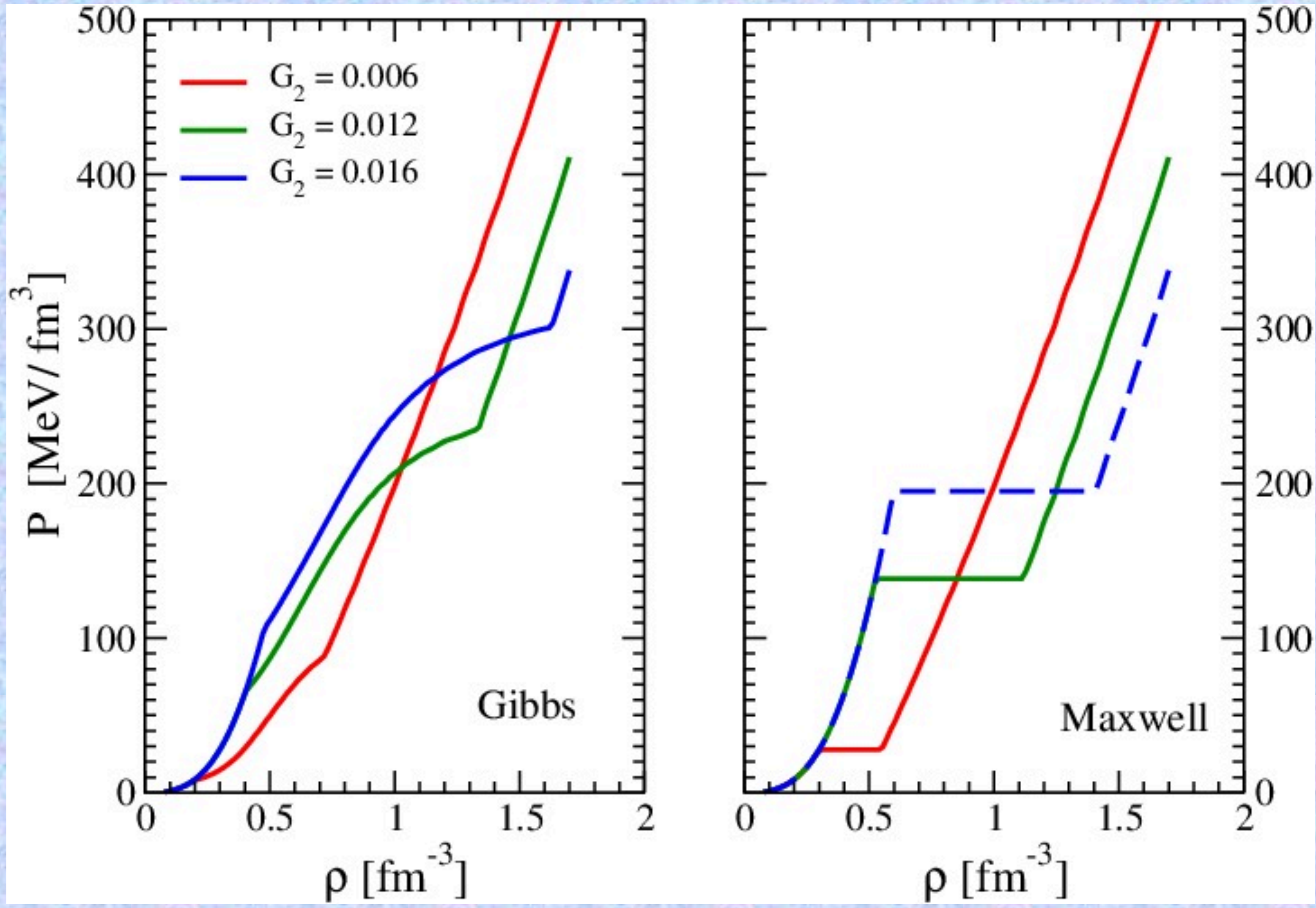
Hyperonic matter (NY)

Deconfinement phase transition

First order transition

Gibbs construction (Glendenning, Phys.Rev.D 46, 1274 (1992)

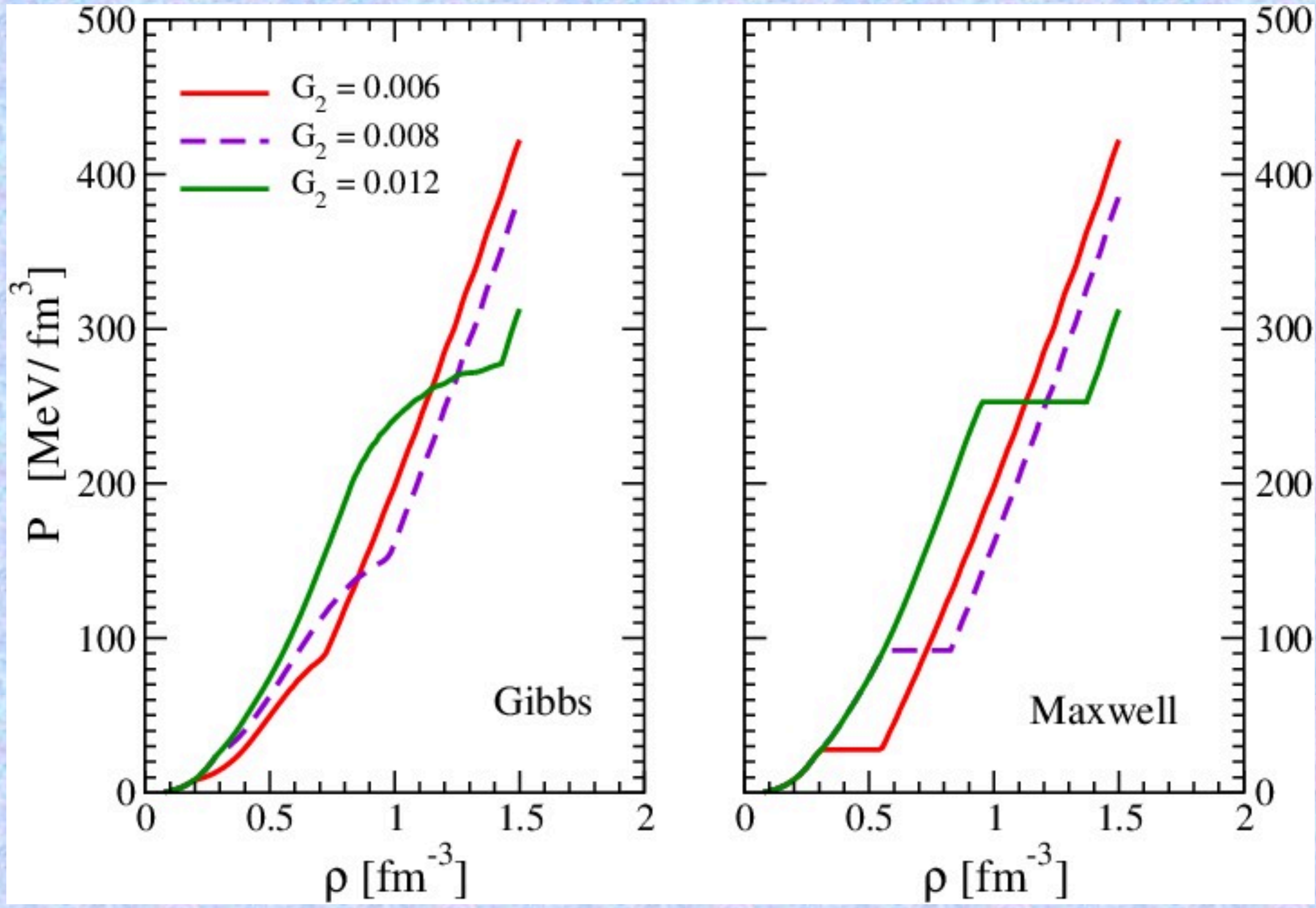
Equation of State



Pure nucleonic matter (N)

V_1= 0.01 GeV

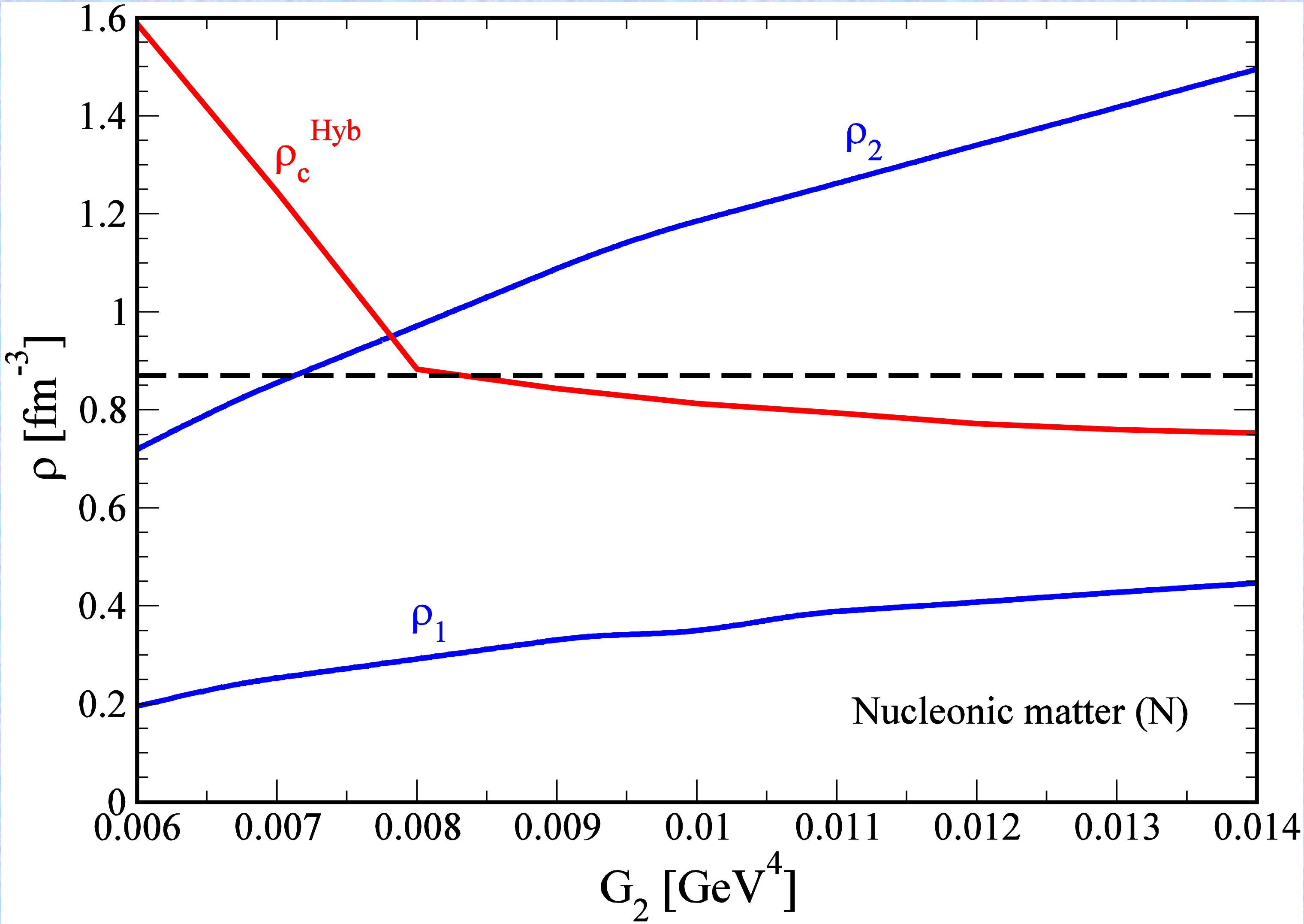
Equation of State



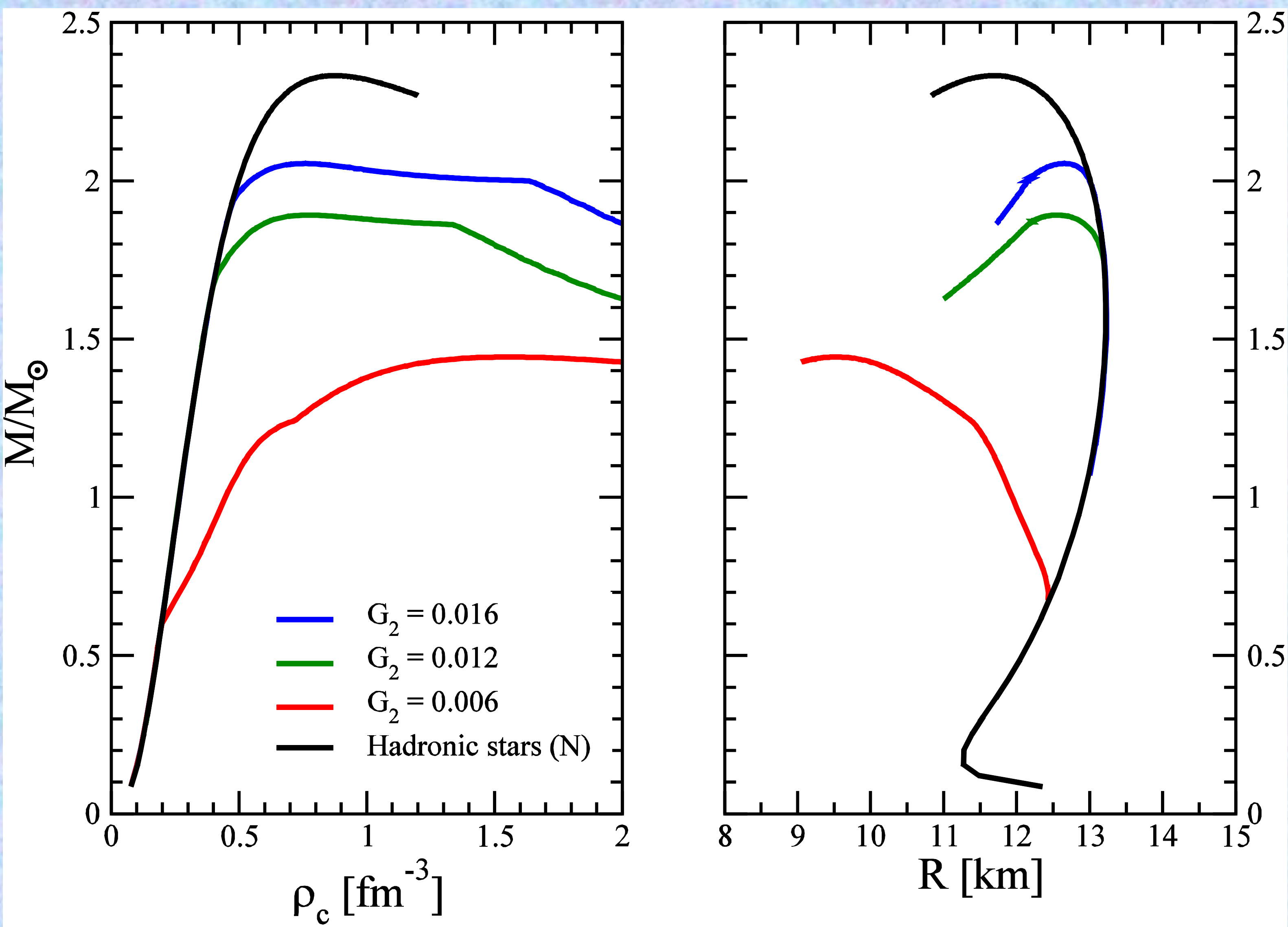
Hyperonic matter (NY) $x_\sigma = 0.6$

$V_1 = 0.01$ GeV

Quark-Hadron phase transition boundaries in β-stable matter

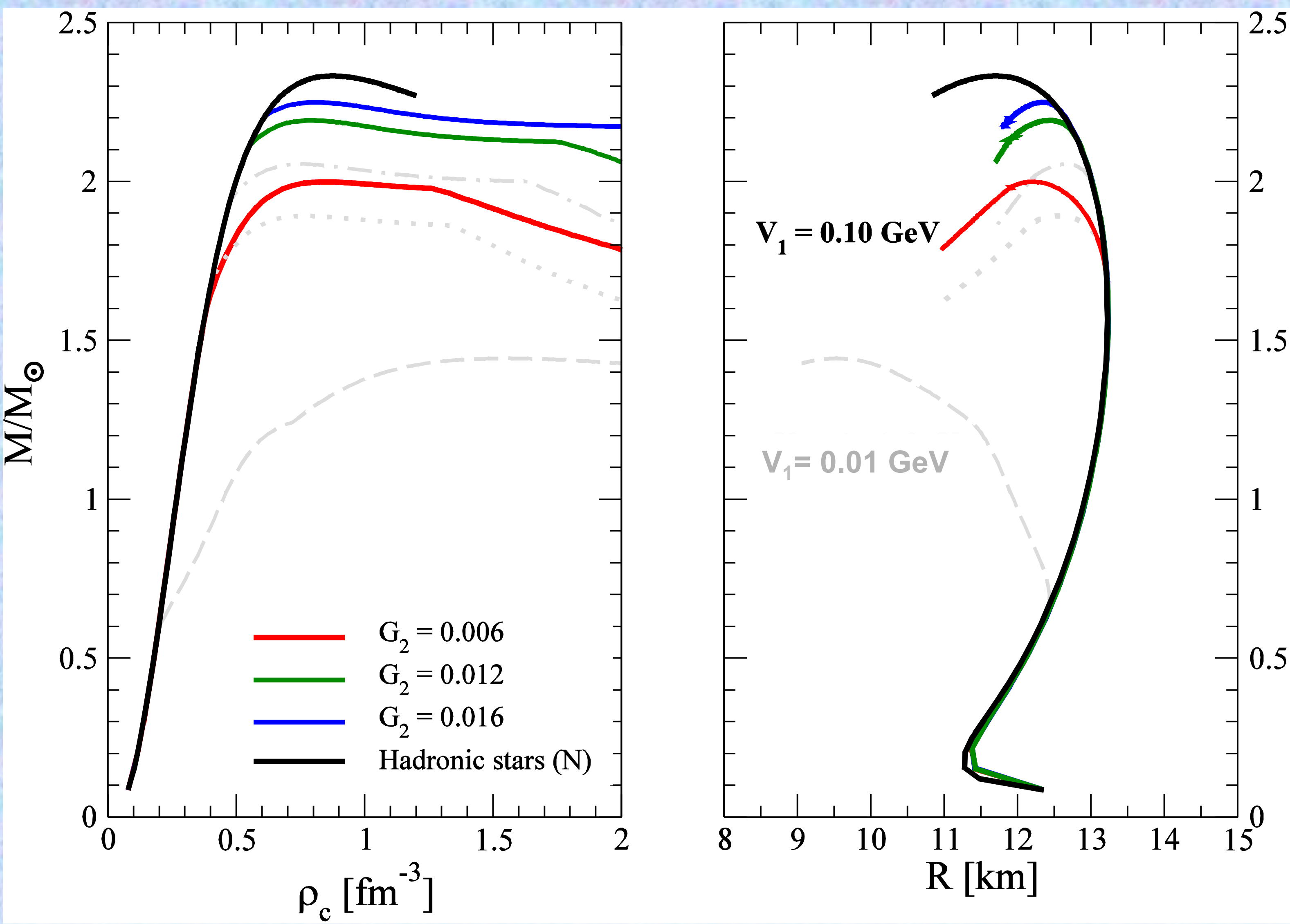


Pure nucleonic matter (N) **V_1= 0.01 GeV**



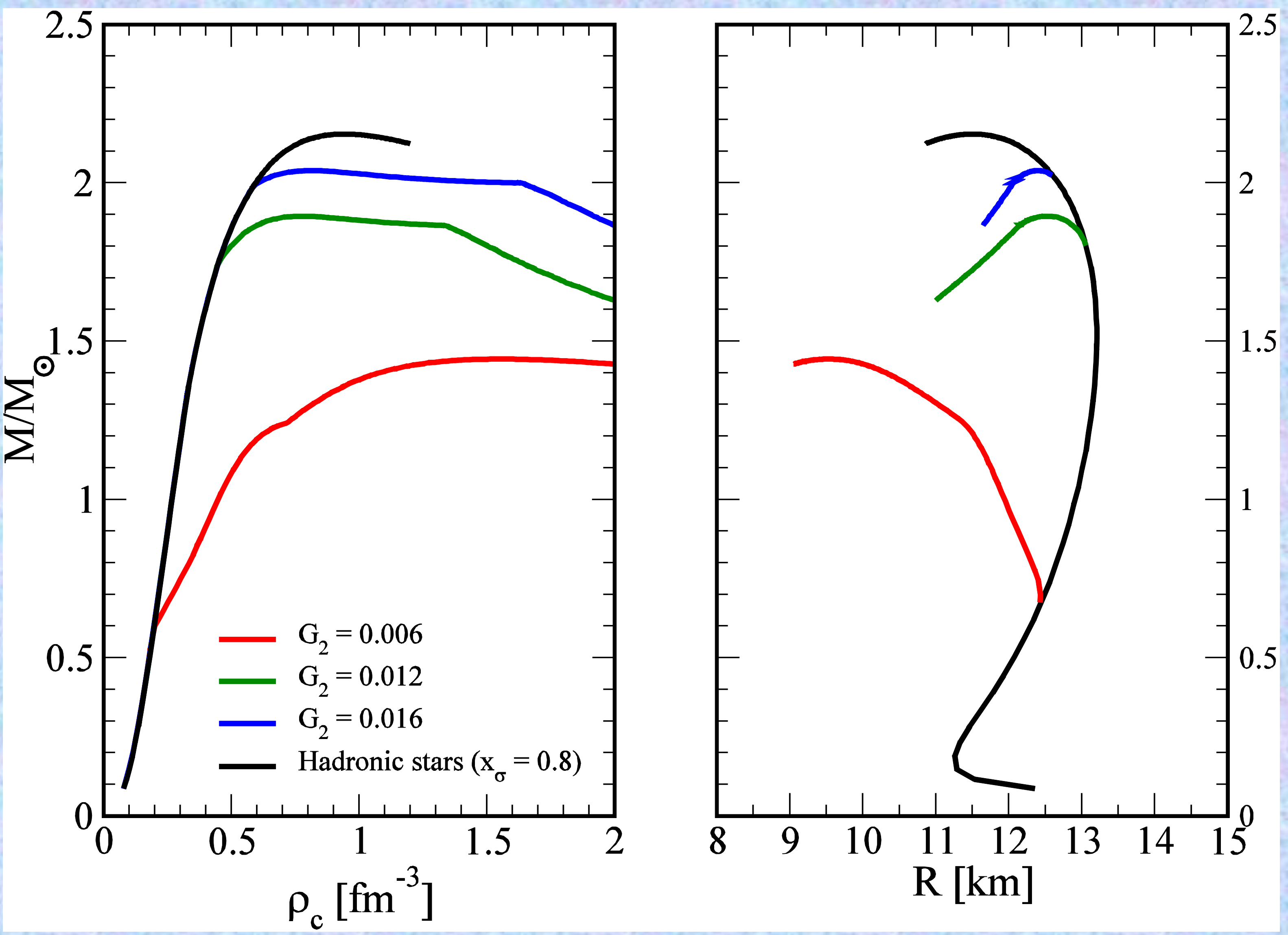
Pure nucleonic matter (N)

V_1= 0.01 GeV



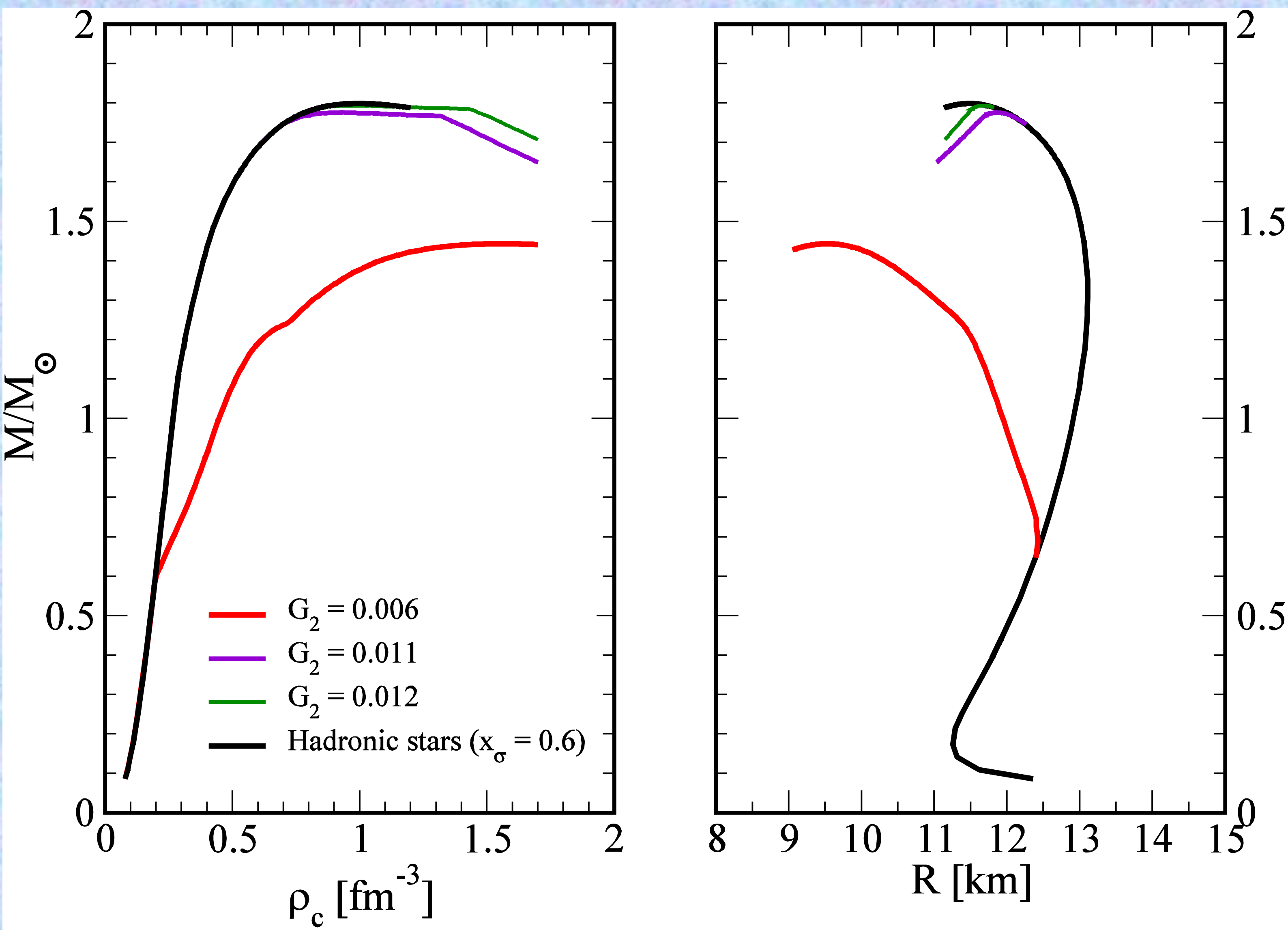
Pure nucleonic matter (N)

V_1= 0.10 GeV



Hyperonic matter (NY) x_σ = 0.8

V_1= 0.01 GeV



Hyperonic matter (NY) x_σ = 0.6

V_1= 0.01 GeV

Properties of the stellar maximum mass configuration

(pure nucleonic matter)

G_2	M_{max}	ρ_c^{hyb}	R	M^{HS}_{max}	ρ_c^{HS}	R^{HS}
0.006	1.44	1.55	9.54			
0.012	1.89	0.77	12.55	2.33	0.87	11.70
0.016	2.05	0.75	12.66			

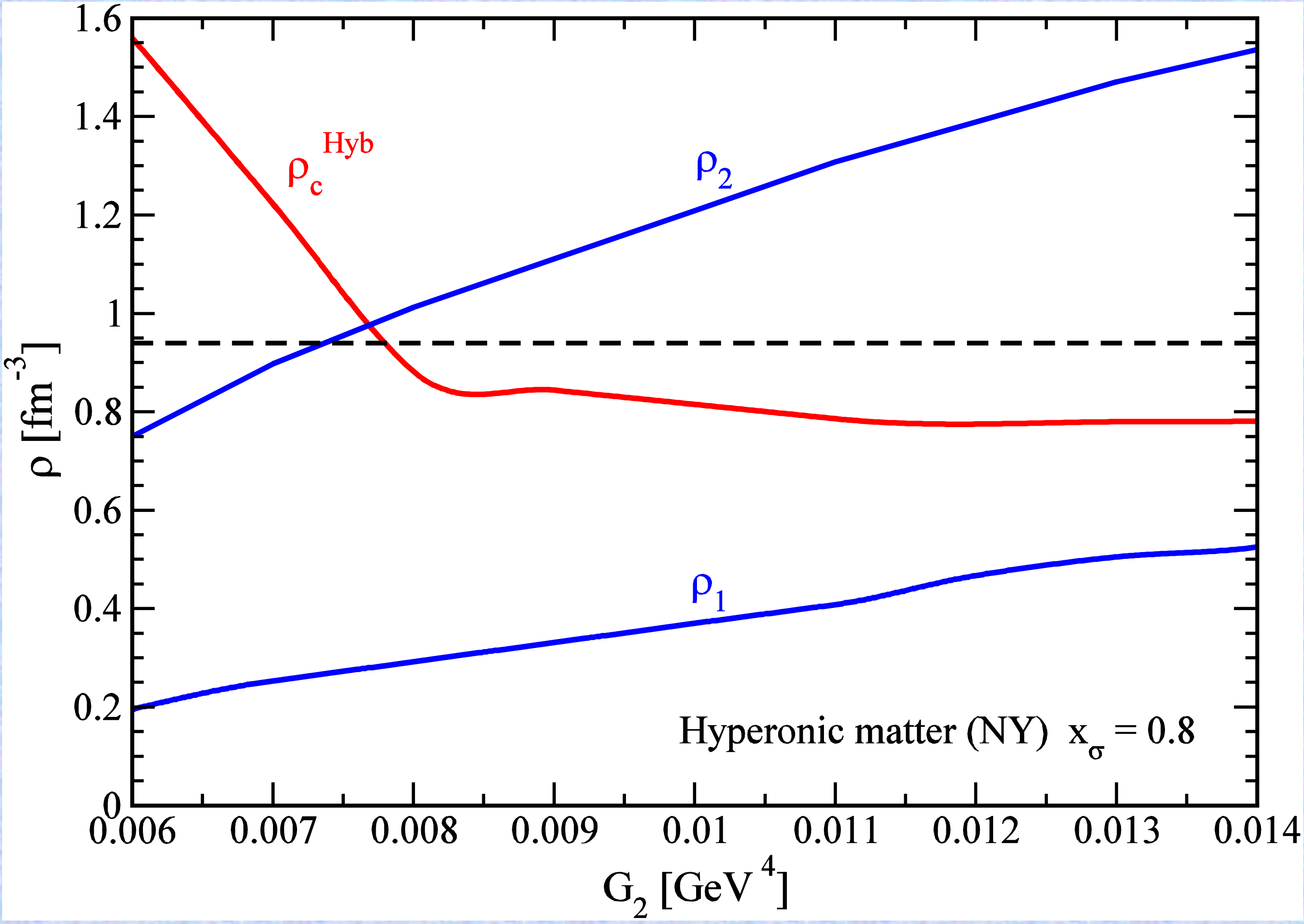
$[G_2] = \mathrm{GeV}^4$ $[R] = \mathrm{km}$ $[M_{max}] = M_{\odot}$ $[\rho_c^{hyb}, \rho_c^{HS}] = \mathrm{fm}^{-3}$

Properties of the stellar maximum mass configuration

x_σ	G_2	M_{max}	ρ_c^{hyb}	R	M^{HS}_{max}	ρ_c^{HS}	R^{HS}
	0.006	1.44	1.55	9.54			
N	0.012	1.89	0.77	12.55	2.33	0.87	11.70
	0.016	2.05	0.75	12.66			
	0.006	1.44	1.56	9.52			
0.8	0.012	1.89	0.77	12.53	2.15	0.94	11.50
	0.016	2.04	0.81	12.40			
	0.006	1.43	1.56	9.51			
0.6	0.010	1.73	0.89	12.00	1.80	1.00	11.49
	0.013	1.80	1.00	11.49			

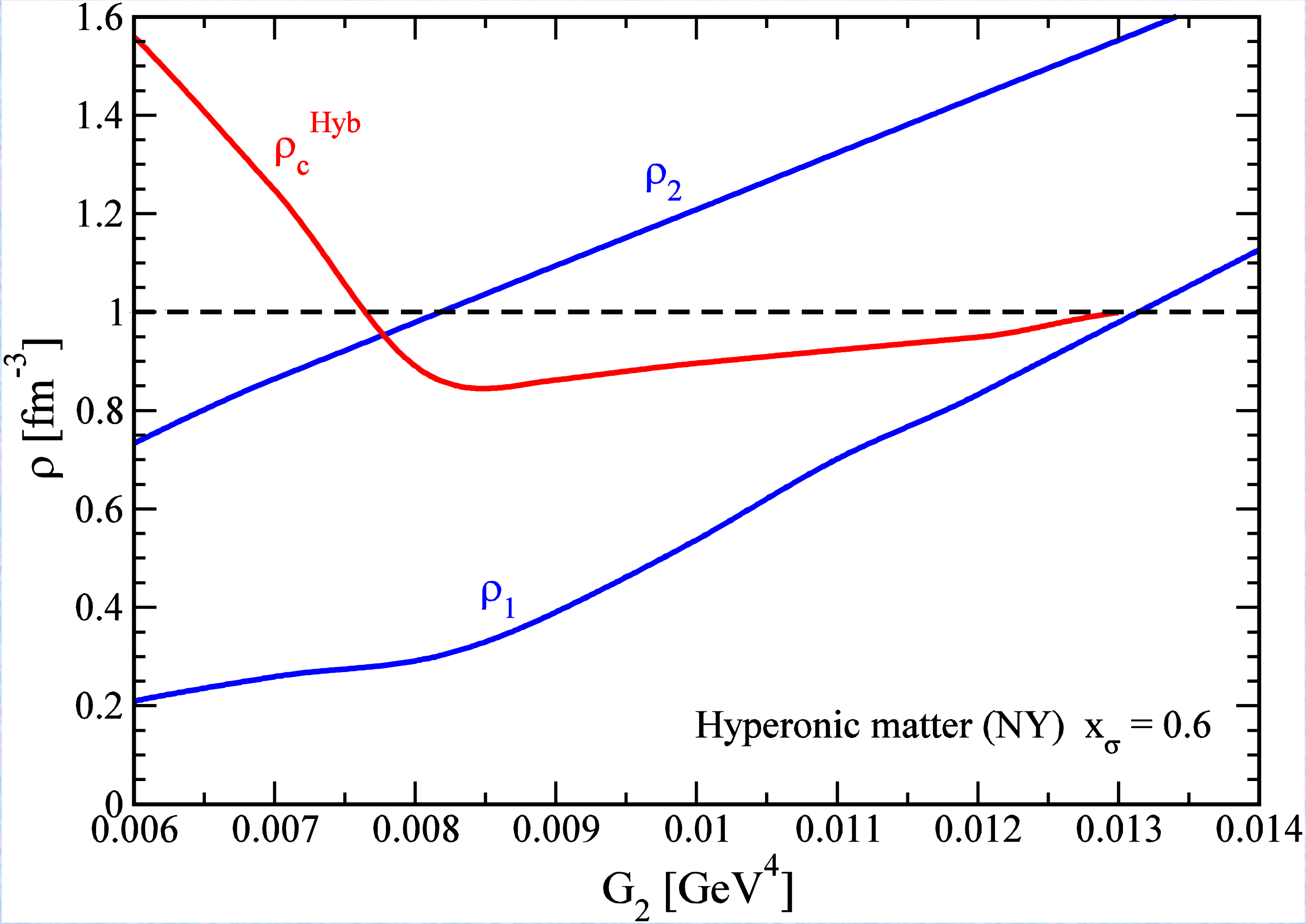
$[G_2]$ = GeV^4 $[R]$ = km $[M_{max}] = M_\odot$ $[\rho_c^{hyb}, \rho_c^{HS}]$ = fm^{-3}

Quark-Hadron phase transition boundaries in β-stable matter



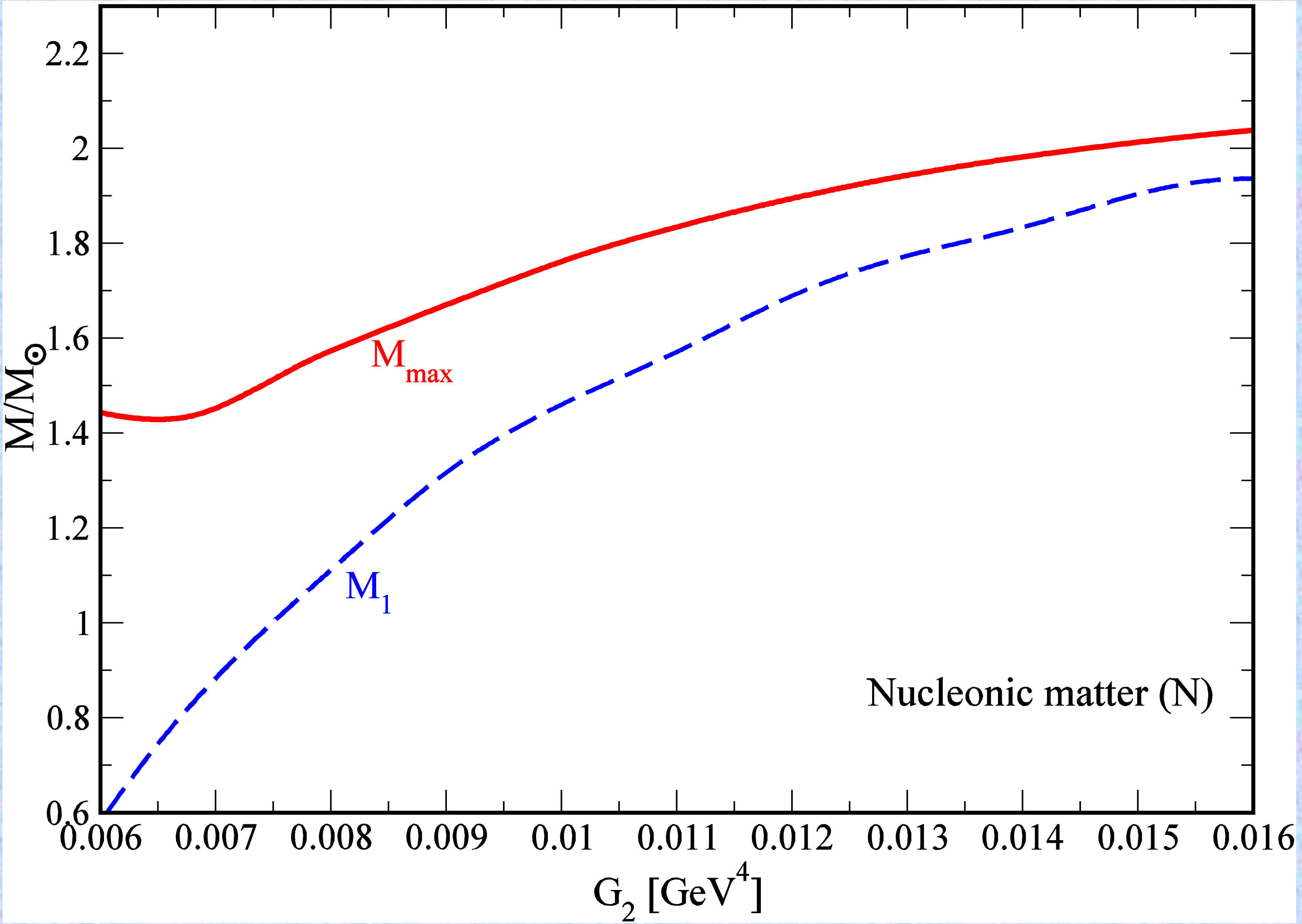
Hyperonic matter (NY) x_σ = 0.8 **V_1= 0.01 GeV**

Quark-Hadon phase transition boundaries in β-stable matter



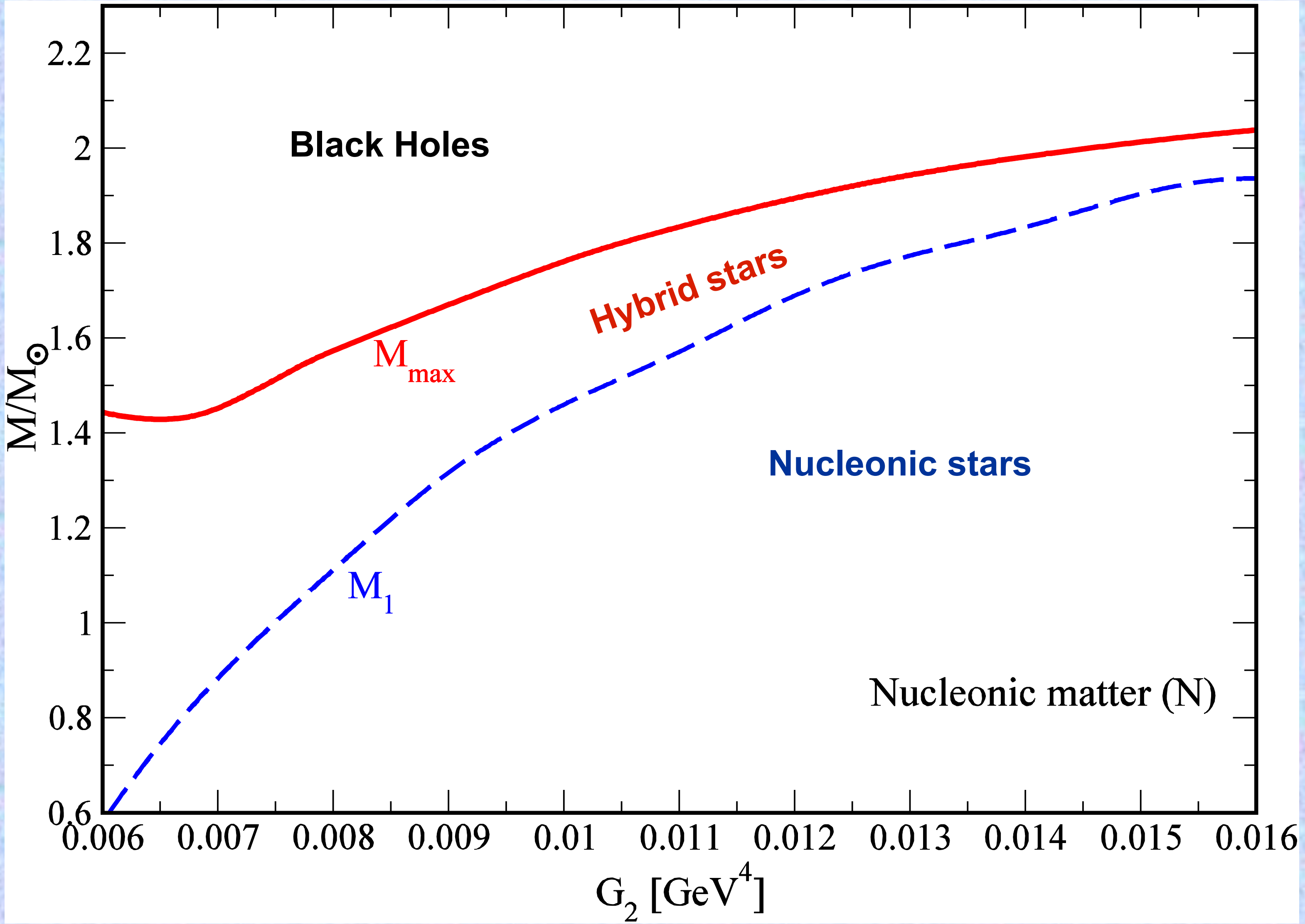
Hyperonic matter (NY) x_σ = 0.6

V_1= 0.01 GeV



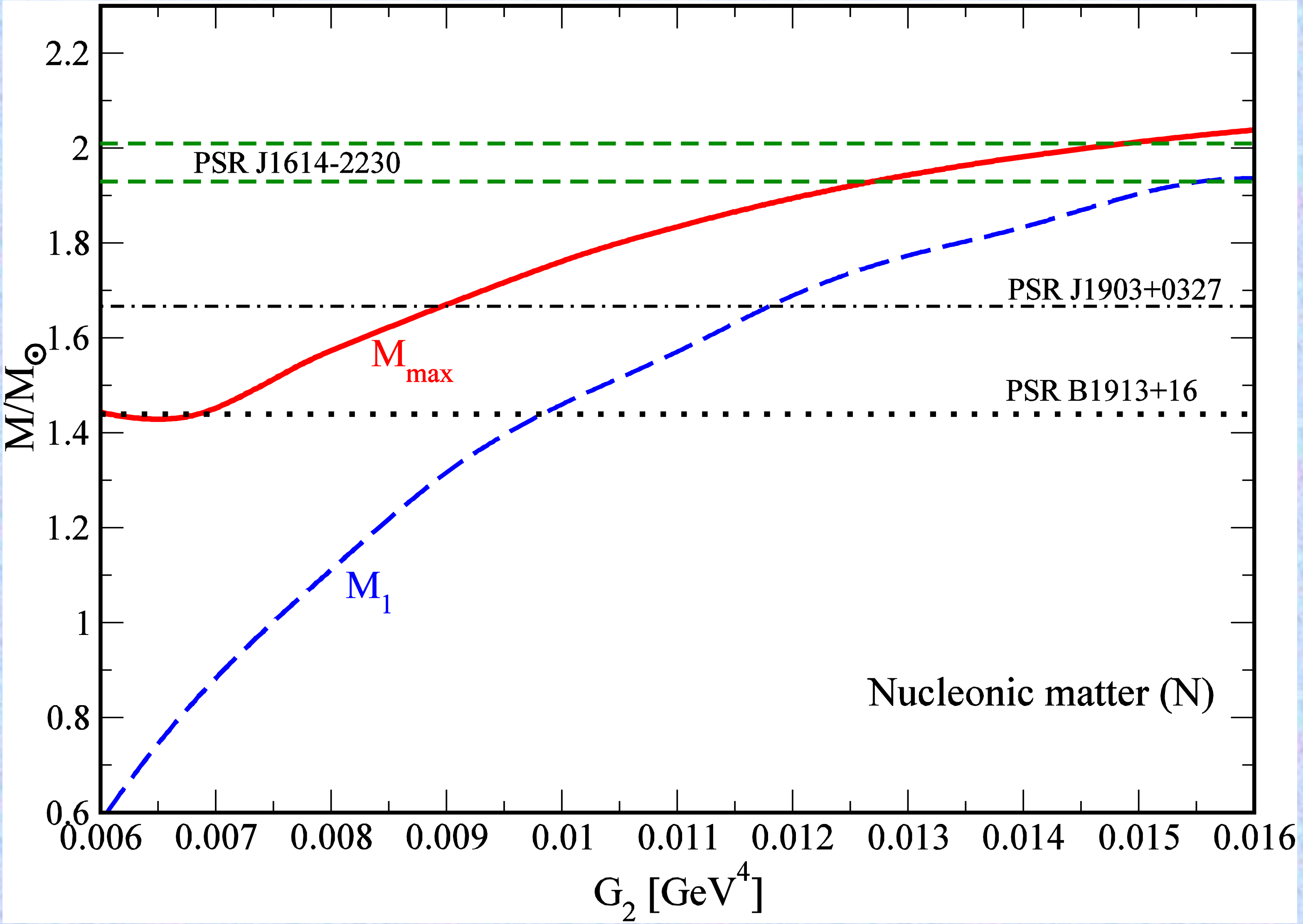
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
M/M⊙
0.006
0.007
0.008
0.009
0.01
0.011
0.012
0.013
0.014
0.015
0.016
G2 [GeV4]
Mmax
M1
Nucleonic matter (N)

V_1= 0.01 GeV



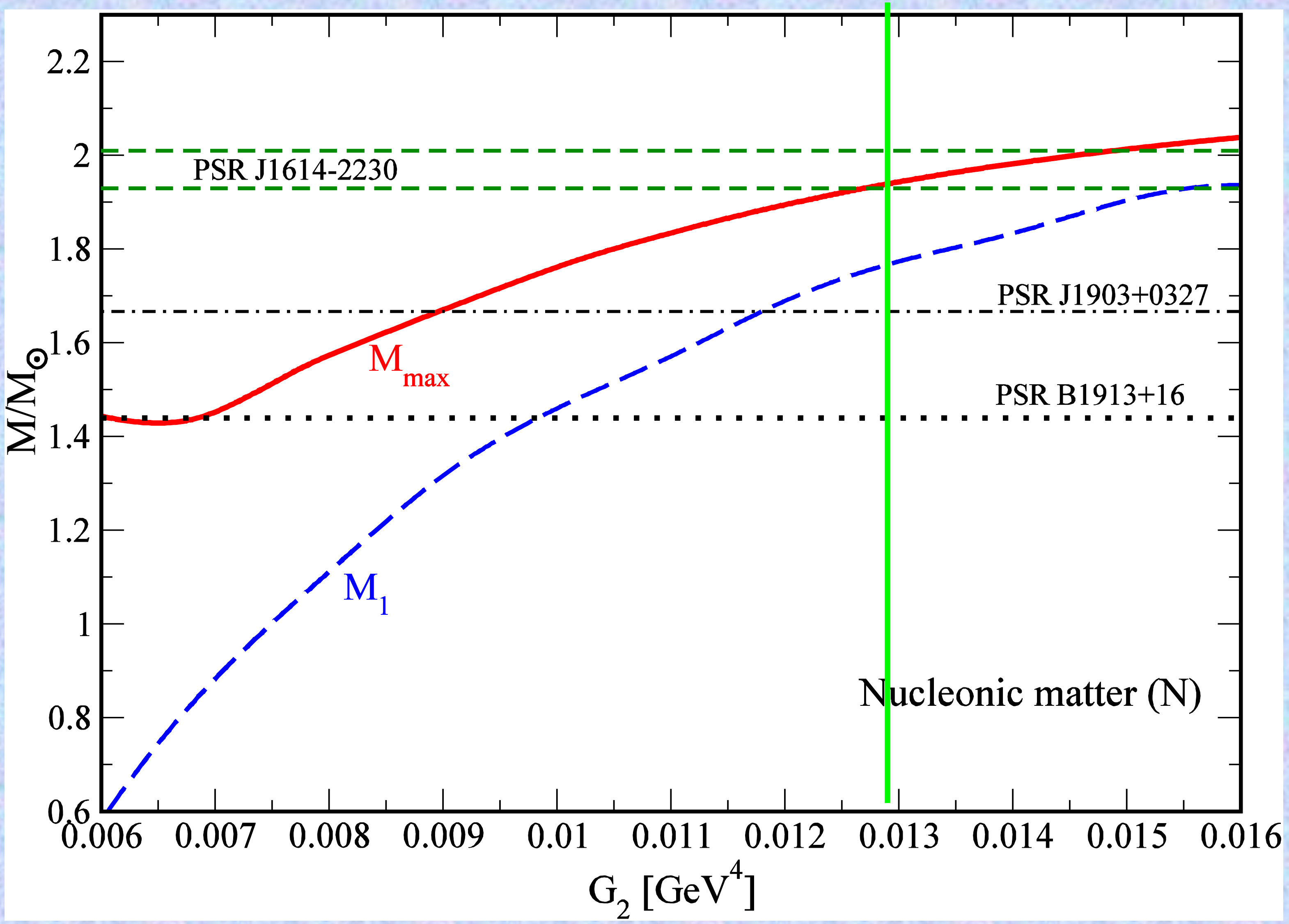
Black Holes
Hybrid stars
M_{max}
Nucleonic stars
M_1
Nucleonic matter (N)
$M/M_\odot$
G_2 [GeV^4]
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.006
0.007
0.008
0.009
0.01
0.011
0.012
0.013
0.014
0.015
0.016

V_1 = 0.01 GeV



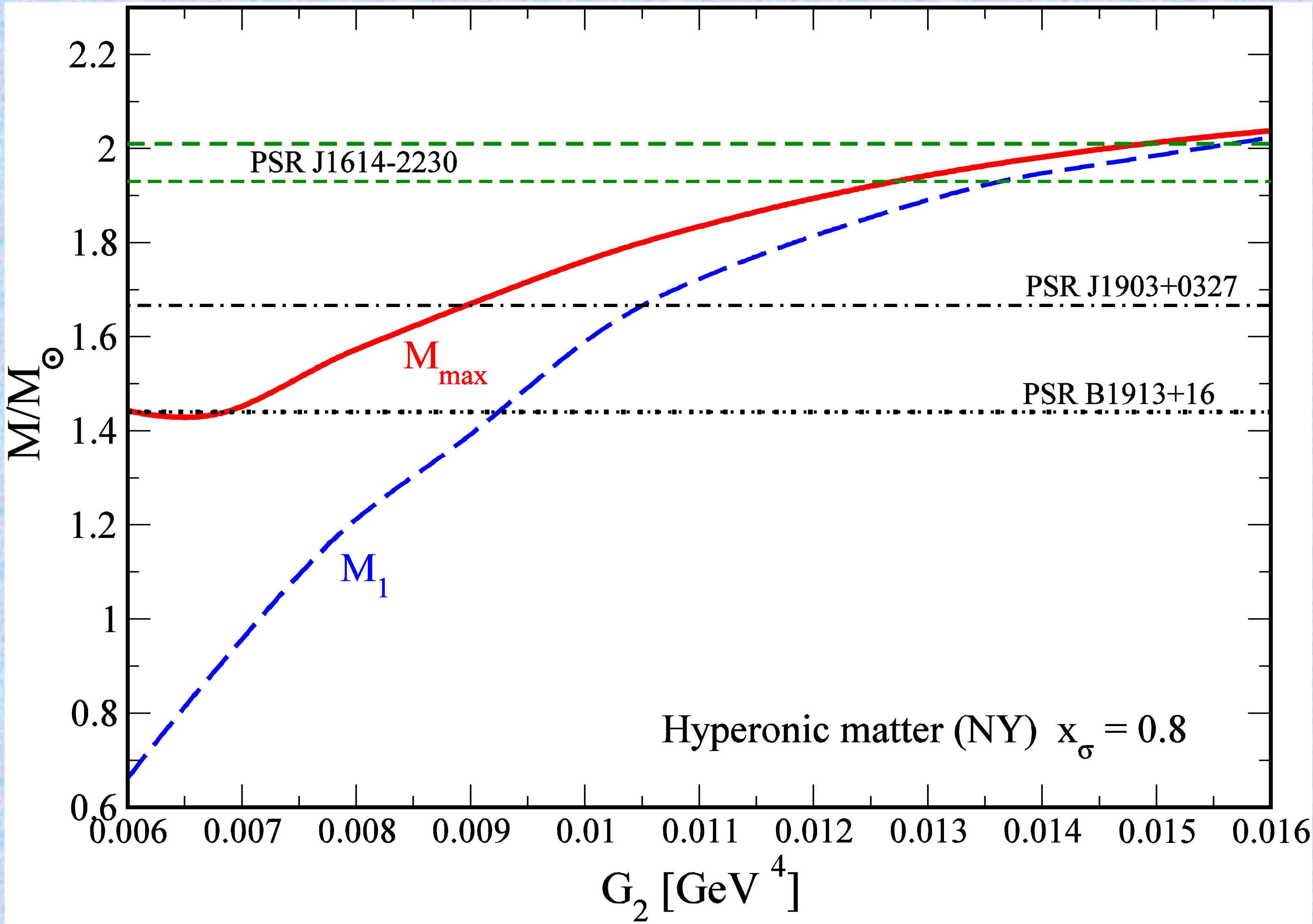
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.006
0.007
0.008
0.009
0.01
0.011
0.012
0.013
0.014
0.015
0.016
G_2 [GeV^4]
$M/M_\odot$
PSR J1614-2230
PSR J1903+0327
PSR B1913+16
M_{max}
M_1
Nucleonic matter (N)

V_1= 0.01 GeV

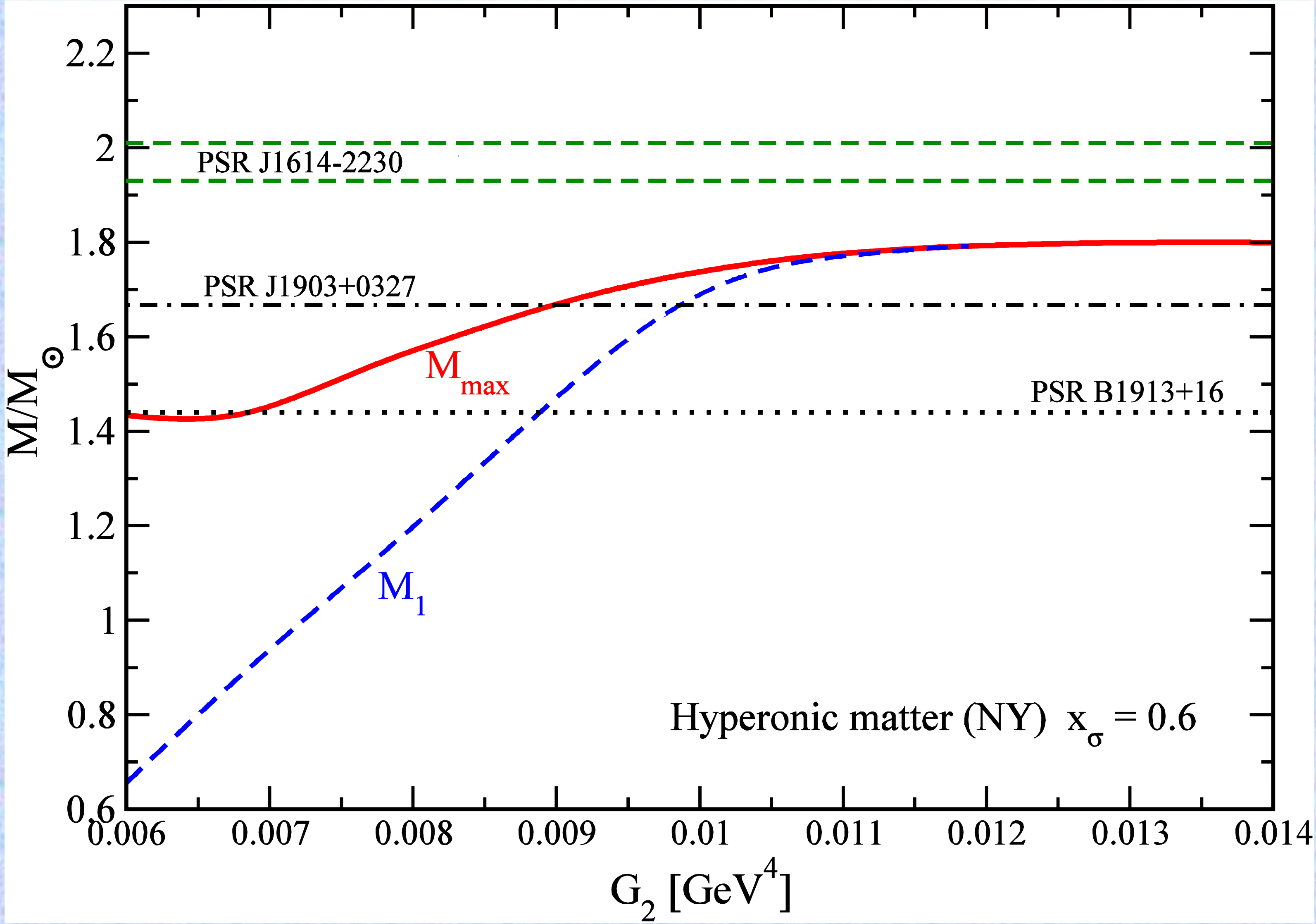


0.0129

V_1= 0.01 GeV

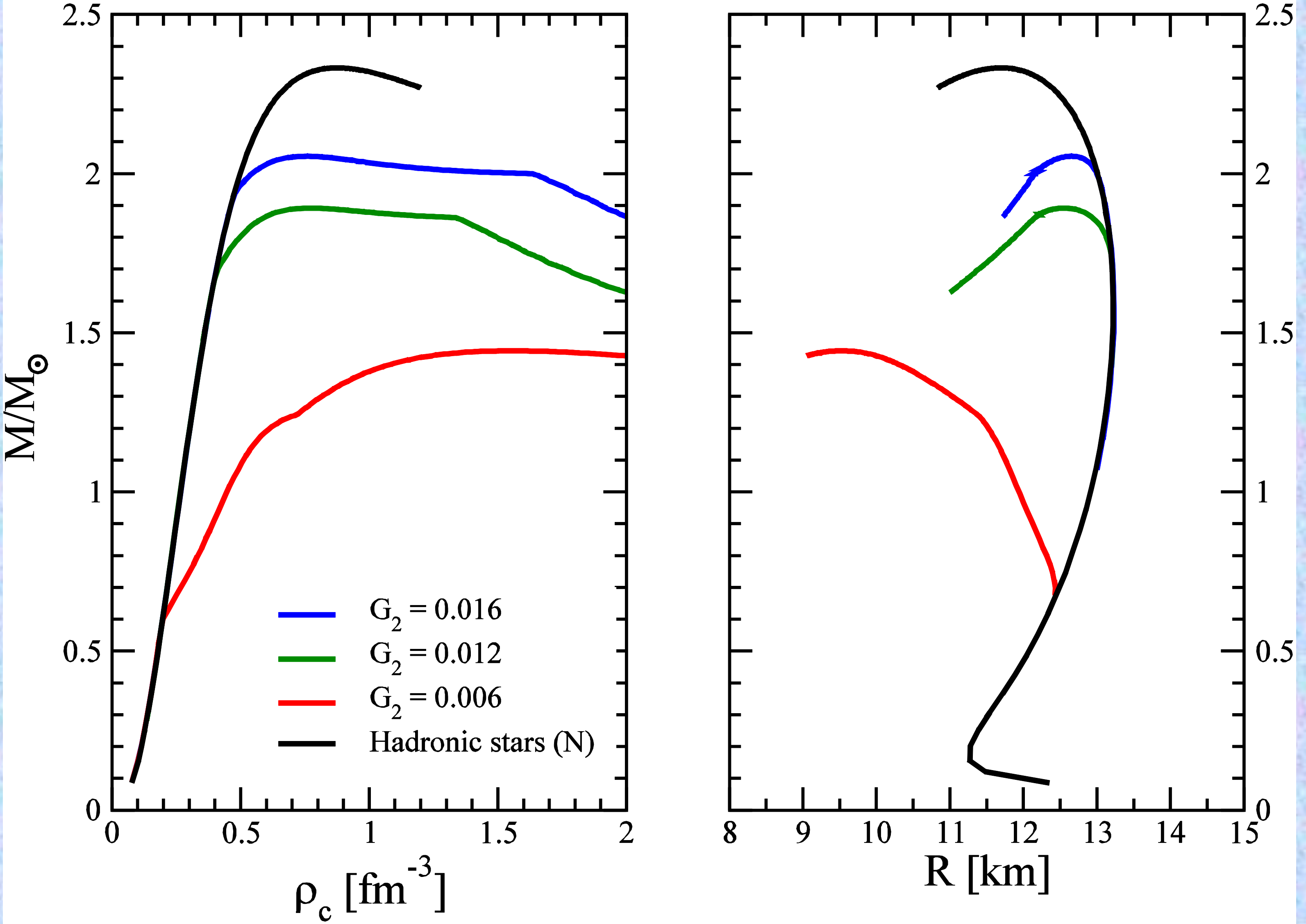


$G_2 \geq 0.0127$ $V_1 = 0.01$ GeV


2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.006
0.007
0.008
0.009
0.01
0.011
0.012
0.013
0.014
PSR J1614-2230
PSR J1903+0327
PSR B1913+16
M_{max}
M_1
Hyperonic matter (NY) $x_\sigma = 0.6$
$M/M_\odot$
G_2 [GeV^4]

V_1= 0.01 GeV

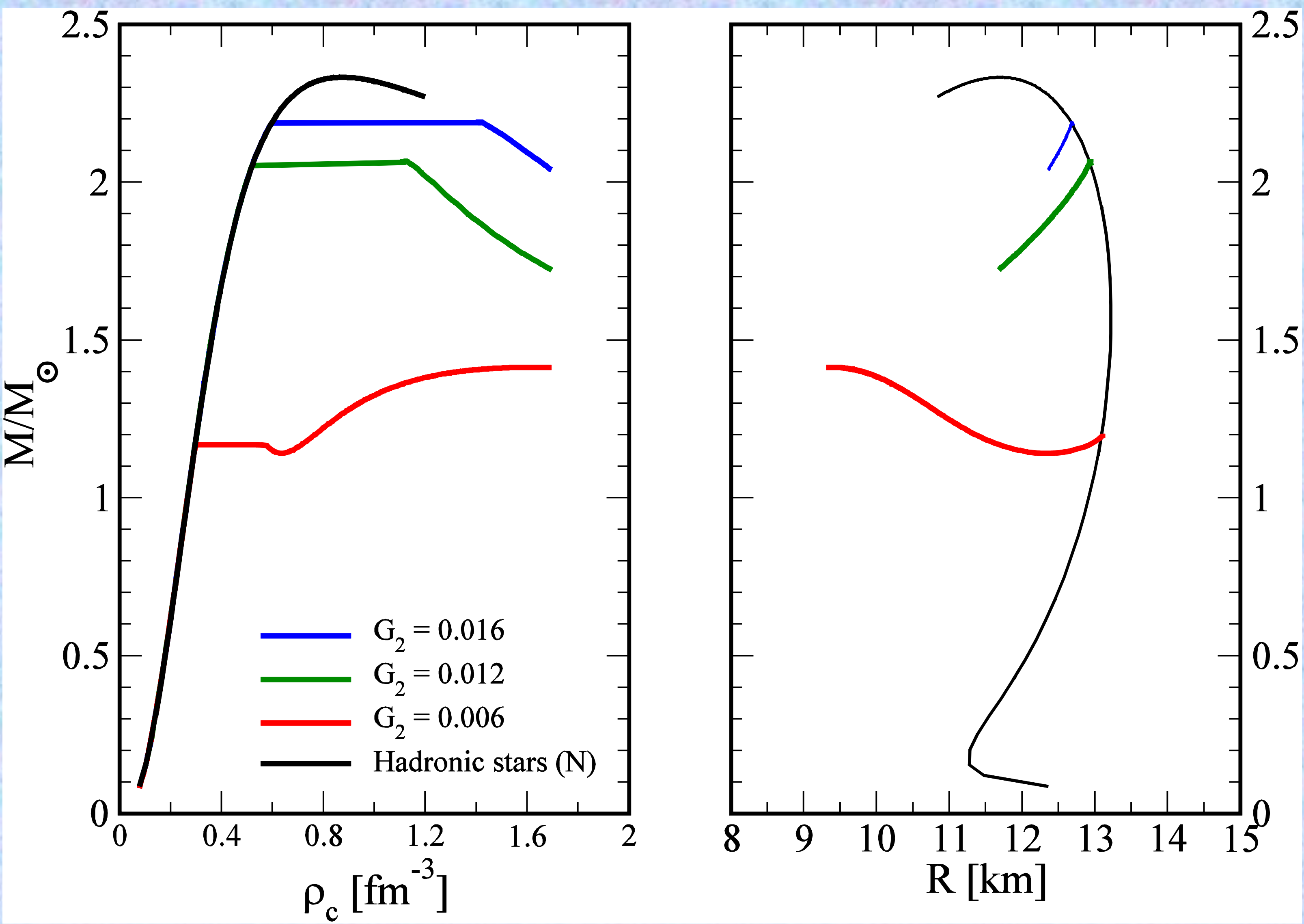
Gibbs construction



Pure nucleonic matter (N)

V_1= 0.01 GeV

Maxwell construction



Pure nucleonic matter (N)

V_1= 0.01 GeV

Deconfinement transition temperature at $\mu_b = 0$ in the FCM

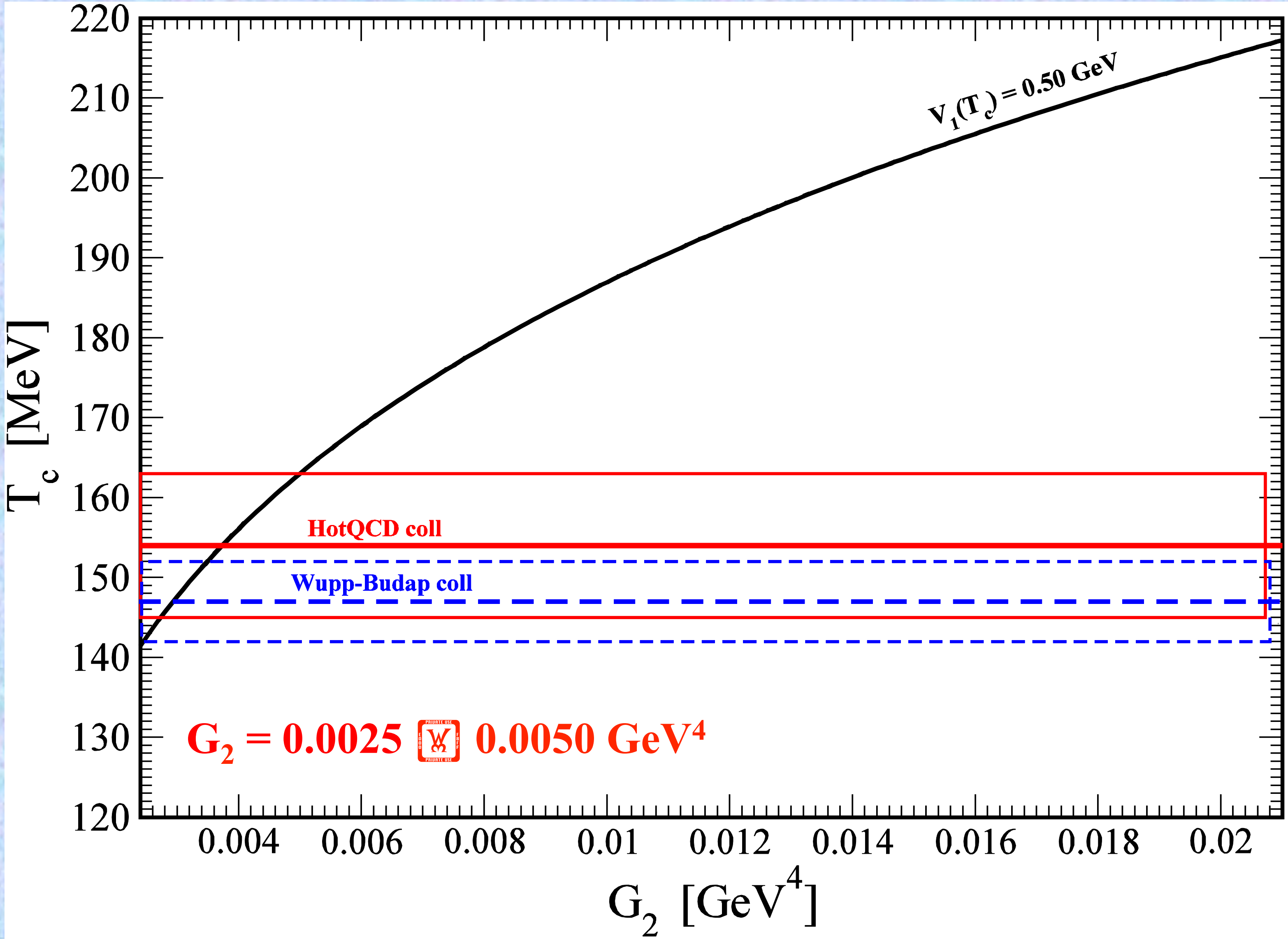
$$T_c = \frac{a_0}{2} G_2^{1/4} \left(1 + \sqrt{1 + \frac{V_1(T_c)}{2 a_0 G_2^{1/4}}} \right)$$

$$N_f = 3 \quad \Rightarrow \quad a_0 = \left(\frac{3\pi^2}{768} \right)^{1/4}$$

$$V_1(T_c) = 0.5 \quad \text{GeV}$$

Yu. Simonov, M.A. Trusov, JEPT Lett. 85, 598 (2007)
Yu. Simonov, M.A. Trusov, Phys.Lett. B 650, 36 (2007)

Deconfinement transition temperature at $\mu_b = 0$ in the FCM



$$V_1(T) = \int_0^{1/T} d\tau \left(1 - \tau T\right) \int_0^{\infty} d\xi \; \xi \; D_1^E\left(\sqrt{\xi^2 + \tau^2}\right)$$

$$D_1^E(x) = D_1^E(0) \exp\left(-\left|x\right|/\lambda\right)$$

Assume $D_1^E(0)$ **does not depend on T**

supported by lattice QCD calcul.s **D’Elia et al., Phys.Rev: D 67, 114504 (2003)**

$$V_1(T) = \int_0^{1/T} d\tau \left(1 - \tau T\right) \int_0^{\infty} d\xi \; \xi \; D_1^E\left(\sqrt{\xi^2 + \tau^2}\right)$$

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Assume $D_1^E(0)$ **does not depend on T**

supported by lattice QCD calcul.s **D'Elia et al., Phys.Rev: D 67, 114504 (2003)**

$$V_1(T) = V_1(0) \left\{1 - \frac{3}{2}\frac{\lambda T}{\hbar c} + \frac{1}{2}\left(1 + 3\frac{\lambda T}{\hbar c}\right) \exp\left(-\frac{\hbar c}{\lambda T}\right)\right\}$$

$$V_1(T) = 0.5\ \text{GeV} \Rightarrow \text{V}_1(0) = 0.85\ \text{GeV}$$

$$\rho_1 > \rho_c^{HS}$$

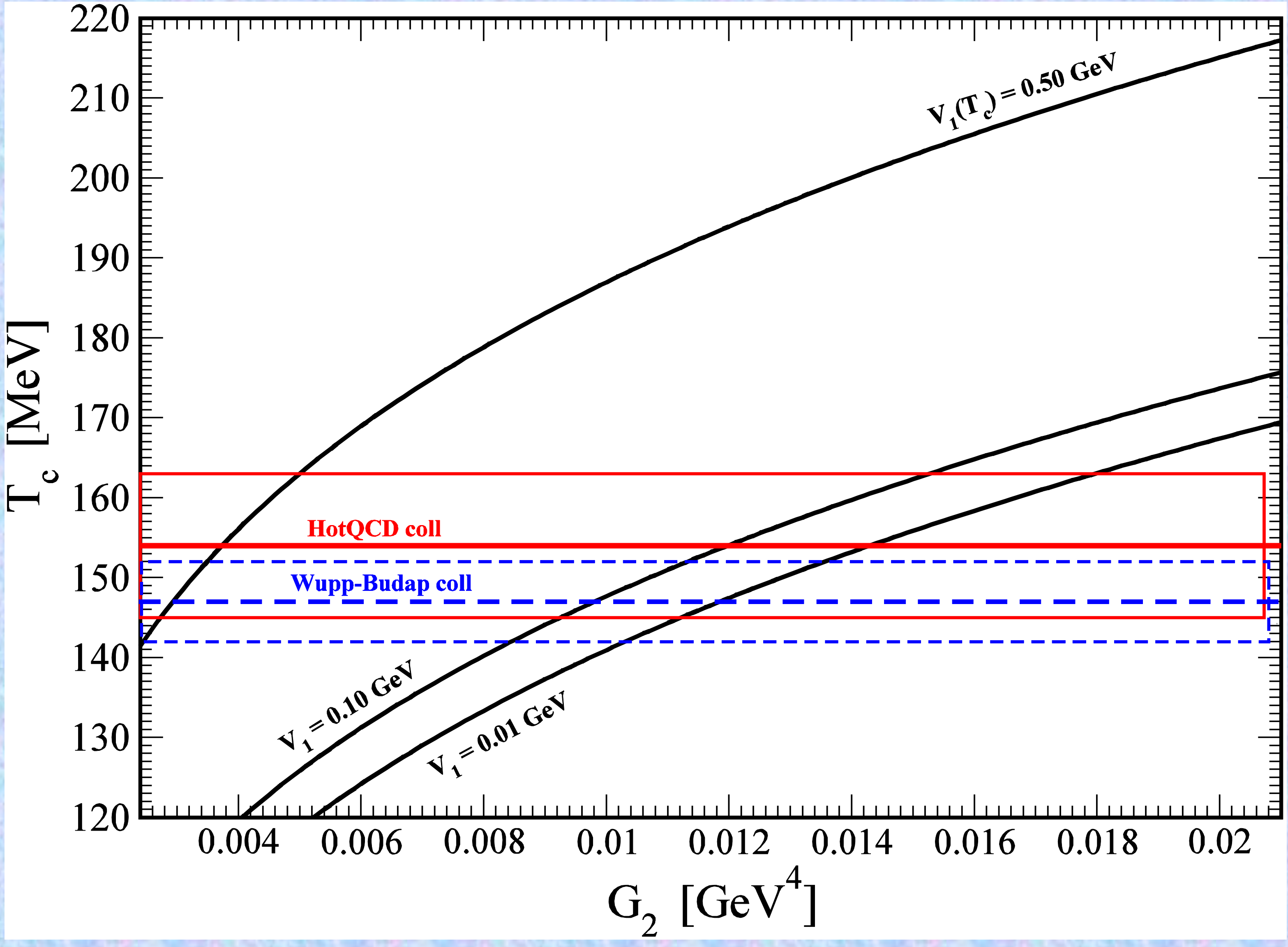
No deconfinement transition in pure Hadronic Stars

Deconfinement transition temperature at $\mu_b = 0$ in the FCM

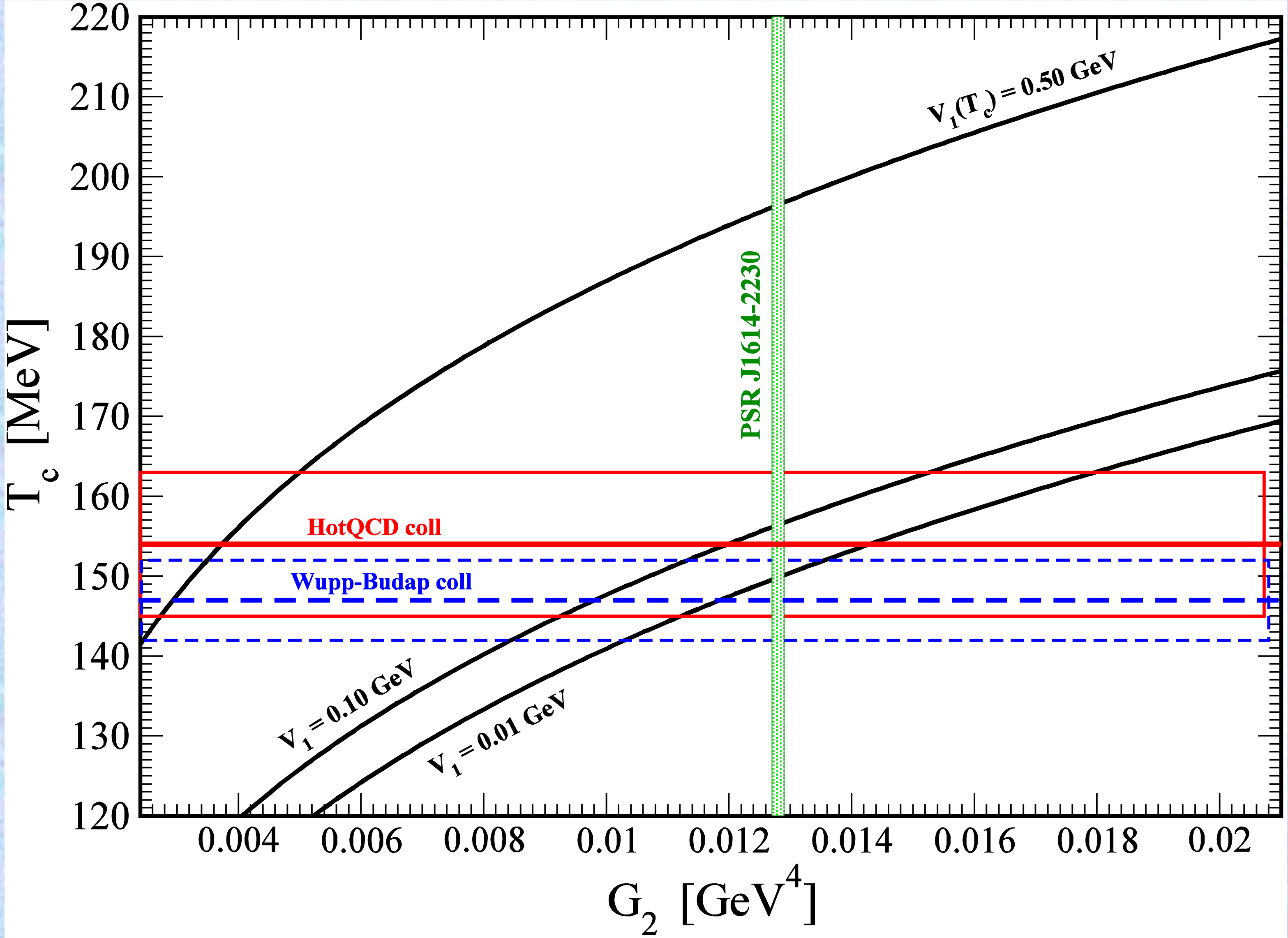
$$T_c = \frac{a_0}{2} G_2^{1/4} \left(1 + \sqrt{1 + \frac{V_1(T_c)}{2 a_0 G_2^{1/4}}} \right)$$

$$V_1(T) = V_1(0) \left\{ 1 - \frac{3}{2} \frac{\lambda T}{\hbar c} + \frac{1}{2} \left(1 + 3 \frac{\lambda T}{\hbar c} \right) \exp\left(- \frac{\hbar c}{\lambda T} \right) \right\}$$

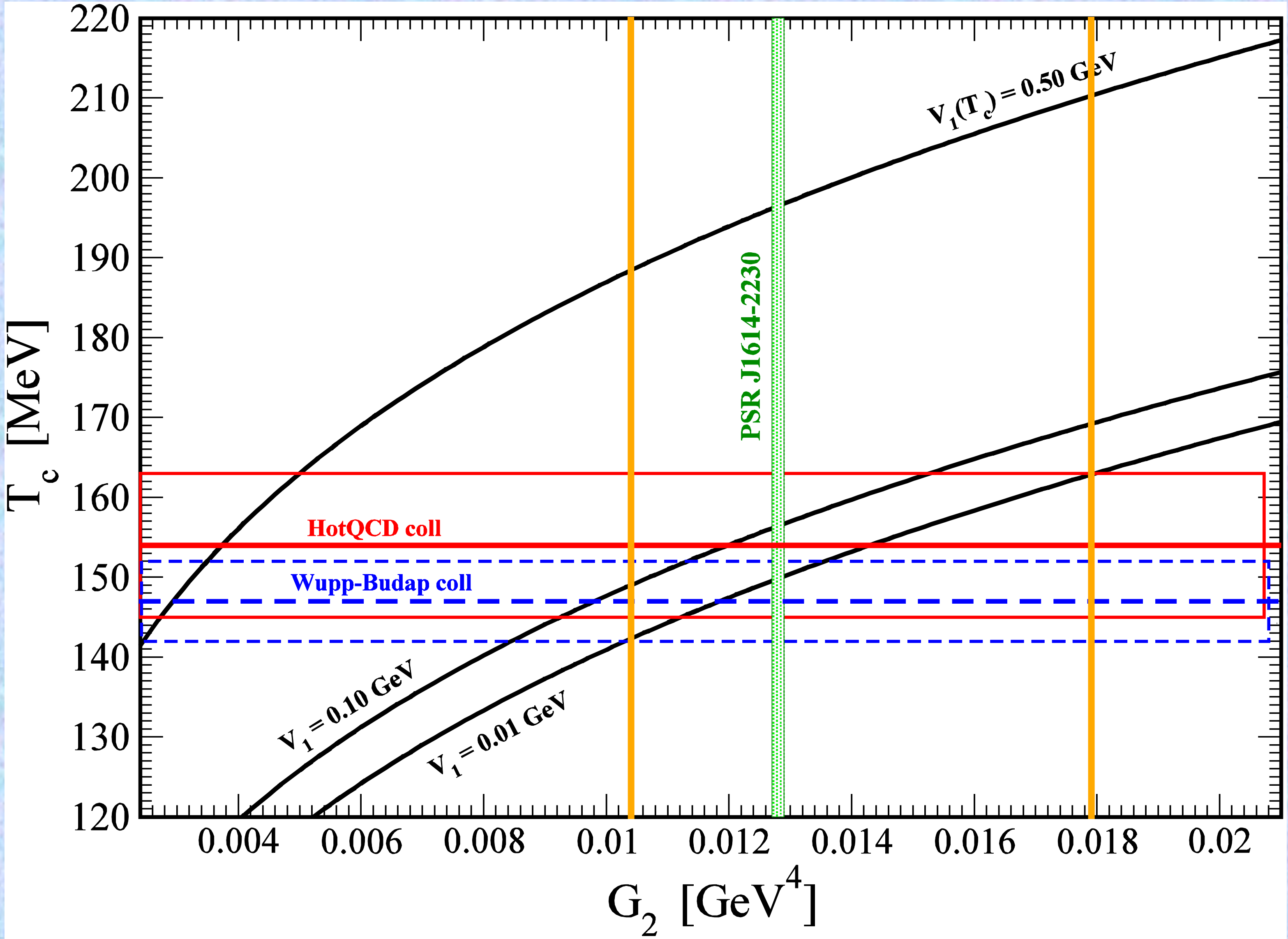
Deconfinement transition temperature at $\mu_b = 0$ in the FCM



Deconfinement transition temperature at $\mu_b = 0$ in the FCM



Deconfinement transition temperature at $\mu_b = 0$ in the FCM



Conclusions

We have established that the values of the gluon condensate extracted within the FCM from lattice QCD calculations of the deconfinement transition temperature, at $\mu_b = 0$, are fully consistent with the value the same quantity derived by the mass measurement of PSR J1614 -2230

The FCM provides a powerful tool to link numerical calculations of QCD on a space-time lattice with Neutron Star physics

$$\sigma^E = \frac{1}{2}\int D^E(x)\ d^2x$$

$$P_q \Big/ T^4 = \frac{1}{\pi^2}\left[\phi_\nu\left(\frac{\mu_q - V_1/2}{T}\right) + \phi_\nu\left(-\frac{\mu_q - V_1/2}{T}\right)\right]$$

$$\phi_\nu(a) = \int_0^\infty du \frac{u^4}{\sqrt{u^2+\nu^2}} \frac{1}{\exp(\sqrt{u^2+\nu^2} - a) + 1}$$

$$V_1(T) = \int_0^{1/T} d\tau \left(1 - \tau T\right) \int_0^\infty d\xi \ \xi \ D_1^E\left(\sqrt{\xi^2 + \tau^2}\right)$$

$$D_1^E(x) = D_1^E(0) \exp\left(-|x|/\lambda\right)$$

$$P_g \Big/ T^4 = \frac{8}{3\pi^2} \int_0^\infty d\chi \ \chi^3 \left[\exp\left(\chi + \frac{9V_1}{8T}\right) - 1\right]^{-1}$$

$$P_{QP} = P_g + \sum_{u,d,s} P_q + \Delta\varepsilon_{vac}$$

$$\Delta\varepsilon_{vac} \approx -\frac{11 - \frac{2}{3}N_f}{32}\frac{G_2}{2}$$

$$N_f = 3 \quad \Rightarrow \quad \Delta\varepsilon_{vac} = -\frac{9}{64}\ G_2$$

Deconfinement transition temperature

$$T_c = \frac{a_0}{2} G_2^{1/4} \left(1 + \sqrt{1 + \frac{V_1(T_c)}{2 a_0 G_2^{1/4}}} \right)$$

$$N_f = 3 \quad \Rightarrow \quad a_0 = \left(\frac{3\pi^2}{768} \right)^{1/4}$$

$$V_1(T) = V_1(0) \left\{ 1 - \frac{3}{2}\lambda T + \frac{1}{2}\left(1 + 3\lambda T\right) \exp\left(- \frac{1}{\lambda T} \right) \right\}$$

$$\Delta^{(2)}_{\mu_1\nu_1\mu_2\nu_2} = \frac{1}{N_c} Tr\left\langle F_{\mu_1\nu_1}(x)\Phi(x,y)F_{\mu_2\nu_2}(y)\Phi(y,x)\right\rangle$$

$$\Phi(x,y) = P \quad \exp\left[i\int_x^y A_\mu dz_\mu\right]$$

$$\frac{1}{N_c} Tr\left\langle E_i(x)\Phi(x,y)E_j(y)\Phi(y,x)\right\rangle =$$

$$\delta_{ij}\left[D^E(z) + D_1^E(z) + z_4^2 \frac{dD_1^E(z)}{dz^2}\right] + z_i z_j \frac{dD_1^E(z)}{dz^2}$$

$$z = x - y$$

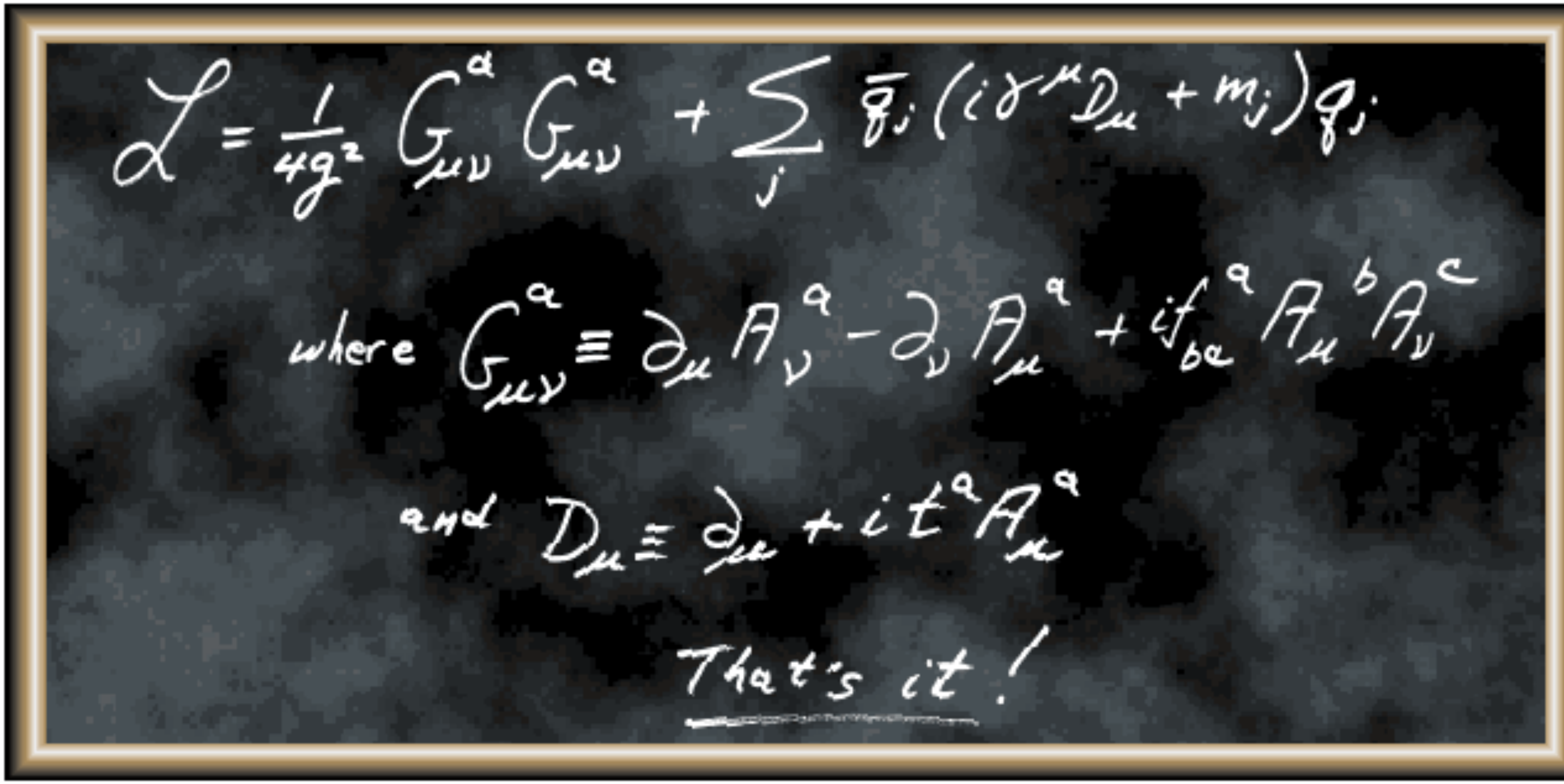


FIGURE 1. THE QCD LAGRANGIAN $\mathcal{L}$ displayed here is, in principle, a complete description of the strong interaction. But, in practice, it leads to equations that are notoriously hard to solve. Here m_j and q_j are the mass and quantum field of the quark of jth flavor, and A is the gluon field, with spacetime indices μ and ν and color indices a, b, c. The numerical coefficients f and t guarantee SU(3) color symmetry. Aside from the quark masses, the one coupling constant g is the only free parameter of the theory.