5.0 Models of neutron star matter

We have seen that, while the calculation of the equation of state (EOS) at $\rho < \rho_0$ implies little ambiguity, the treatment of the region of supernuclear density necesarily involves assumptions. On the other hand, as over 90 % of the neutron star mass is at $\rho \geq \rho_0$, this region plays a crucial role in determining the static properties of the star.

We will now briefly describe different models of neutron star matter at supernuclear density, developed using the theoretical approaches described in the previous lectures.

5.1 Pure neutron matter, nonrelativistic (model A, 1971)

Neutron star matter at $\rho \geq \rho_0$ is described as a uniform neutron fluid, whose dynamics is dictated by the nonrelativistic hamiltonian

$$H = \sum_i \frac{\mathbf{p}^2}{2m} + \sum_{j>i} v_{ij} \ . \tag{1}$$

The long range behavior of v_{ij} is obtained from Yukawa's theory of pion exchange interactions, while the intermediate and short range components result from a 1968 fit to nucleon-nucleon (NN) data.

For any matter density ρ the ground state energy per particle

$$\frac{E_0}{N} = \frac{1}{N}\langle H \rangle \tag{2}$$

is calculated using a variational wave function. Energy density and pressure are then obtained from E_0/N according to

$$\epsilon(\rho) = \left(\frac{E_0}{N} + m\right) \tag{3}$$

$$P(\rho) = \rho^2 \frac{\partial}{\partial \rho}\frac{E_0}{N} \ , \tag{4}$$

and the EOS $P = P(\epsilon)$ can finally be written eliminating ρ from eqs.(3) and (4).

5.2 β-stable matter, nonrelativistic (model APR1, 1998)

In this model, matter in the neutron star's interrior is assumed to consist of neutrons, protons, electrons and muons, in equilibrium with respect to the processes

$$n \leftrightarrow p + e \quad , \quad n \leftrightarrow p + \mu \ . \tag{5}$$

Equilibrium and charge neutrality require

$$\mu_n - \mu_p = \mu_e = \mu_\mu \tag{6}$$

and

$$n_p = n_e + n_\mu \ , \tag{7}$$

where μ_α and n_α denote the chemical potential and number density of particle α ($\alpha = p, n, e, \mu$), respectively.

At any given matter density, eqs.(6) and (7) determine the proton fraction $x_p = n_p/n_B$, $n_B = (n_n + n_p)$ being the baryon number density.

The nuclear hamiltonian used in this model explicitely includes three-body interactions, and can be written

$$H = \sum_i \frac{\mathbf{p}^2}{2m} + \sum_{j>i} v_{ij} + \sum_{k>j>i} V_{ijk} \ . \tag{8}$$

The two- and three-nucleon potentials, v_{ij} and V_{ijk}, have been obtained from a 1997 fit to the properties of two- and three-nucleon data. Unlike the potential models employed in the past, the v_{ij} of model APR1 takes into account small charge symmetry breaking contributions.

The energy per particle is obtained using a variational wave function, and the EOS is constructed as in model A.

5.3 Hyperonic matter, nonrelativistic (model B, 1971)

Besides, β-decay, at large enough density different processes, leading to the appearance of heavier baryons, become energetically favored. For example, the reactions

$$p + e \rightarrow \Lambda + \nu$$

$$n + e \rightarrow \Sigma^- + \nu \ ,$$

with $m_\Lambda \sim 1115$ MeV and $m_\Sigma \sim 1190$ MeV, are driven by weak interactions, while

$$p + n \rightarrow \Delta^+ + n$$

$$n + n \rightarrow \Delta^0 + p \ ,$$

with $m_\Delta \sim 1230$ MeV, proceed through strong interactions.

The above reactions begin to occur as soon as the mass difference between the particles in the final state and those in the initial state is balanced by the kinetic energy carried by the initial particles.

Inclusion of n, p, e, μ, Λ, $\Sigma^{\pm 0}$ and $\Delta^{\pm 0}$, requires

$$\mu_n = \mu_\Lambda = \mu_{\Sigma^0} = \mu_{\Delta^0}$$

$$\mu_p = \mu_{\Delta^+} = \mu_{\Sigma^+} = \mu_n - \mu_e$$

$$\mu_{\Sigma^-} = \mu_{\Delta^-} = \mu_n + \mu_e$$

For any value of ρ, equilibrium and charge neutrality determine the abbundances of the different particles.

The Hamiltonian of this model is written in the form

$$H = \sum_i \frac{\mathbf{p}^2}{2m} + \sum_{j>i} \left[v_{ij}^{NN} + v_{ij}^{BN} + v_{ij}^{BB} \right] \ ,$$

where the interaction term includes nulceon-nucleon (NN), baryon-nucleon (BN) and baryon-baryon (BB) potentials. The baryon-nucleon potential v_{ij}^{BN} has been obtained combinimg information from both phenomenology and quark models of baryon structure. As the heavier baryons have low density, the contribution of v_{ij}^{BB} is neglected.

The energy per particle and the EOS are calculated as in models A and APR1.

5.4 β-stable matter, semirelativistic (model APR2, 1998)

This model is the same as model APR1, but includes relativistic corrections to v_{ij} up tp order $(v/c)^2$, v being the velocity of the center of mass of the two interacting nucleons.

The fits to NN scattering data that determine v_{ij} are carried out in the NN center of mass frame, in which, by definition, $\mathbf{P} = (\mathbf{p}_i + \mathbf{p}_j)/2 = 0$. In general, however, the interaction between two particles depends upon their total momentum, and can be written

$$v_{ij}(\mathbf{P}) = v_{ij} + \delta v(\mathbf{P}) \ ,$$

with

$$\delta v(\mathbf{P}) = \frac{\mathbf{P}^2}{8m^2} v_{ij} + \frac{1}{8m^2}(\mathbf{P}\cdot\mathbf{r})(\mathbf{P}\cdot\nabla v_{ij}) \ ,$$

and $\mathbf{r} = \mathbf{r}_i - \mathbf{r}_j$. Model APR2 can be obtained from model APR1 replacing

$$H \to H^* = \sum_i \frac{\mathbf{p}^2}{2m} + \sum_{j>i} [v_{ij} + \delta v(\mathbf{P})] + \sum_{k>j>i} \widetilde{V}_{ijk} \ .$$

Note that, in order to maintain the global fit to two- and three-nucleon data, any modification of the two-nucleon interaction must be accompanied by a redefinition of the three-nucleon potential. This process leads to a new three-nucleon potential $\widetilde{V}_{ijk} << V_{ijk}$, implying that δv includes a significant fraction of the old V_{ijk}, obtained from a fully nonrelativistic fit.

5.4 Pure neutron matter, semirelativistic (model L, 1975)

Model L is a hybrid model. Neutron interactions are described by exchange of σ, ω and ρ mesons. However, while the heavy vector mesons, i.e. the ρ ($m_\rho \sim 770$ MeV) and the ω ($m_\omega \sim 780$ MeV) are treated in the nonrelativistic approximation using static potentials, the light scalar meson is treated in the mean field approximation discussed in Lecture 4. The argument underlying this approach is that the nonrelativistic approximation appears to be more appropriate than the mean field approximation for heavy meson, whose Compton wavelength ($\sim$ .25 fm) is much smaller than the average NN distance ($\sim$ 1.2 fm at $\rho = \rho_0$).

The energy per particle is written in the form

$$\frac{E_0}{N} = T + V + \frac{1}{2} m_\sigma \frac{\langle\phi\rangle}{\rho} - g\langle\phi\rangle \ ,$$

where $\langle\phi\rangle$ denotes the ground state expectation value of the σ field. In the above equation, T and V are the neutron kinetic energy and the energy associated with heavy meson exchange, respectively, that have been calculated using variational wave functions. The remaining two terms are the mass term associated with the σ meson and the $\sigma - n$ interaction. The parameters of the model have been fixed by a fit to the empirical inuclear matter properties.

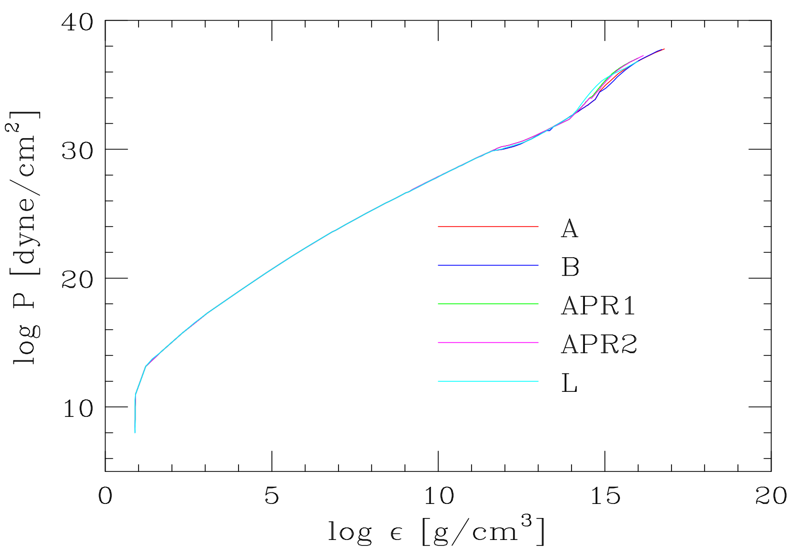

FIG. 1. EOS of the models of neutron star matter described in the text

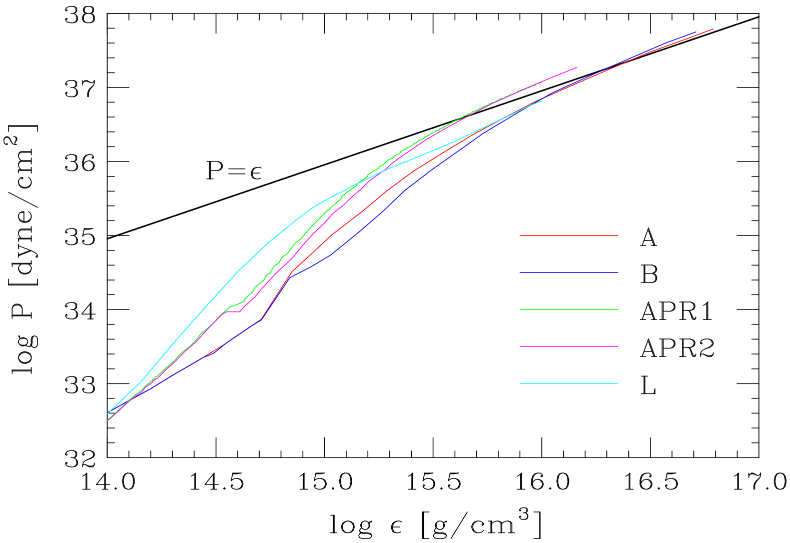

FIG. 2. Behavior of the EOS of fig.1 at supernuclear density. The stiffest equation of state compatible with causality, $P = \epsilon$, is also shown for comparison