Momento magnetico anomalo

Forma più generale della funzione di vertice

$$\bar{u}'\Lambda^\mu u = \bar{u}'\left[A_1(q^2)\gamma^\mu + A_2(q^2)p^\mu + A_3(q^2)p'^\mu + A_4(q^2)\sigma^{\mu\nu}p_\nu + A_5(q^2)\sigma^{\mu\nu}p'_\nu\right]u$$

$p^2 = p'^2 = m^2 \qquad q = p - p' \qquad \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$

$$q_\mu(\bar{u}'\Lambda^\mu u) = 0$$

$$q_\mu \bar{u}'\Lambda^\mu u = \bar{u}'\left[A_1(q^2)\, q_\mu\gamma^\mu + A_2(q^2)(p-p')_\mu p^\mu + A_3(q^2)(p-p')_\mu p'^\mu + A_4(q^2)\sigma^{\mu\nu}(p-p')_\mu p_\nu + A_5(q^2)\sigma^{\mu\nu}(p-p')_\mu p'_\nu\right]u$$

$$= \bar{u}\left\{A_1(q^2)q_\mu\gamma^\mu + \left[A_2(q^2) - A_3(q^2)\right]\left[m^2 - (pp')\right] + \left[A_5(q^2) + A_4(q^2)\right]\underbrace{(p_\mu p'_\nu)}_{\to\,\sigma^{\mu\nu}}\right\}u$$

$A_2(q^2) = A_3(q^2) \qquad A_4(q^2) = -A_5(q^2)$

$$\bar{u}'\Lambda^\mu u = \bar{u}'\left[A_1(q^2)\gamma^\mu + A_2(q^2)(p+p')^\mu + A_4(q^2)\sigma^{\mu\nu}q_\nu\right]u$$

Ermiticità $A_1(q^2)$ e $A_2(q^2)$ reali $A_4(q^2)$ immaginario

Identità di Gordon

$$2m\,\bar{u}'\gamma^\mu u = \bar{u}'\left[(p+p')^\mu + i\sigma^{\mu\nu}q_\nu\right]u$$

$$\bar{u}'\Lambda^\mu u = \bar{u}'\left\{A_1(q^2)\gamma^\mu + 2m\,A_2(q^2)\gamma^\mu - \left[A_2(q^2)+i\,A_4(q^2)\right]i\sigma^{\mu\nu}q_\nu\right\}u$$

$$= \bar{u}'\left\{F_1(q^2)\gamma^\mu + \frac{1}{2m}F_2(q^2)\,i\sigma^{\mu\nu}q_\nu\right\}$$

$$\left.\begin{aligned} F_1(q^2) &= A_1(q^2) + 2m\,A_2(q^2) \\ F_2(q^2) &= -2m\left[i\,A_4(q^2) + A_2(q^2)\right] \end{aligned}\right\}\ \text{Reali}$$

Sappiamo che

$$\bar{u}'\Lambda^\mu u = L\gamma^\mu + \bar{u}'\Lambda^\mu_c u$$

$p \to p'$ $\quad \bar{u}\Lambda^\mu u = L\bar{u}\gamma^\mu u \qquad \bar{u}\Lambda^\mu_c u = 0$

ma

$$\bar{u}\Lambda^\mu u = F_1(0)\,\bar{u}'\gamma^\mu u$$

quindi

$F_1(0) = L$. Expand $F_{1,2}(q^2) = F_{1,2}(0) + O(q^2)$

$$\bar{u}'\Lambda u = \bar{u}'\left[L\gamma^\mu + F_2(0)\frac{i}{2m}\sigma^{\mu\nu}q_\nu\right]u + O(q^2)$$

Includendo tutte le correzioni del secondo ordine che contribuiscono alla rinormalizzazione della carica %

$$ie_0\bar{u}'\gamma^\mu u \to ie\bar{u}'\left[\gamma^\mu + \frac{e^2}{2m}F_2(0)\,i\sigma^{\mu\nu}q_\nu\right]u$$
$$+O(q^2)$$

Identità di Gordon, ancora

$$ie\bar{u}'\gamma^\mu u \to ie\bar{u}'\left\{\frac{p^\mu + p'^\mu}{2m} + \frac{1}{2m}\left[1+e^2F_2(0)\right]i\sigma^{\mu\nu}q_\nu\right\}u$$

momento magnetico $\qquad \frac{e}{2m}\left[1+e^2F_2(0)\right]2\vec{S}$

$$= g\,\frac{e}{2m}\vec{S}$$

$$g = 2\left[1+e^2F_2(0)\right]$$

$$e^2F_2(0) = \frac{g-2}{2}$$

Vogliamo calcolare $e^2F_2(0)$ (correzione $O(e^2)$) per $p\sim p'$, che implica $q\sim 0$. Quindi trascuriamo termini quadratici in q.

Inoltre, siccome siamo interessati solo alla correzione finita, trascuriamo i termini proporzionali a γ^μ.

$$\bar{u}'\Lambda^\mu u = \bar{u}'(-i)\int \frac{d^4k}{(2\pi)^4}\frac{N^\mu(k,p,p')}{(k^2+i\varepsilon)[(p'-k)^2-m^2+i\varepsilon][(p-k)^2-m^2+i\varepsilon]}\, u$$

$$N^\mu(k,p,p') = \gamma^\alpha(\not{p}'-\not{k}+m)\gamma^\mu(\not{p}-\not{k}+m)\gamma_\alpha$$

definizioni

$$q^2=(p'-p)^2 = 2m^2-2(pp')$$

$$(p+p')^2 = 2m^2+2(pp') = 4m^2-q^2$$

$$Q=\frac{p+p'}{2} \quad\rightarrow\quad p'=Q+\frac{q}{2} \qquad p=Q-\frac{p}{2}$$

$$Q^2=\frac{(p+p')^2}{4} = m^2-\frac{q^2}{4} = m^2+O(q^2) \qquad \%$$

denominatore

$$[(p'-k)^2-m^2+i\varepsilon][(p-k)^2-m^2+i\varepsilon]$$

$$=[k^2-2(p'k)+i\varepsilon][k^2-2(pk)+i\varepsilon]$$

$$=\left[k^2-2\left(Q+\frac{q}{2}\right)k+i\varepsilon\right]\left[k^2-2\left(Q-\frac{q}{2}\right)k+i\varepsilon\right]$$

$$=(k^2-2Qk+i\varepsilon)^2-q^2k^2 \approx (k^2-2Qk+i\varepsilon)^2$$

Quindi

$$\bar{u}'\Lambda^\mu u = -i\int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}' N^\mu(k,p,p')u}{(k^2+i\varepsilon)(k^2-2Qk+i\varepsilon)^2} + O(q^2)$$

Parametrizzazione di Feynman

$$\frac{1}{D_1^2 D_2} = 2\int_0^1 dz \frac{z}{[D_2+(D_1-D_2)z]^3}$$

$$D_1 = (k^2-2Qk+i\varepsilon)^2 \qquad D_2 = k^2+i\varepsilon$$

$$\frac{1}{(k^2+i\varepsilon)(k^2-2Qk+i\varepsilon)^2} = 2\int_0^1 dz$$

$$\times \frac{z}{[(k^2+i\varepsilon)-2z(kQ)+i\varepsilon]^3}$$

$$= 2\int_0^1 dz \frac{z}{[(k-zQ)^2-z^2Q^2+i\varepsilon]^3}$$

$$\approx 2\int_0^1 dz \frac{z}{[\underbrace{(k-zQ)}_{t}{}^2-z^2m^2+i\varepsilon]^3}$$

$$t = k-zQ \qquad k = t+zQ$$

$$\bar{u}'\Lambda u = -i\int_0^1 2z\,dz \int \frac{d^4t}{(2\pi)^4} \frac{\bar{u}'\,N(t+zQ,p,p')\,u}{(t^2-z^2m^2+i\varepsilon)^3}$$

N^μ contiene termini in t^0, t^1 e t^2

- t^1 contributo nullo all'integrale, per simmetria

- t^2 termini $\sim \gamma^\mu$ perché vengono da

$$\gamma^\alpha \not{t}\gamma^\mu \not{t}\gamma_\alpha$$

$$= \not{t}\gamma^\mu\not{t} = t_\alpha t_\beta \gamma^\alpha\gamma^\mu\gamma^\beta$$

$$= t_\alpha t_\beta \gamma^\alpha (2g^{\beta\mu} - \gamma^\beta\gamma^\mu)$$

$$= 2\not{t}\,t^\mu - t^2\gamma^\mu$$

$$\gamma^\alpha (2\not{t}t^\mu - t^2\gamma^\mu)\gamma_\alpha$$

$$= 2t^\beta t^\mu \gamma^\alpha\gamma_\beta\gamma_\alpha - t^2\gamma^\alpha\gamma^\mu\gamma_\alpha$$

$$= 2t^\beta t^\mu \gamma^\alpha (2g_{\alpha\beta} - \gamma_\alpha\gamma_\beta)$$

$$-t^2 (2g^{\alpha\mu} - \gamma^\mu\gamma^\alpha)\gamma_\alpha$$

$$= \underbrace{4t_\alpha t^\mu\gamma^\alpha - 4t^\beta t^\mu\gamma_\beta}\; - 2t^2\gamma^\mu$$

$$= \underbrace{2\not{t}t^\mu - 4t^\mu\not{t}} \qquad + 4t^2\gamma^\mu$$

$$N^\mu(k,p,p') = \gamma^\alpha(\not p' - \not k + m)\gamma^\mu(\not p - \not k + m)\gamma_\alpha$$

$$= \gamma^\alpha(\not p' - \not t - z\not Q + m)\gamma^\mu(\not p - \not t - z\not Q + m)\gamma_\alpha$$

Termini in t^0 ok

t^1 danno contributo nullo

t^2 proporzionali a γ^μ

$$N^\mu(t+zQ,\ p,p') \to N^\mu(\mathrm{const} + zQ,\ p,p')$$

il termine const da solo contributi $\propto \gamma^\mu$

$$\to N^\mu(zQ,p,p')$$

Quindi

$$\frac{1}{(2\pi)^4}\frac{-i\pi^2}{2}\frac{1}{z^2m^2}$$

$$\bar u' \Lambda^\mu u = -i\int_0^1 2z\,dz\ \bar u' N^\mu(zQ,p,p')\,u \times \int \frac{d^4t}{(2\pi)^4}\frac{1}{(t^2 - z^2m^2 + i\varepsilon)^3}$$

$$= \frac{-i}{(2\pi)^4}\int_0^1 2\cancel{z}\,dz\ \bar u' N^\mu(zQ,p,p')\,u\ \frac{-i\pi^2}{2z^2m^2}$$

$$= \frac{-1}{16\pi^2 m^2}\int_0^1 \frac{dz}{z}\ \bar u' N^\mu\left(z\frac{p+p'}{2}, p\,p'\right)u$$

$$\bar{u}'\gamma^\alpha\Big(\underbrace{\not p' - z\,\frac{\not p + \not p'}{2}}_{\not a} + m\Big)\gamma^\mu\Big(\underbrace{\not p - z\,\frac{\not p + \not p'}{2}}_{\not b} + m\Big)\gamma_\alpha u$$

uso $\gamma^\alpha\gamma^\rho\gamma^\mu\gamma^\sigma\gamma_\alpha = -2\gamma^\sigma\gamma^\mu\gamma^\rho$

$$a_\rho b_\sigma\,\gamma^\alpha\gamma^\rho\gamma^\mu\gamma^\sigma\gamma_\alpha = -2\not b\,\gamma^\mu\not a$$

$$\checkmark\ \gamma^\alpha\gamma^\rho\gamma^\mu\gamma_\alpha = 4g^{\rho\mu} = \gamma^\alpha\gamma^\mu\gamma^\rho\gamma_\alpha$$

$$a_\rho\,\gamma^\alpha\gamma^\rho\gamma^\mu\gamma_\alpha = 4a^\mu$$

$$b_\sigma\,\gamma^\alpha\gamma^\mu\gamma^\sigma\gamma_\alpha = 4b^\mu$$

$$m^2\gamma^\alpha\gamma^\mu\gamma_\alpha = m^2(2g^{\mu\alpha} - \gamma^\mu\gamma^\alpha)\gamma_\alpha$$

$$= m^2(2\gamma^\mu - 4\gamma^\mu) = -2m^2\gamma^\mu$$

non contribuisce

$$\bar{u}'\Big\{-2\left[\not p-\frac{z(\not p+\not p')}{2}\right]\gamma^\mu\left[\not p'-\frac{z(\not p+\not p')}{2}\right]$$
$$+4m\left[p'^\mu-\frac{z(p+p')^\mu}{2}+p^\mu-\frac{z(p+p')^\mu}{2}\right]\Big\}u$$

$$=\bar{u}'\Big\{-2\left[\not p-\frac{z(\not p+\not p')}{2}\right]\gamma^\mu\left[\not p'-z\left(\frac{\not p+\not p'}{2}\right)\right]$$
$$+4m(p+p')^\mu(1-z)\Big\}u$$

Ora usiamo $\not q=\not p'-\not p \rightarrow \not p=\not p'-\not q$

$$\bar{u}'\left[\not p-\frac{z(\not p+\not p')}{2}\right]$$

$$=\bar{u}'\left[\not p'-\not q-z\,\frac{2\not p'-\not q}{2}\right]$$
$$=\bar{u}'\left[m-\not q-z\left(m-\frac{\not q}{2}\right)\right]$$
$$=\bar{u}'\left[m(1-z)-\not q\left(1-\frac{z}{2}\right)\right]$$

$\not p'=\not q+\not p$

$$\left[\not p'-z\left(\frac{\not p+\not p'}{2}\right)\right]u$$

$$=\left[\not p+\not q-z\,\frac{2\not p+\not q}{2}\right]u$$

$$= \left[m + \not{q} - z\left(m - \frac{\not{q}}{2}\right) \right] u$$

$$= \left[m(1-z) + \not{q}\left(1 - \frac{z}{2}\right) \right] u$$

$$\bar{u}' \Big\{ -2\left[m(1-z) - \not{q}\left(1-\frac{z}{2}\right)\right] \gamma^\mu \left[m(1-z) + \not{q}\left(1-\frac{z}{2}\right)\right]$$

$$+ 4m(p+p')^\mu (1-z) \Big\} u$$

$$= \bar{u}' \left[2m(1-z)\left(1-\frac{z}{2}\right)\left(\not{q}\gamma^\mu - \gamma^\mu \not{q}\right) \right.$$

$$\left. + 4m(p+p')^\mu (1-z) \right] u$$

Gordon

$$= \bar{u}' \left[4m(1-z)\left(1-\frac{z}{2}\right)\frac{1}{2}[\not{q}, \gamma^\mu] + 4m(p+p')^\mu (1-z) \right] u$$

$$= \bar{u}' \left[4m(1-z)\left(1-\frac{z}{2}\right) \underbrace{\frac{1}{2}[\gamma^\nu, \gamma^\mu] q_\nu}_{+i\sigma^{\mu\nu} q_\nu} - 4m \overset{(1-z)}{} i\sigma^{\mu\nu} q_\nu \right] u$$

$$= \bar{u}' \; 4m \, i\sigma^{\mu\nu} q_\nu (1-z) \underbrace{\left(1 - \frac{z}{2} - 1\right)}\; u \qquad (1-z)\frac{z}{2}$$

$$= -2m \, z(1-z) \, \bar{u}' i\sigma^{\mu\nu} q_\nu u$$

Gordon $\bar{u}'(p+p')^\mu u = \bar{u}'(2m\gamma^\mu - i\sigma^{\mu\nu} q_\nu) u$

Integrazione su z (pag. 7)

$$\int_0^1 \frac{dz}{z}\, z(1-z) = \int_0^1 dz(1-z)$$

$$= 1 - \frac{z^2}{2}\Big|_0^1 = 1 - \frac{1}{2} = \frac{1}{2}$$

$$e^2\, \bar{u}'\, \Lambda^\mu\, u = e^2 \frac{1}{16\pi^2 m^2}\; \frac{1}{2}\; (-2m)\; \bar{u}'\, i\sigma^{\mu\nu} q_\nu\, u$$

$$= \frac{e^2}{8\pi^2}\; \frac{1}{2m}\; \bar{u}'\, i\sigma^{\mu\nu} q_\nu\, u$$

$$e^2 F_2(0) = \frac{e^2}{8\pi^2} = \frac{\alpha}{2\pi} \qquad \text{ok}$$

$$\frac{\alpha}{2\pi} = 0.00116 \qquad \text{Expt}\quad 0{,}00119 \pm 0.00005$$

Compare

$$\frac{e^2 F_2(0)}{2m} = \frac{e^2}{8\pi^2}\;\frac{1}{2m}$$

$$e^2 F(0) = \frac{e^2}{8\pi^2} = \frac{\alpha}{2\pi}$$