Neutron star matter EOS: where do we stand ?

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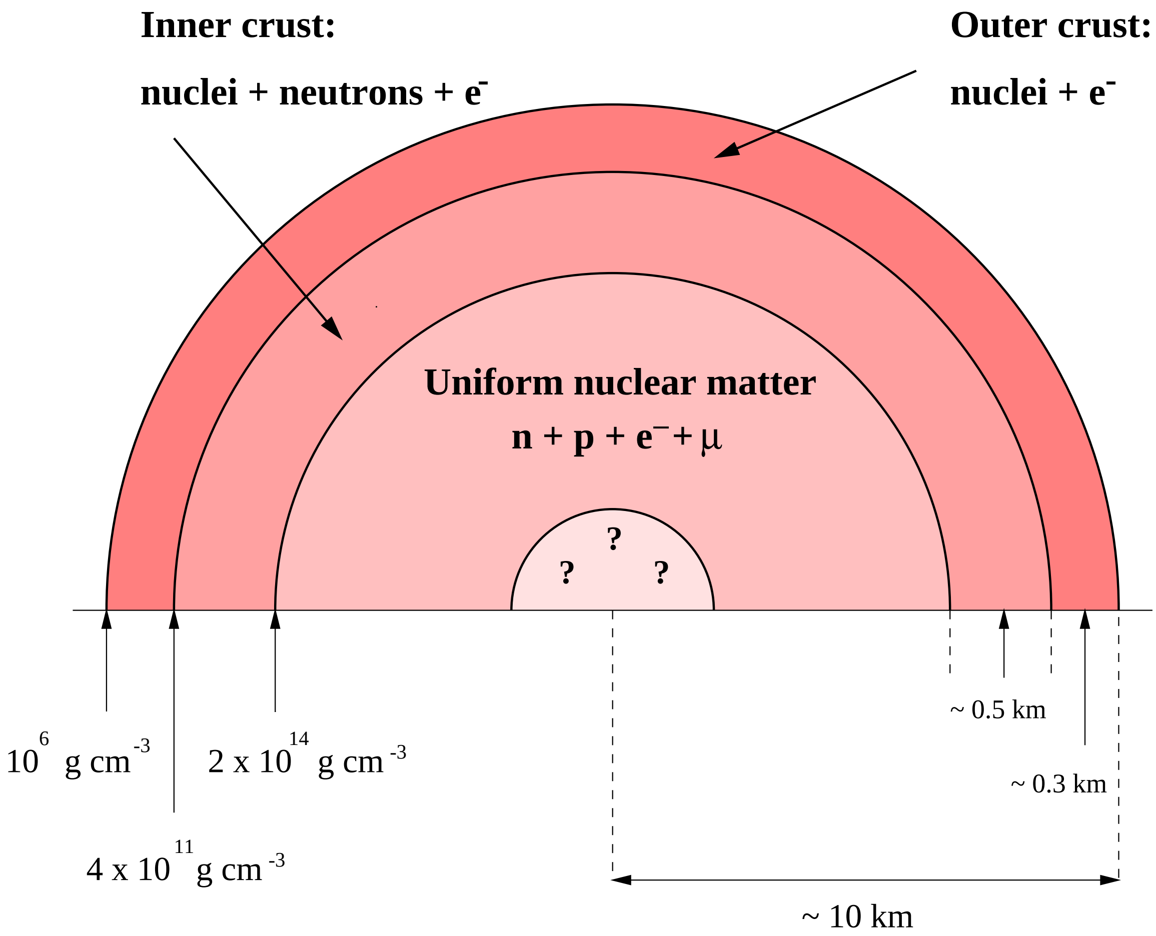
Outline

- Overview of neutron star structure
- The nuclear EOS within nonrelativistic many-body theory
- Inclusion of more exotic degrees of freedom
- Constraints from neutron star observations
- Prospects & perspectives

Overview of Neutron Stars' Structure

- recall: $\rho_0 \approx 0.16$ nucl/fm$^3 = 2.67 \times 10^{14}$ g/cm^3

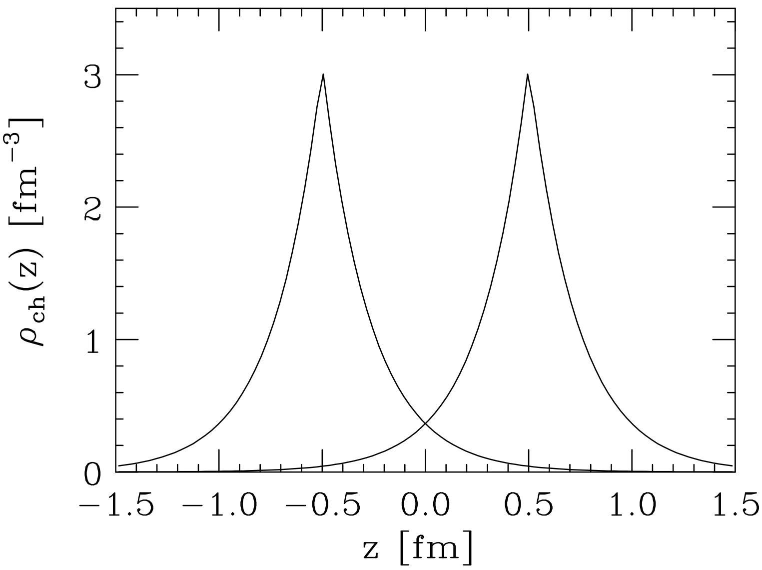


- thermal effects negligible (T $\ll e_F$)
- properties of matter it the outer crust can be obtained from data on neutron rich nuclei
- in the region of supranuclear density, $\rho > \rho_0$, one has to resort to extrapolations of theoretical models, constrained by the observed neutron star properties

Modeling the EOS @ $\rho > \rho_0$

- ▷ what are the right degrees of freedom ?
- ▷ proton charge distribution extracted from a dipole fit to the charge form factor, measured by electron scattering

$$F(q) = \frac{1}{[1 + (q/q_0)^2]^2} \quad , \quad q_0 = 0.84 \text{ GeV/c}$$



- ▷ two nucleons separated by 1 fm still look pretty much like individual objects
- ▷ average NN separation in nuclear matter

$$1.2 \gtrsim r_0 \gtrsim 0.8 \quad @ \quad \rho_0 \lesssim \rho \lesssim 4\rho_0$$

Models based on hadronic degrees of freedom

- Nonrelativistic many-body theory

Pointlike nucleons interacting through the hamiltonian

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{j>i} v_{ij} + \sum_{k>j>i} V_{ijk}$$

with

$$V_{ijk} \ll v_{ij}$$

▷ v_{ij} : π exchange + phenomenological short and intermediate range interaction, strongly constrained by NN data (deuteron properties and $\sim$ 4000 NN scattering phase-shifts, fitted with $\chi^2/N \sim 1$)

▷ V_{ijk} : needed to reproduce the empirical equilibrium density of nuclear matter and the measured binding energies of the three-nucleon bound states

- Relativistic boost correction to v_{ij}

▷ the v_{ij} obtained from phase shift analysis describes the interaction in the NN center of mass frame, where

$$\mathbf{P}_{ij} = \mathbf{p}_i + \mathbf{p}_j = 0$$

▷ at $\mathbf{P} \neq 0$ replace

$$v_{ij} \to v_{ij} + \delta v_{ij}(\mathbf{P})$$

▷ the boost correction, evaluated to order $\mathbf{P}^2/m^2$, reads (Krajcic & Foldy (1974), Friar (1975))

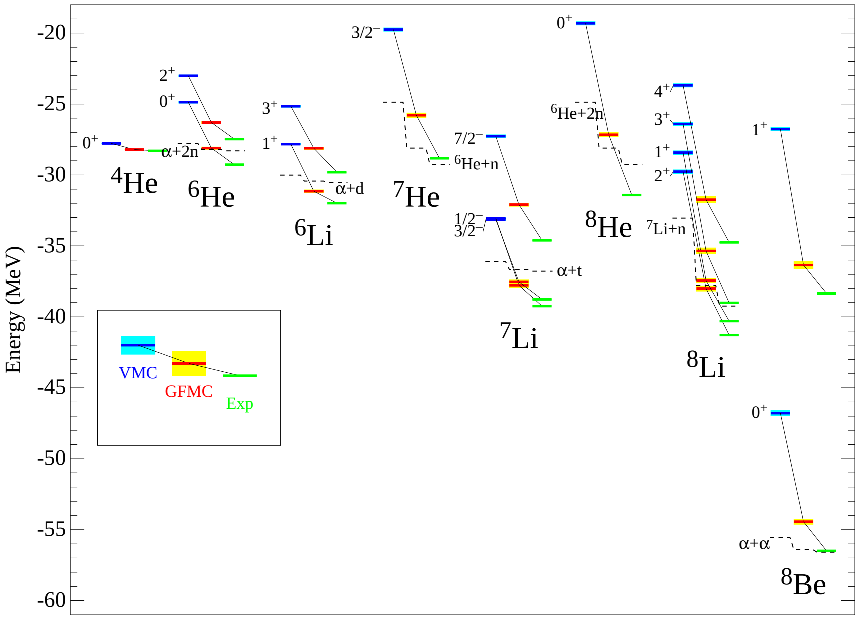
$$\delta v_{ij}(\mathbf{P}) = -\frac{\mathbf{P^2}}{8m^2} v_{ij} + \frac{1}{8m^2}[\mathbf{P}\cdot\mathbf{r}\,\mathbf{P}\cdot\boldsymbol{\nabla}, v_{ij}]$$

$$+\frac{1}{8m^2}[(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)\times\mathbf{P}\cdot\boldsymbol{\nabla}, v_{ij}].$$

▷ $\langle \delta v_{ij} \rangle \sim$ 7% in ^{4}He

▷ $\langle \delta v_{ij} \rangle \sim$ 12 − 15% in neutron matter @ $\rho \sim (1-2)\rho_0$

▷ the ground state energy of nuclei with $A < 8$ is obtained using the Green Function Monte Carlo method to solve the Schrödinger equation



$$\Delta = \frac{|E_{GFMC} - E_{exp}|}{E_{exp}} \lesssim 5\%$$

▷ the total energy per baryon of nuclear matter in β-equilibrium is calculated using a variational approach

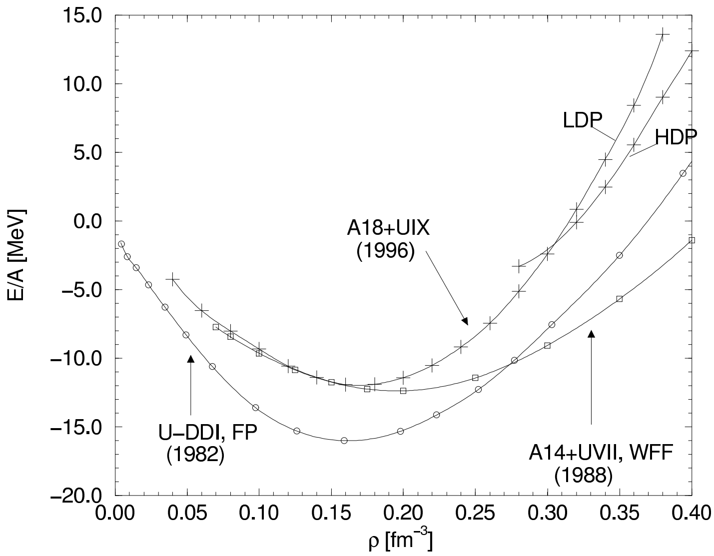
▷ the EOS is obtained by eliminating ρ from

$$\epsilon(\rho) = \rho E(\rho) \quad , \quad P(\rho) = \rho^2 \frac{dE(\rho)}{d\rho}$$

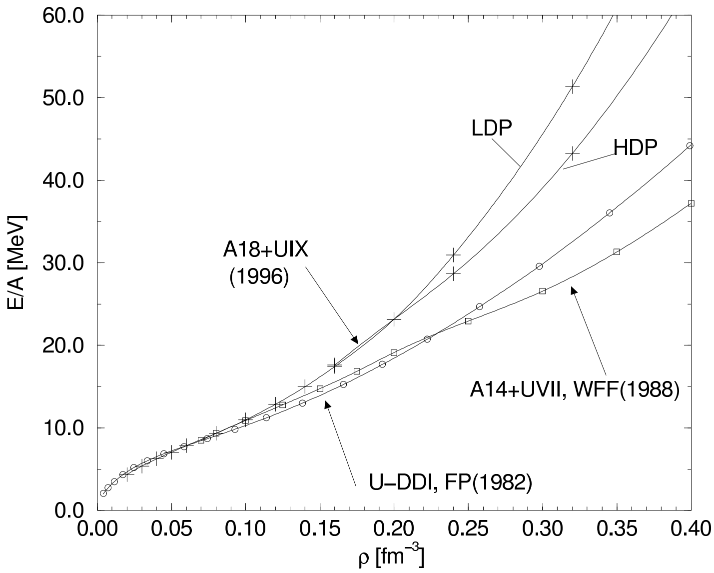
Energy per baryon of nucleon matter

(Akmal, Pandharipande & Ravenhall (1998))

- symmetric nuclear matter

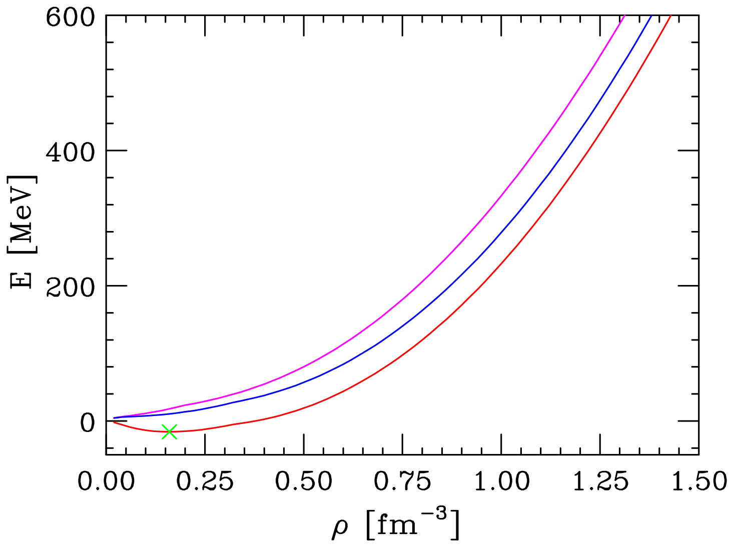


- pure neutron matter

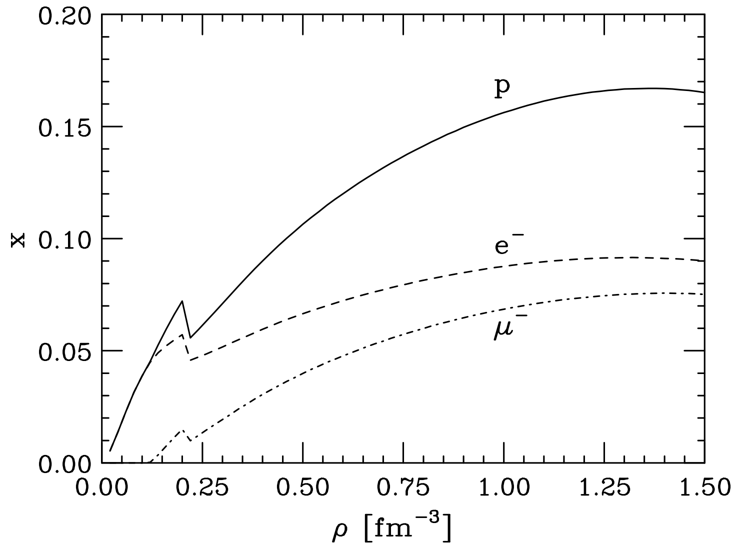


β-stable matter: n, p, e^- and μ^-

- energy-density



- proton and lepton fractions



- Relativistic approaches

Pointlike nucleons and mesons described by a lagrangian density (simplest implementation: Walecka, 1976)

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_\omega(x) + \mathcal{L}_\sigma(x) + \mathcal{L}_{int}$$

$$\mathcal{L}_N(x) = \overline{\psi}(x)\left(i\partial\!\!\!/ - m\right)\psi(x)$$

$$\mathcal{L}_\omega(x) = -\frac{1}{4}F^{\mu\nu}(x)F_{\mu\nu}(x) + \frac{1}{2}m_\omega^2 V_\mu(x)V^\mu(x)$$

$$F_{\mu\nu}(x) = \partial_\mu V_\nu(x) - \partial_\nu V_\mu(x)$$

$$\mathcal{L}_\sigma(x) = +\frac{1}{2}\partial_\mu\phi(x)\partial^\mu\phi(x) - \frac{1}{2}m_\sigma^2\phi(x)^2$$

$$\mathcal{L}_{int}(x) = g_\sigma\phi(x)\overline{\psi}(x)\psi(x) - g_\omega V_\mu(x)\overline{\psi}(x)\gamma^\mu\psi(x)$$

▷ the formalism is manifestely covariant

▷ extension to T $\neq$ 0 feasible

▷ dynamics constrained by nuclear matter properties

▷ field equations tractable only in the mean field approximation, whose applicability is questionable as

$$\frac{1}{m_\sigma} \ , \ \frac{1}{m_\omega} \ll r_0$$

Possible appearance of mesons and heavy baryons

- At $\rho \gg \rho_0$ a different form of matter, containing hadrons other than nucleons may have lower energy
- neutron star matter may contain K^- mesons, or baryons other than n and p, such as Σ^- Δ^- or Λ
- For example, Σ^- and K^- can appear through the processes

$$n + e \rightarrow \Sigma^- + \nu_e \quad , \quad n \rightarrow p + K^-$$

as soon as

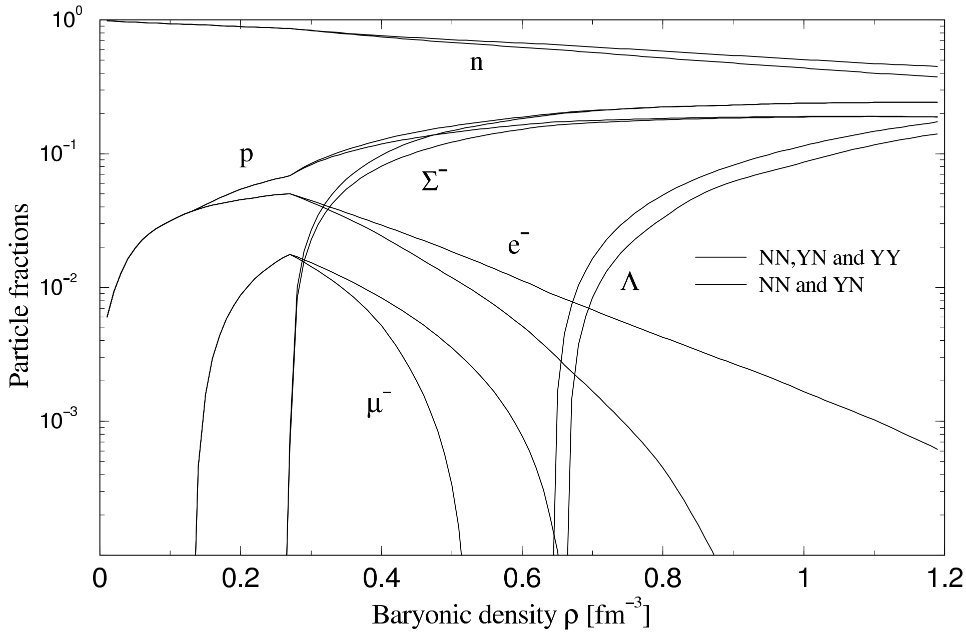
$$\mu_n + \mu_e = \widetilde{M}_{\Sigma^-} \quad , \quad \mu_n - \mu_p = \mu_e = \widetilde{M}_{K^-}$$

where $\widetilde{M}_{\Sigma^-}$ and $\widetilde{M}_{K^-}$ denote the masses of a Σ^- and a K^- embedded in the nuclear medium

- Theoretical estimates of the critical densities strongly affected by the poor knowledge of the dynamics driving interaction effects

▷ the results of a recent calculation (Carlson *et al* (2000)) suggest that K^- condensation may occur at $\rho \sim 5\rho_0$

▷ nonrelativistic calculations of hyperonic matter, carried out using the Nijmegen baryon-baryon potential, (Vidana *et al* (2000)) predict Σ^- apearance at $\rho \sim 2\rho_0$



▷ NOTE : as most processes leading to the appearance of negatively charged mesons and heavy baryons exploit the large electron chemical potential (~ 100 MeV), only one of them (if any) may occur

From hadronic matter to quark matter

- at a high enough density ($\sim 10^{15}$ g/cm^3) there will eventually be a transition to a new form of matter, in which quarks are no longer clustered into nucleons or hadrons

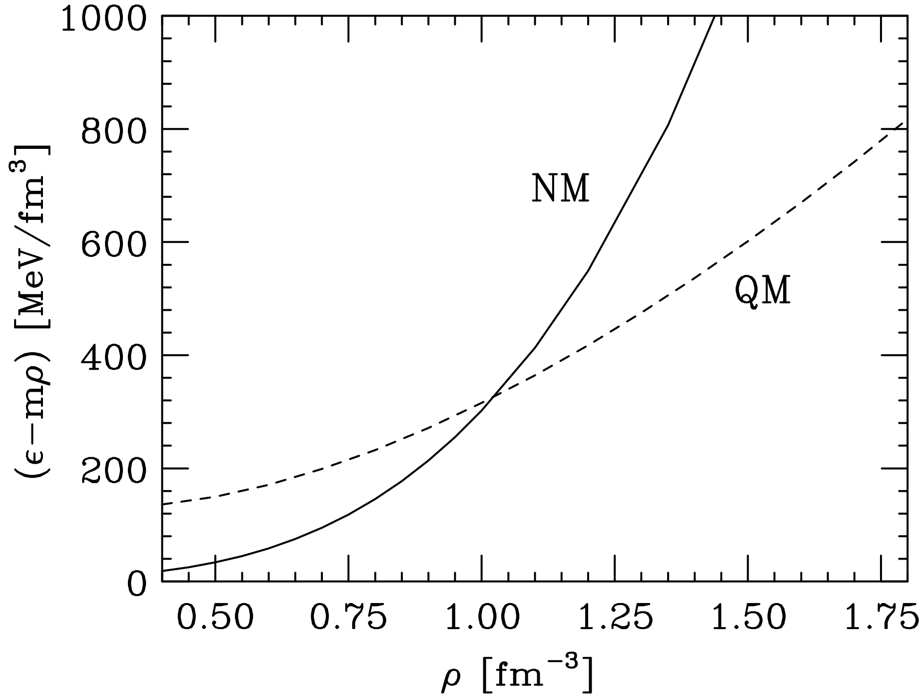
- The EOS of quark matter can be estimated using the simple bag model
  - noninteracting quarks confined to a finite region of space (the bag)
  - bag volume limited by a pressure B (the bag constant)

$$\epsilon = B + \frac{3}{4\pi^2} \sum_{i=1}^{N_f} p_{F_i}^4$$

$$P = -B + \frac{1}{4\pi^2} \sum_{i=1}^{N_f} p_{F_i}^4$$

- interaction effects associated with gluon exchange between quarks can be easily included using perturbation theory

Nuclear matter vs quark matter



- as $\rho \rightarrow \infty$

$$E_{NM} \propto \rho \quad , \quad E_{QM} \propto \rho^{1/3}$$

- a transition from nucleon to quark matter may occur in the inner core of neutron stars
- combining their nuclear matter EOS and the bag model (with $B = 200\text{MeV}$) Akmal Pandharipande and Ravenhall find a transition region

$$5.4 \leq (\rho/\rho_0) \leq 9.8$$

Self-bound strange quark matter

- deconfined u and d quarks can convert to other flavors via weak interactions, in order to lower the Fermi energy by increasing the degeneracy
- for example, the process

$$d + u \rightarrow u + s$$

is energetically favored as soon as

$$\mu_d \geq m_s \sim 150 \text{ MeV}$$

- in the 80s Witten conjectured that quark matter in equilibrium with the weak interaction containing comparable numbers of u, d and s quarks may be absolutely stable
- Witten's hypotesis suggests the possible existence of a new family of compact stars, made entirely of self-bound strange quark matter

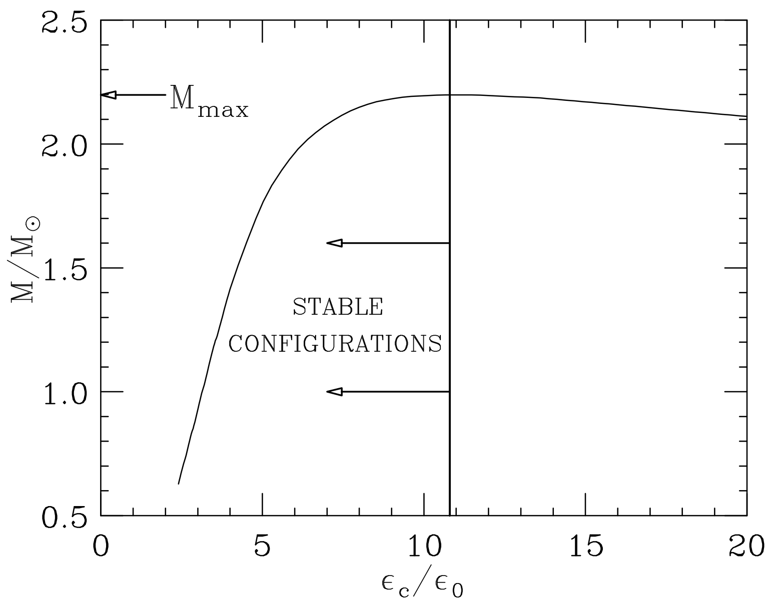
Maximum mass of a neutron star

- Given the EOS, mass and radius of a neutron star can be obtained from the Tolman-Oppenheimer-Volkov equations (hydrostatic equilibrium + Einstein's eqns)

$$\frac{dP(r)}{dr} = -G\,\frac{\left[\epsilon(r) + P(r)/c^2\right]\left[M(r) + 4\pi r^2 P(r)/c^2\right]}{r^2\left[1 - 2GM(r)/rc^2\right]}$$

$$M(r) = 4\pi \int_0^r r'^2 dr' \epsilon(r') \quad , \quad \epsilon(r=0) = \epsilon_c$$

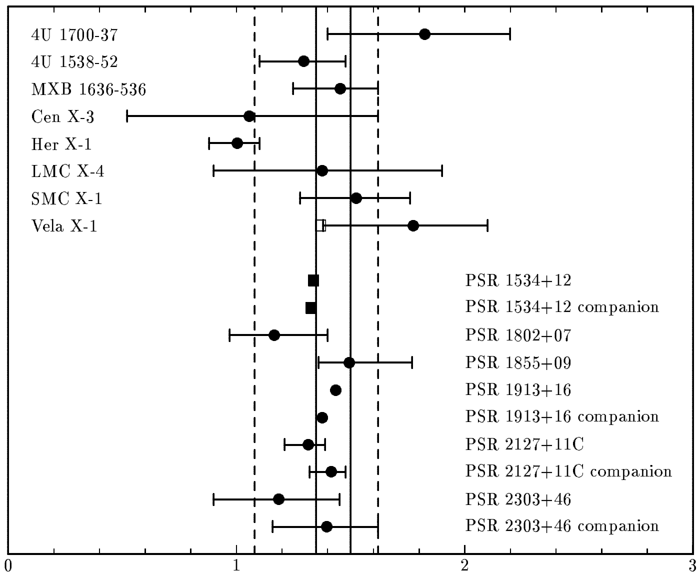
- typical mass-central energy-density curve



- the maximum mass is given by

$$M_{max} = M(\overline{\epsilon}_c) \quad , \left(\frac{dM}{d\epsilon_c}\right)_{\epsilon_c=\overline{\epsilon}_c} = 0$$

Predicted maximum masses vs data



- Basic experimental facts

  ▷ Mass of the Hulse-Taylor bynary pulsar: $M = 1.4411 \pm 0.0007 M_\odot$

  ▷ ~ 20 accurate measurements from bynary systems yield $M = 1.35 \pm 0.1 M_\odot$

  ▷ recent suggested evidences of heavier neutron stars

  - mass of Vela X-1: $M = 1.87^{+0.23}_{-0.17} M_\odot$
  - mass of Cygnus X-2: $M = 1.78 \pm 0.23 M_\odot$
  - QPO of galactic X-ray sources seem to indicate that they include neutron stars with $M =\sim 2 M_\odot$

- Theoretical predictions
  - ▷ bottom line: most EOS support a stable neutron star of mass ~ 1.4 $M_\odot$
  - ▷ EOS based on nucleonic degrees of freedom predict maximum masses typically $\gtrsim 2$ $M_\odot$
  - ▷ the presence of a core of deconfined quark matter lowers the maximum mass by $\sim 10\%$
  - ▷ the appearance of K^- or hyperons makes the EOS significantly softer, leading to $M_{max} \lesssim 1.6 M_\odot$
  - ▷ typical maximumm masses resulting from models of strange stars are $\lesssim 1.6 M_\odot$

- Theory vs data
  - ▷ the mass suggested by QPO would rule out softer, more exotic EOS
  - ▷ the mass suggested by Vela X-1 and Cygnus X-2 leaves some room for exotic effects

New observational developments

- The Iron and Oxygen transitions recently observed in the spectra of 28 bursts of the X-ray binary EXO0748-676 correspond to a gravitational redshift

$$z = 0.35$$

(J. Cottam *et al*, Nature, **420**(2002)51)

- The redshift can be related to the mass-radius ratio through (J. Van Paradijs, APJ **234**(1979)609)

$$R_\infty = R(1+z) = R\left(1 - \frac{2GM}{c^2}\frac{1}{R}\right)^{-1/2}$$

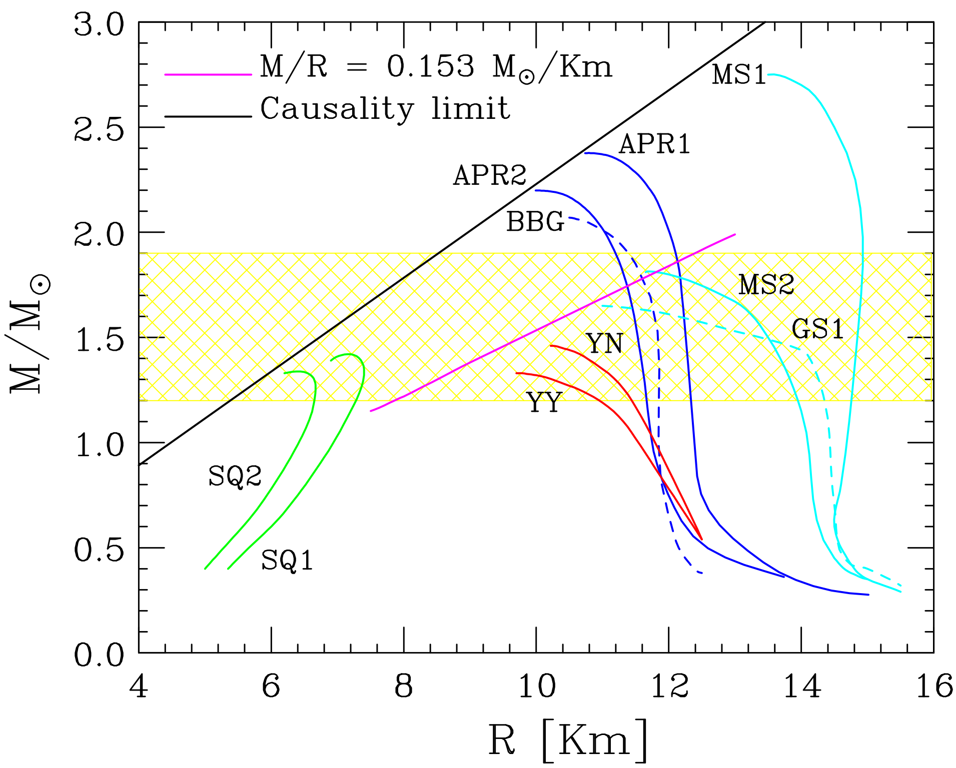
yielding

$$\frac{M}{R} = 0.153\ \frac{M_\odot}{\text{Km}}$$

i.e.

$$1.4 \lesssim M/M_\odot \lesssim 1.8 \iff 9 \lesssim R \lesssim 12\ \text{Km}$$

How do the M/R ratios predicted by different EOS compare to data ?



- ▷ SQ1, SQ2 : self-bound strange matter
- ▷ MS1, MS2 : nucleon matter. Relativistic mean field approach
- ▷ APR1, APR2, BBG : nucleon matter. Nonrelativistic many-body theory
- ▷ YN, YY : hyperonic matter. Nonrelativistic many-body theory
- ▷ GS1 : nucleon matter with kaon condensation. Relativistic mean field approach

Prospects & perspectives

- ▷ Nonrelativistic many-body theory (NMBT) provides a fully consistent framework to calculate properties of hadronic systems (energies, form factors, static and dynamical responses . . .).
- ▷ In the nucleon sector NMBT is fully determined by the nuclear hamiltonian, whose parameters are obtained fitting the experimetal data on the two- and three-nucleon systems. There are no adjustable parameters.
- ▷ Relativistic corrections to the NN potential can be systematically included.
- ▷ Exact calculations are feasible in nuclei with A $\leq$ 9 and pure neutron matter. Extension to symmetric nuclear matter and β-stable matter is under way.
- ▷ Extension to hyperonic matter requires models of the Λ- and Σ^--N interactions
- ▷ Neutron star properties predicted by NMBT are consistent with all the available empirical information