Equation of State of Neutron Star Matter and Gravitational Wave Emission

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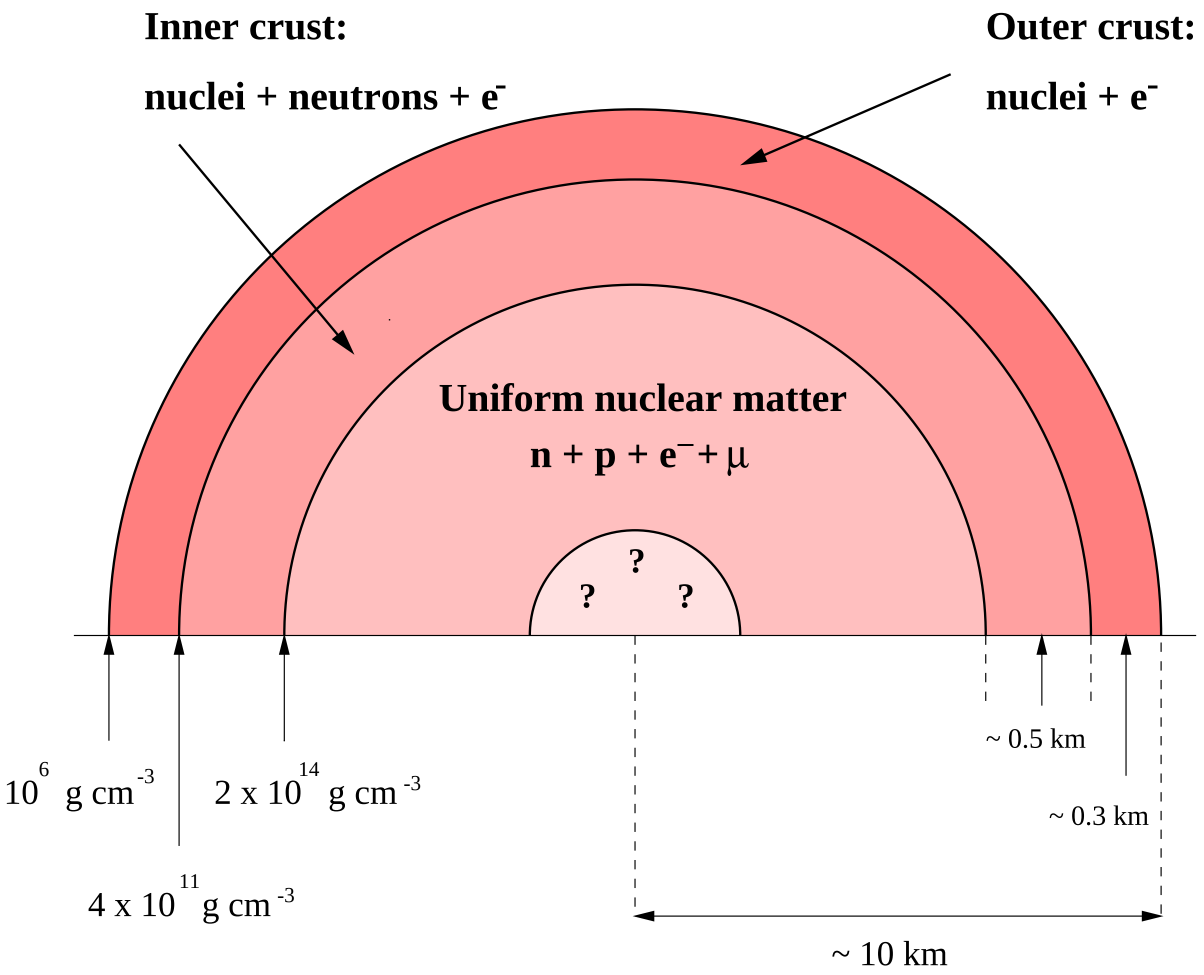
Outline

- ▷ Overview of neutron star structure
- ▷ Models of the equation of state (EOS) of neutron star matter
- ▷ Constraints from static properties of neutron stars
- ▷ Nonradial oscillations and gravitational wave (GW) emission from neutron stars
- ▷ Prospects and perspectives

Overview of Neutron Star Structure

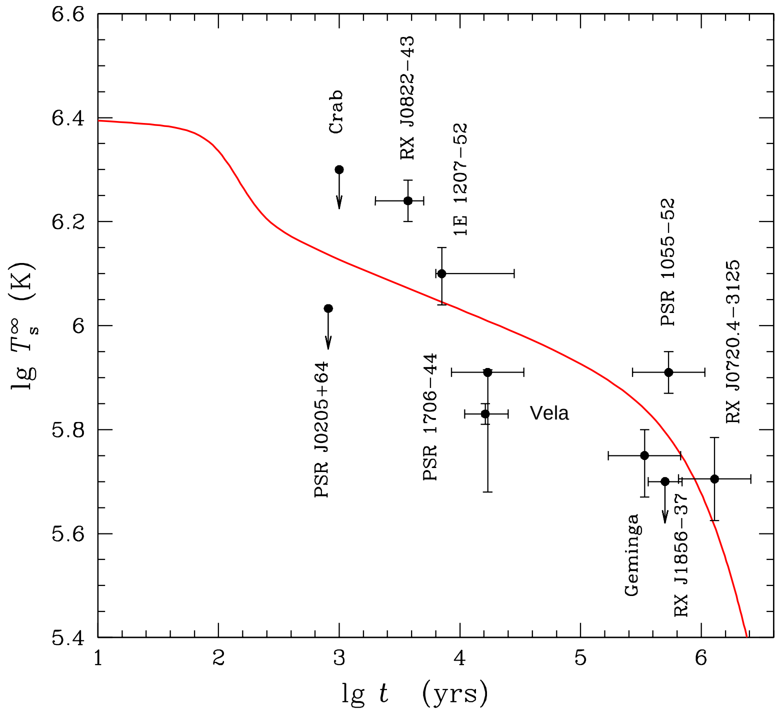
- recall: $\rho_0 \approx 0.16 \text{ nucl/fm}^3 = 2.67 \times 10^{14} \text{ g/cm}^3$



- ▷ ???: hyperons, π-condensate, K-condensate, quark matter . . .
- ▷ note: most of the neutron star mass is in the region $\rho > \rho_0$

Basic assumptions

- cold matter: $T = 0\,^\circ K$



▷ typical neutron star temperature $\sim 10^9\,^\circ K$

▷ to be compared to average kinetic energies in the range

$$10^{11} < \langle T \rangle < 10^{12}\,^\circ K$$

$$@\quad \rho_0 < \rho < 4\rho_0$$

- transparency to neutrino:

$$\lambda_\nu >> 10\ \mathrm{Km}\quad @\quad T \sim 10^9\,^\circ K$$

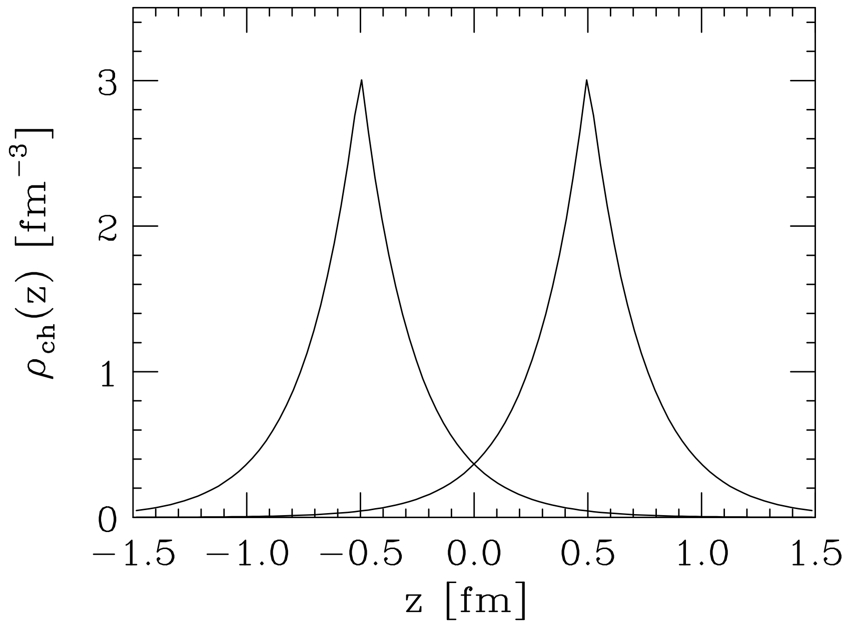
Modeling neutron star matter @ $\rho > \rho_0$

- Bottom line
  - properties of matter it the outer and inner crust can be inferred from data on neutron rich nuclei
  - in the region of supranuclear density, $\rho > \rho_0$, one has to resort to extrapolations of theoretical models
  - the Fermi gas model is incompatible with all measured neutron star masses
  - strong and weak interactions dynamics must be taken into account

What are the right degrees of freedom ?

- proton charge distribution from a dipole fit of the measured form factor

$$F(q) = \frac{1}{[1+(q/q_0)^2]^2} \quad , \quad q_0 = 0.84 \text{ GeV}$$



- two nucleons separated by 1 fm still look like individual objects
- average NN separation in nuclear matter

$$1.2 \gtrsim r_0 \gtrsim 0.8 \;\; @ \;\; \rho_0 \lesssim \rho \lesssim 4\rho_0$$

Models based on nucleon degrees of freedom

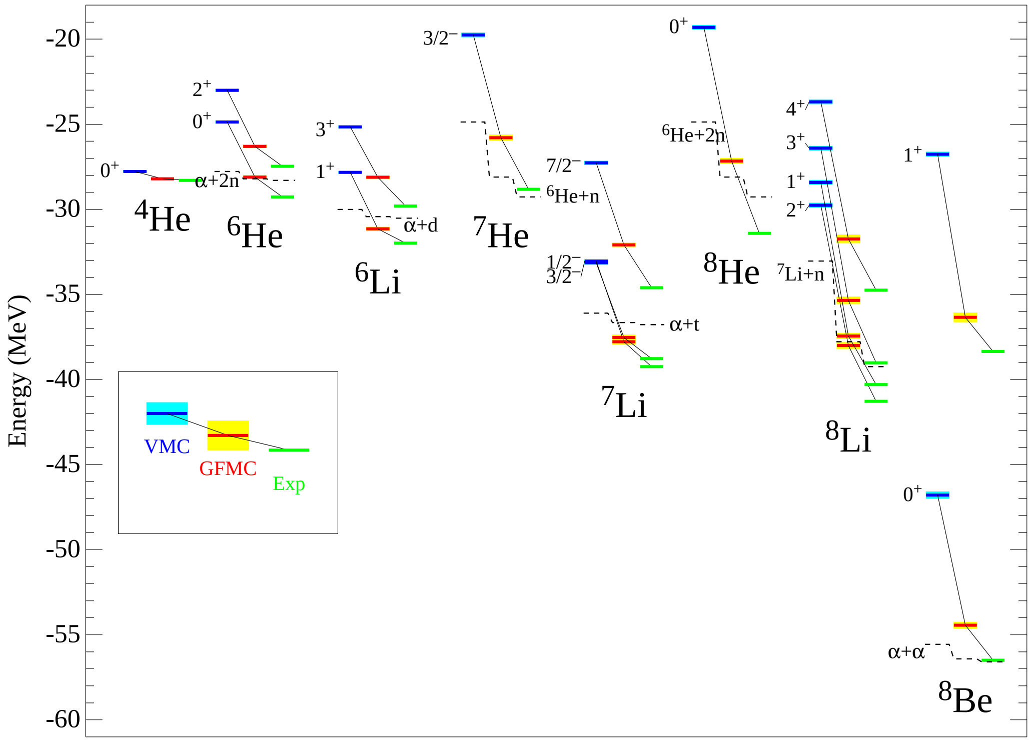
- Nonrelativistic approach
  - ▷ pointlike nucleons interacting through the hamiltonian

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{j>i} v_{ij} + \sum_{k>j>i} V_{ijk}$$

  - ▷ v_{ij}: π exchange + phenomenological short and intermediate range interaction, strongly constrained by NN data (deuteron properties and $\sim$ 4000 NN scattering phase-shifts, fitted with $\chi^2/N \sim 1$)
  - ▷ $V_{ijk} (\ll v_{ij})$: needed to reproduce the empirical equilibrium density of nuclear matter and the measured binding energies of the three-nucleon bound states

Results of NMBT

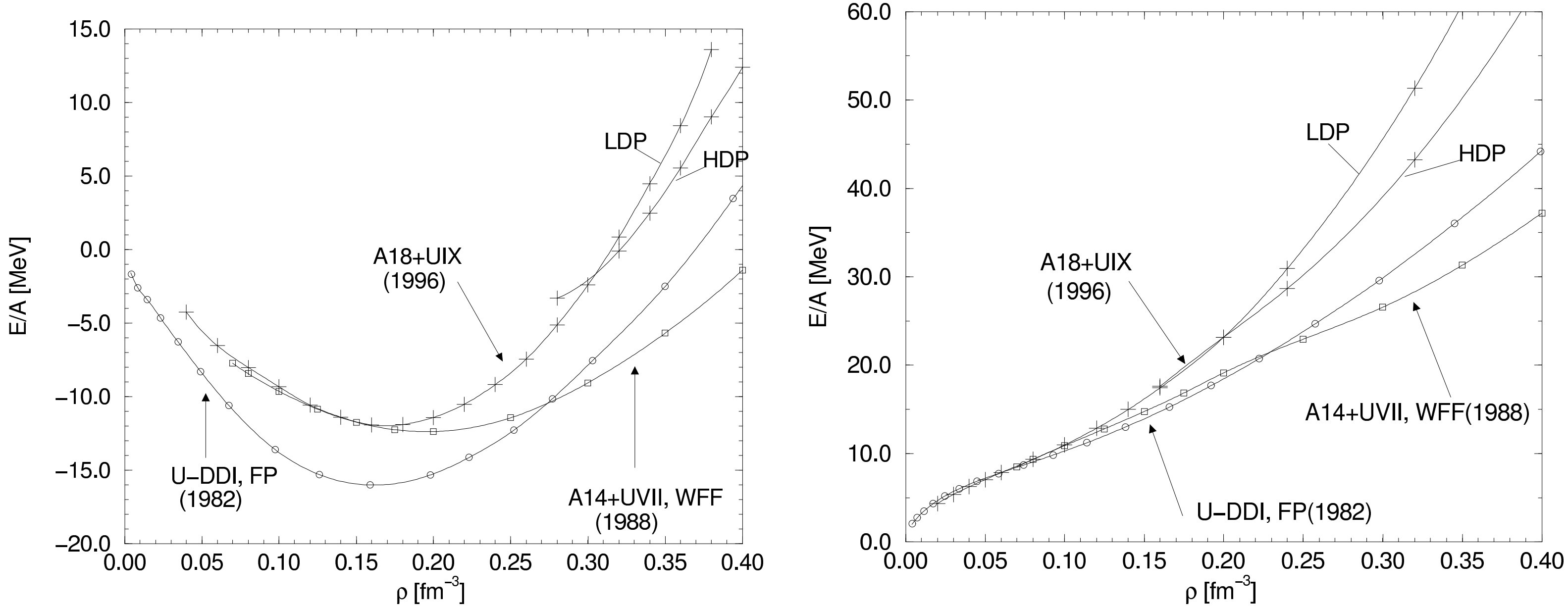
- the energies of nuclei with $A < 8$ have been obtained using the Green Function Monte Carlo (GFMC) method to solve the Schrödinger equation (Pieper & Wiringa, 2001)



$$\Delta = \frac{|E_{GFMC} - E_{exp}|}{E_{exp}} \lesssim 5\%$$

Results of NMBT (continued)

- energy per baryon of nucleon matter (Akmal, Pandharipande & Ravenhall, 1998)



- note: calculated relativistic boost corrections to v_{ij} are $\sim 7\%$ in ^{4}He and $\sim 12-15\%$ in neutron matter @ $\rho \sim (1-2)\rho_0$

β-stable nucleon matter

- n, p and e in equilibrium with respect to the process

$$n \leftrightarrow p + e$$

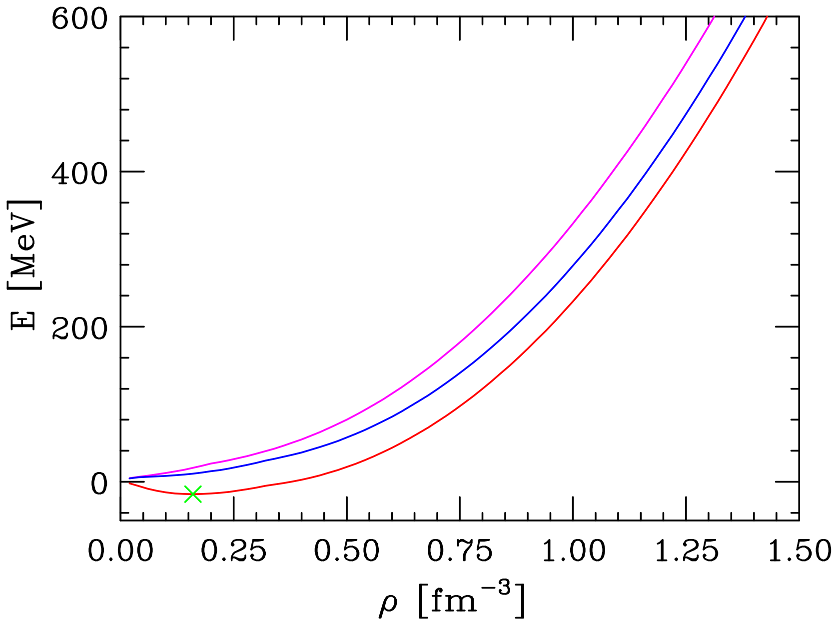
$$\mu_n - \mu_p = \mu_e \qquad\qquad \rho_p = \rho_e$$

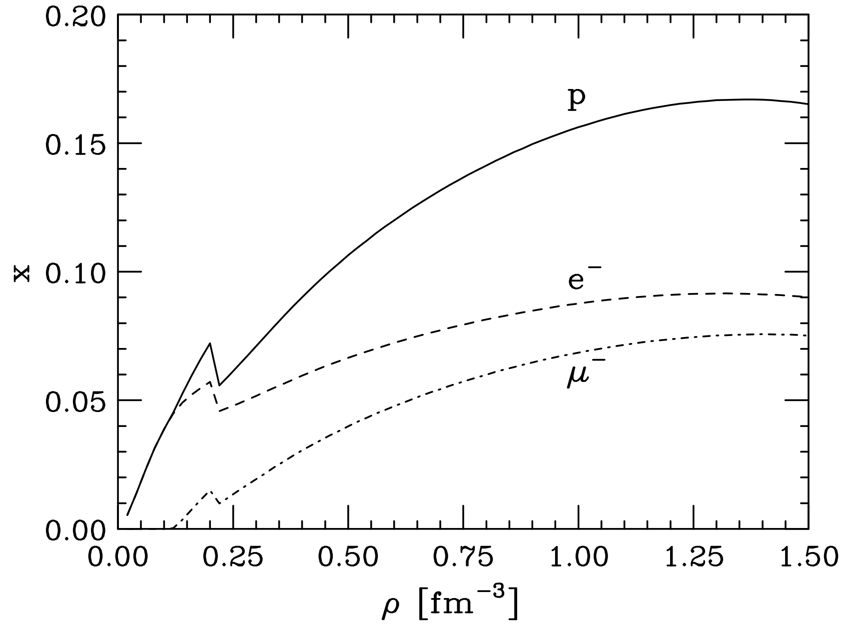
equilibrium   charge neutrality

- as ρ increases (typically just above ρ_0) $\mu_e > m_\mu$ and muons also appear through $n \leftrightarrow p + \mu$, with $\mu_n - \mu_p = \mu_e = \mu_\mu$ and $\rho_p = \rho_e + \rho_\mu$
- at any given ρ equilibrium and charge neutrality determine the proton fraction $x_p = \rho_p/\rho$

Results of NMBT (continued)

- β-stable matter of n, p, e and μ: energy-density and proton fraction as a function of baryon number density (Akmal, Pandharipande & Ravenhall, 1998)





More complex models of hadronic matter

- appearance of heavier strange baryons through

$$p + e \longrightarrow \Lambda^0 + \nu_e$$

$$n + e \longrightarrow \Sigma^- + \nu_e$$

  may become energetically favoured at large ρ

- example: equilibrium in matter including

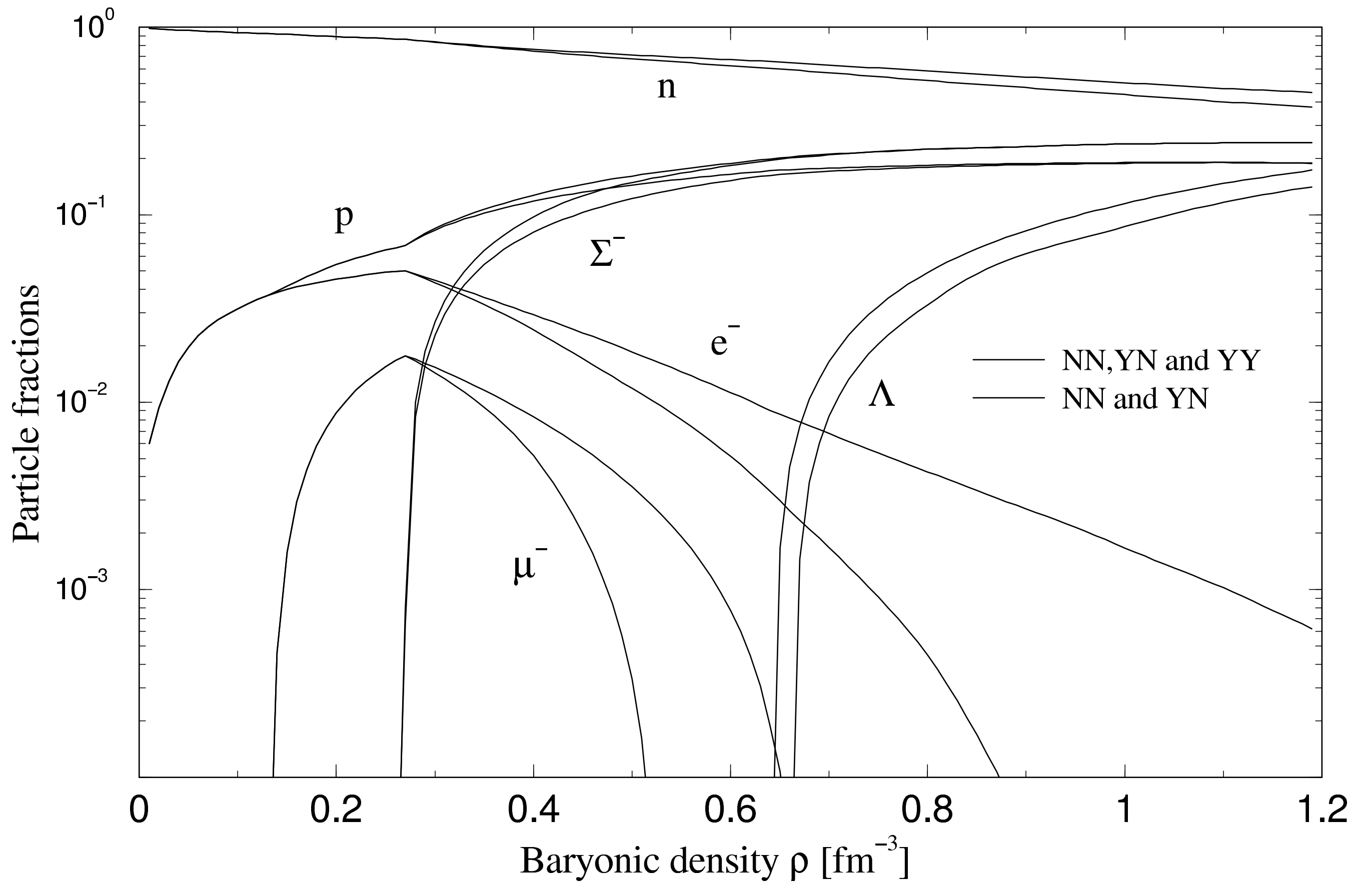
$$n\ ,\ p\ ,\ e\ ,\ \mu,\ \Lambda^0\ ,\ \Sigma^-$$

  requires

$$\mu_n = \mu_{\Lambda^0} \qquad \mu_p = \mu_n - \mu_e \qquad \mu_{\Sigma^-} = \mu_n + \mu_e$$

Including hyperons in NMBT

- Problem: YN and YY interactions largely unknown
- Example: using the Nijmegen potential Σ^- appearance has been predicted at $\rho \sim 2\rho_0$ (Vidana et al, 2000)



EOS of hadronic matter

- for any given value of baryon number density the EOS is determined by minimizing the energy E

- energy density and pressure are trivially related to E through

$$\epsilon(\rho) = \frac{E}{V} = \rho\,\frac{E}{N}$$

$$P(\rho) = \rho^2\,\frac{\partial}{\partial\rho}\frac{E}{N}$$

- EOS can be classified according to their "stiffness"

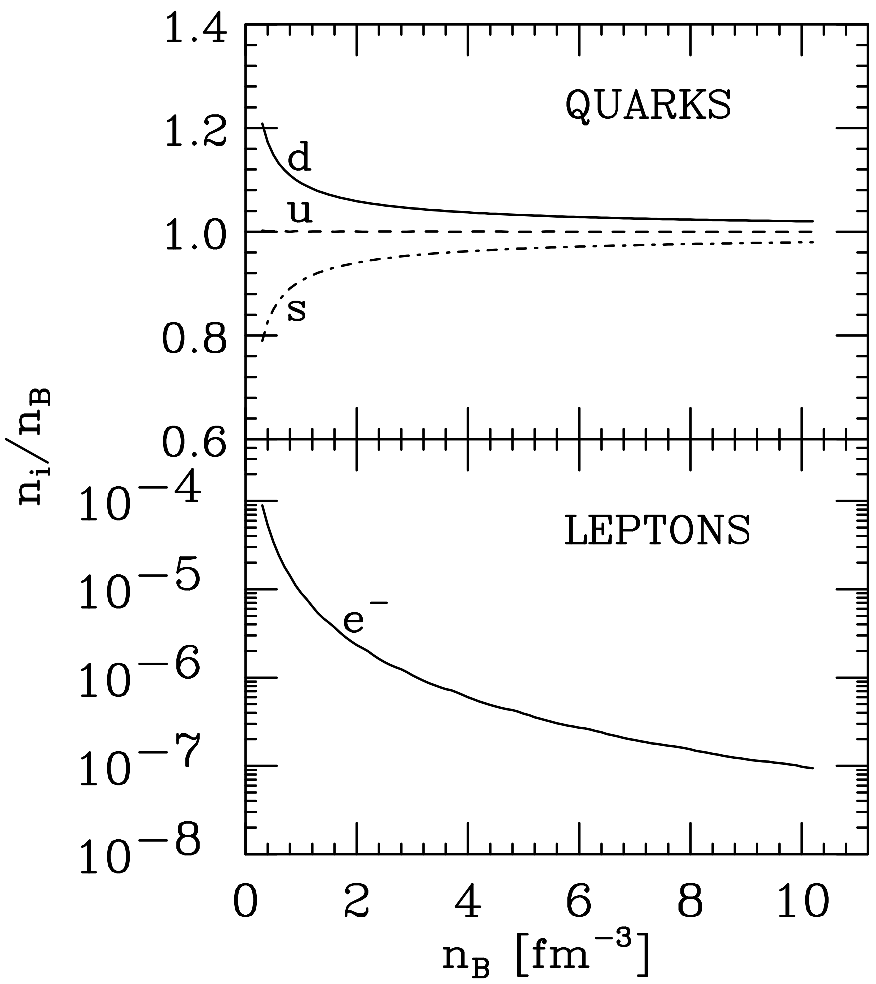
$$\Gamma = \frac{d\ln P}{d\ln\rho}$$

From hadronic matter to quark matter

- @ $\rho \sim 10^{15}$ g/cm^3 quarks are no longer clustered into nucleons or hadrons
- the EOS of quark matter can be estimated using the bag model
  - noninteracting quarks confined to a finite region of space (the bag)
  - bag volume limited by a pressure B (the bag constant)
- example: noninteracting massless quarks

$$\epsilon = B + \frac{3}{4\pi^2}\sum_{i=1}^{N_f} p_{F_i}^4 \quad , \quad P = -B + \frac{1}{4\pi^2}\sum_{i=1}^{N_f} p_{F_i}^4$$

Quark and lepton densities as a function of ρ



- baryon number density

$$\rho = 3\sum_q \rho_q$$

- charge neutrality

$$\sum_q Q_q \rho_q = \sum_\ell Q_\ell \rho_\ell$$

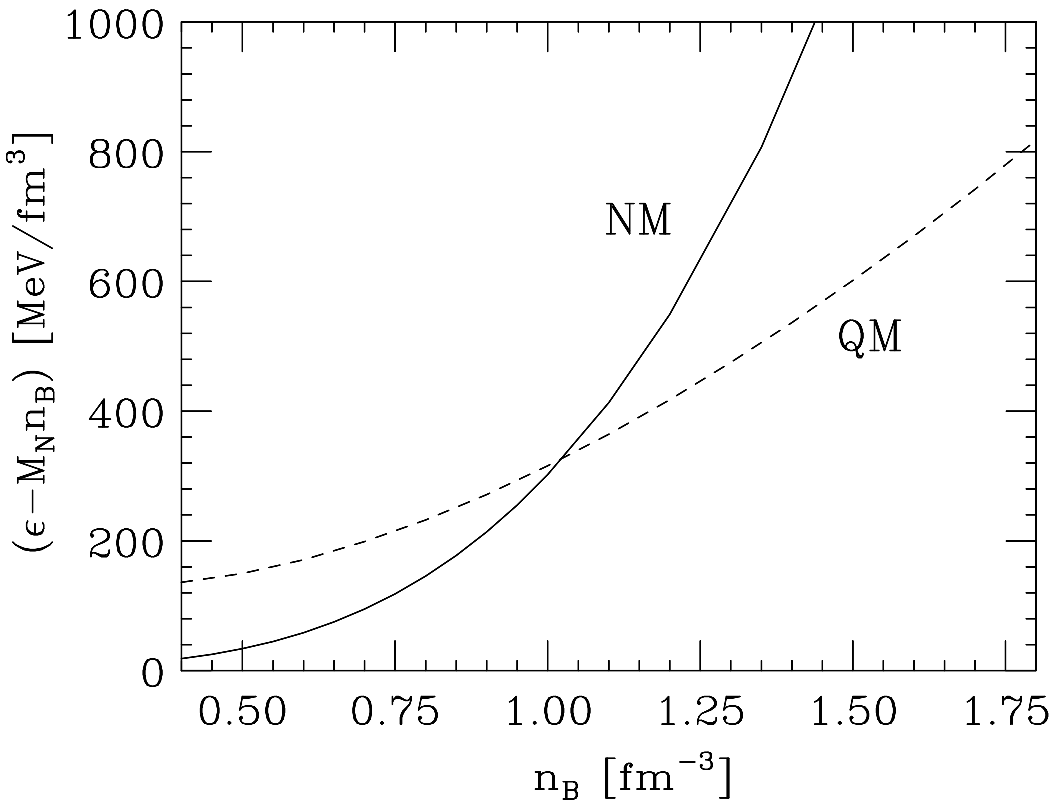
- weak equilibrium

$$\mu_d - \mu_u = \mu_e = \mu_\mu$$

$$\mu_s = \mu_d$$

From hadronic matter to quark matter (continued)

- nucleon matter vs quark matter



- as $\rho \longrightarrow \infty$

$$\left(\frac{E}{N}\right)_{NM} \propto \rho$$

$$\left(\frac{E}{N}\right)_{QM} \propto \rho^{1/3}$$

- The transition takes place either at constant pressure or with formation of a mixed phase

EOS and properties of nonrotating neutron stars

- given the EOS, mass and radius of a nonrotating star can be obtained from the Tolman-Oppenheimer-Volkov (TOV) equations (hydrostatic equilibrium + Einstein eqs)

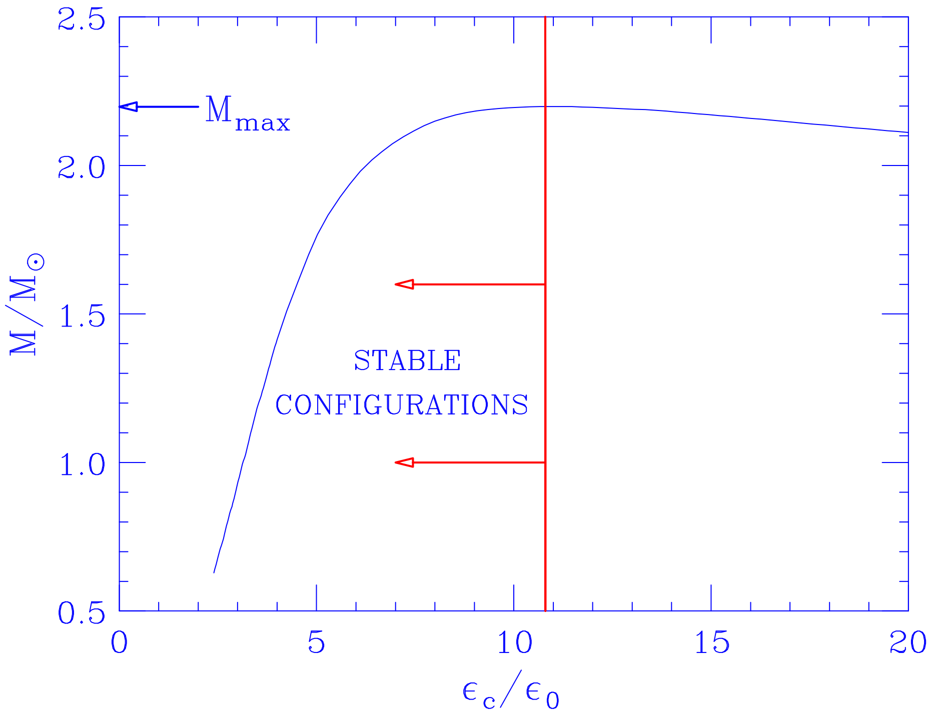
$$\frac{dP(r)}{dr} = -G \, \frac{\left[\epsilon(r) + P(r)/c^2\right]\left[M(r) + 4\pi r^2 P(r)/c^2\right]}{r^2\left[1 - 2GM(r)/rc^2\right]}$$

$$M(r) = 4\pi \int_0^r r'^2 dr' \epsilon(r') \quad , \quad \epsilon(r=0) = \epsilon_c$$

- solving TOV equations one obtains a set of neutron star configurations, characterized by the radius R, defined through $P(R) = 0$, and the mass $M = M(R)$

Maximum neutron star mass

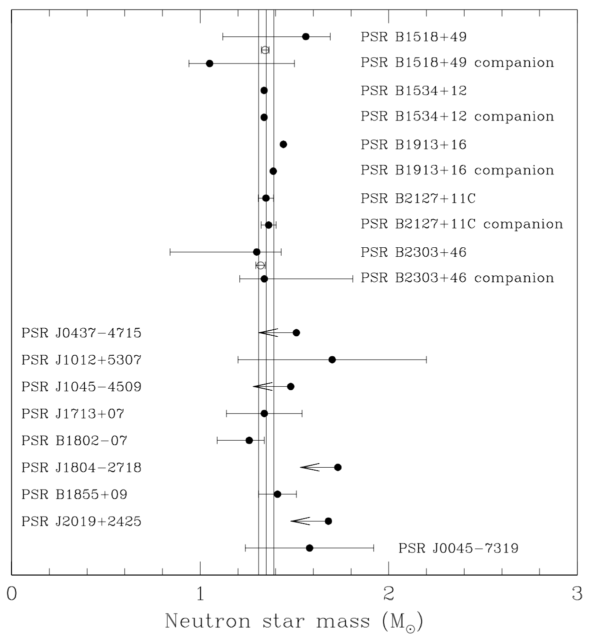
- typical mass-central energy-density curve



- maximum mass given by

$$M_{max} = M(\bar{\epsilon}_c) \quad , \quad \left(\frac{dM}{d\epsilon_c}\right)_{\epsilon_c=\bar{\epsilon}_c} = 0$$

Compilation of measured neutron star masses



- Hulse & Taylor: binary pulsar $M = 1.441 \pm .0007 M_\odot$
- $\sim$ 20 accurate measurements of bynary systems yield $M = 1.35 \pm 0.1 M_\odot$
- a recent determination of the mass of the X-ray pulsar Vela X-1 yields $M = 1.87^{+0.23}_{-0.17} M_\odot$

Predicted maximum masses vs data

- bottom line: most EOS support a stable neutron star of mass $\sim 1.4\ \mathrm{M}_\odot$
- EOS based on nucleonic degrees of freedom predict maximum masses typically $\geq 2\ \mathrm{M}_\odot$
- the presence of a core of deconfined quark matter lowers the maximum mass by $\sim 10\%$
- the appearance of hyperons makes the EOS significantly softer, typically leading to $M_{max} < 1.5 M_\odot$
- if confirmed, the measured mass of Vela X-1 will rule out soft EOS, thus leaving little room for the occurrence of "exotic" matter

New observational developments

- Iron and Oxygen transitions recently observed in the spectra of 28 bursts of the X-ray binary EXO0748-676 correspond to a gravitational redshift $z = 0.35$ (Cottam et al, 2002)

- z is related to the mass-radius ratio through

$$R(1+z) = R\left(1 - \frac{2GM}{c^2}\frac{1}{R}\right)^{-1/2}$$

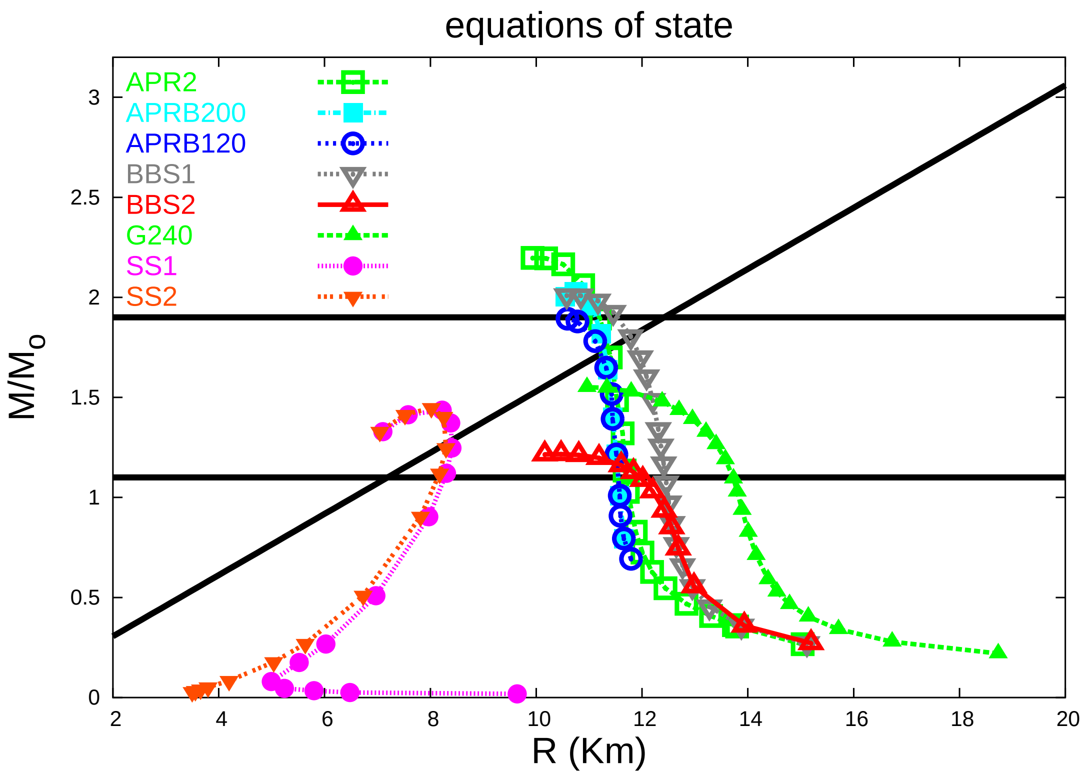
yielding

$$\frac{M}{R} = 0.153\,\frac{M_\odot}{\mathrm{Km}}$$

i.e.

$$1.4 \lesssim M/M_\odot \lesssim 1.8 \iff 9 \lesssim R \lesssim 12\ \mathrm{Km}$$

Predicted M/R ratios vs data



- ▷ APR2, BBS1: nucleons only, nonrelativistic ; APRB120, APR2B200: APR2 + quark matter core
- ▷ BBS2: nucleons + hyperons, nonrelativistic ; G_{240}: nucleons + hyperons, relativistic mean fi eld ; SS1, SS2: strange stars (with and without crust)

Gravitational waves from neutron stars

- a neutron star emits GW at the (complex) frequencies of its quasi-normal modes
  - g-modes: main restoring force is the buoyancy force
  - p-modes: main restoring force is pressure
  - f-modes: intermediate between g- and p-modes
  - w-modes: pure space-time modes
  - r-modes: main restoring force is the Coriolis force

$$\{\omega_{gn}\} < \omega_f < \{\omega_{pn}\} < \{\omega_{wn}\}$$

- in newtonian theory the frequency of the f-mode is proportional to the average density of the star

GW emission and EOS

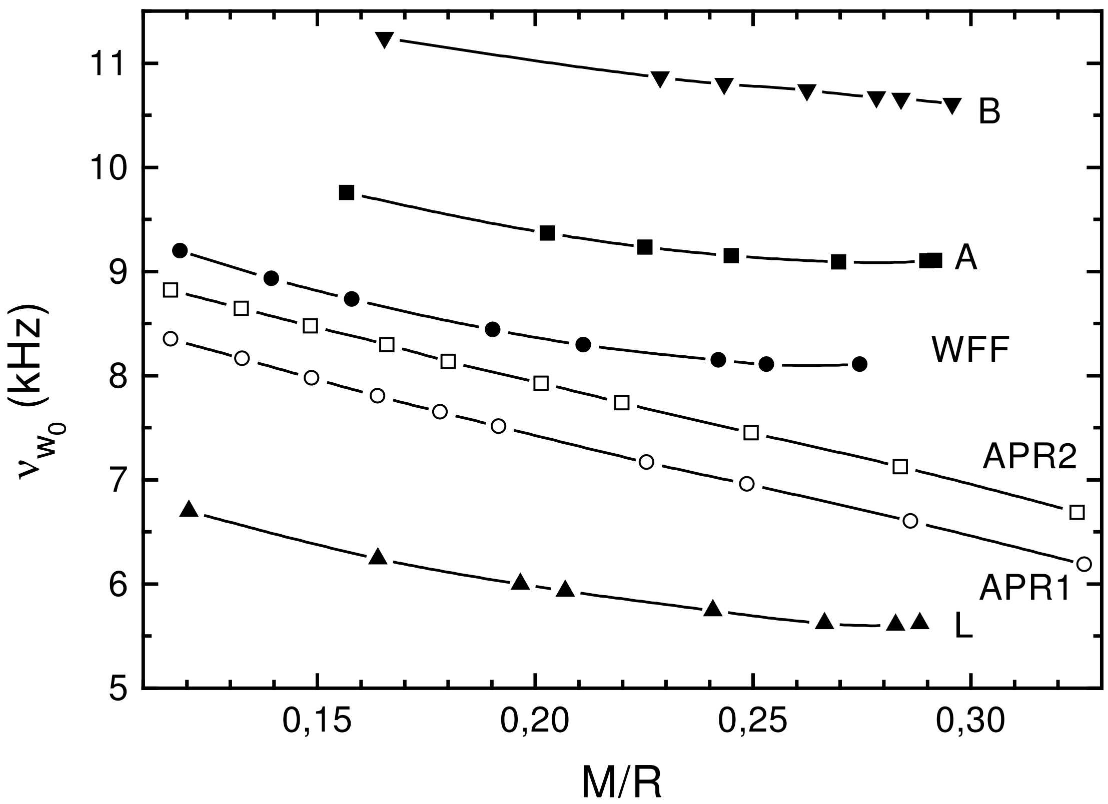
- how do neutron star oscillation modes associated with GW emission depend upon the EOS ?

- example: the frequencies of axial (odd parity) w-modes are eigenvalues of a Schrödinger-like equation, whose potential $V_\ell(r)$ explicitly depends upon the EOS

$$V_\ell(r) = \frac{\mathrm{e}^{2\nu(r)}}{r^3} \left\{\ell(\ell+1)r + r^3\left[\epsilon(r) - P(r)\right] - 6M(r)\right\}$$

$$\frac{d\nu}{dr} = -\frac{1}{\left[\epsilon(r) + P(r)\right]}\frac{dP}{dr}$$

GW emission and EOS (continued)



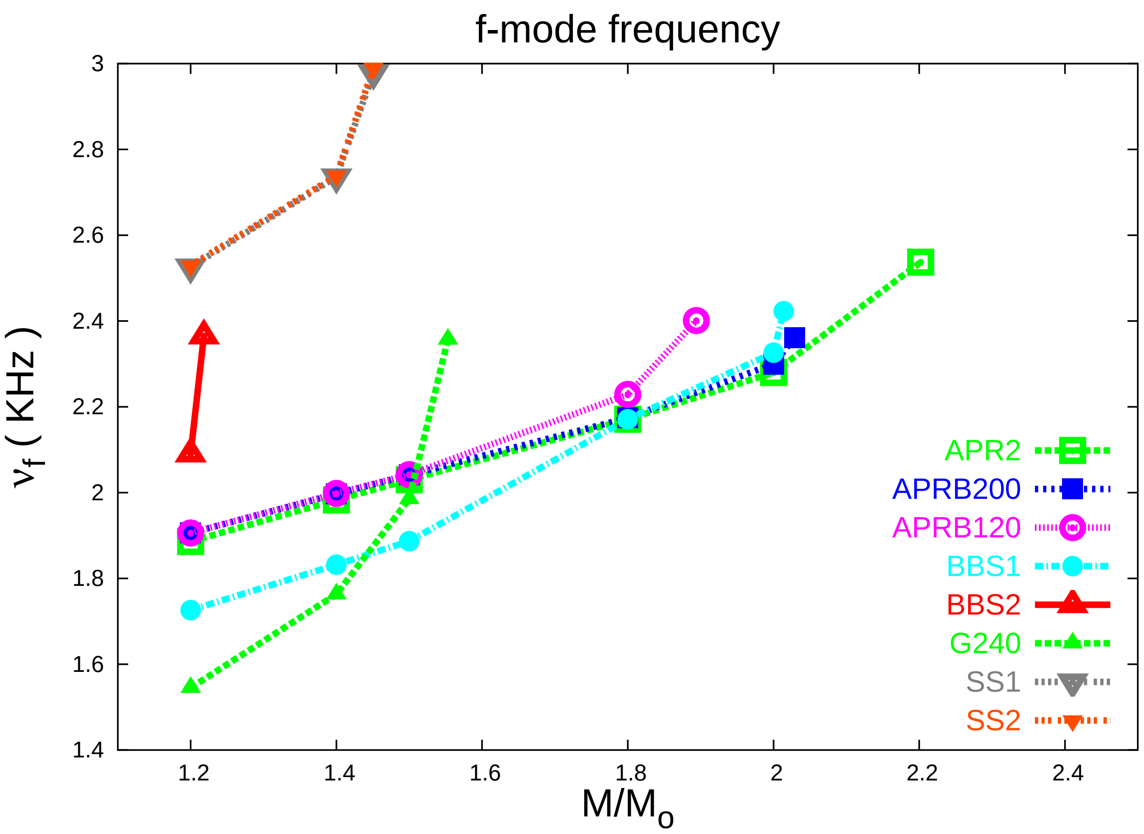
- ▷ frequency of the 1st w-mode vs star compactness (Benhar, Berti & Ferrari, 1999)

- ▷ the pattern of frequencies reflects the stiffness of the EOS. Softer EOS correspond to higher frequencies

- ▷ for a given EOS, the frequency depends weakly upon M/R

GW emission and EOS (continued)

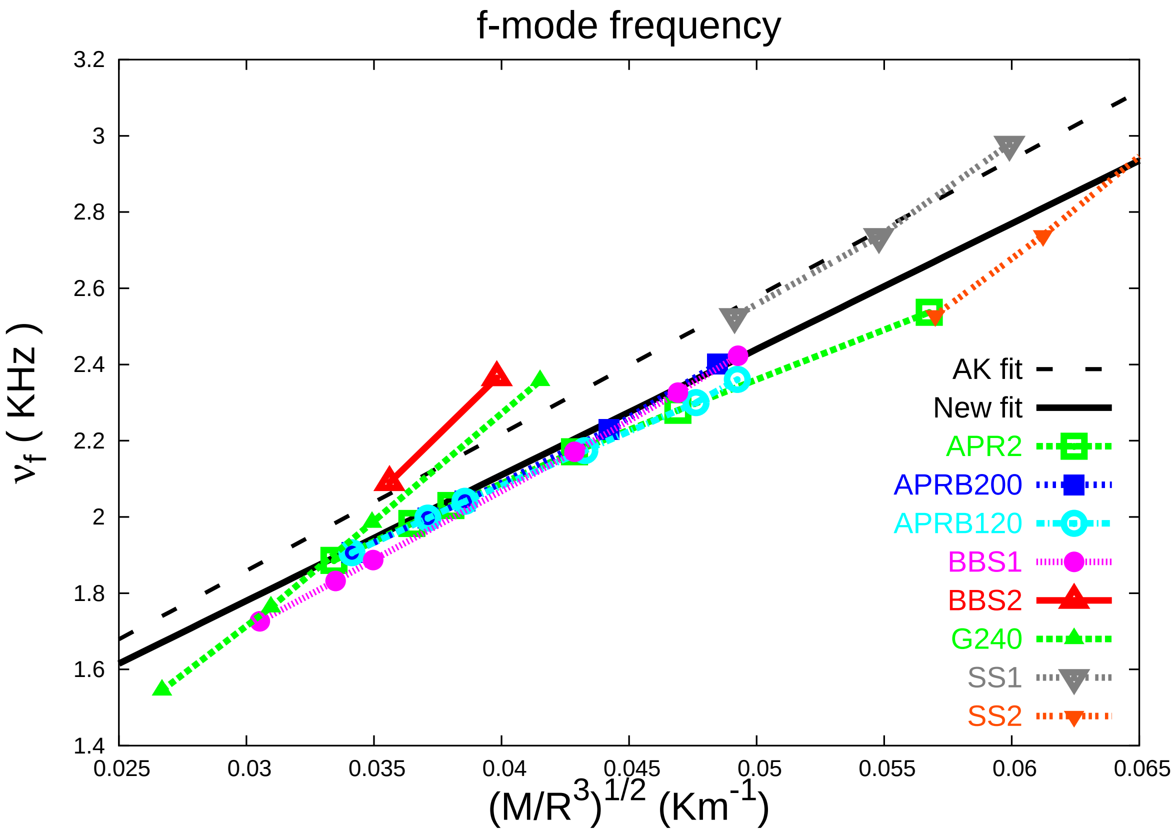
- f-mode frequency as a function of the neutron star mass (Benhar, Ferrari & Gualtieri, 2004)



- stars containing hyperons and strange stars have much higher frequencies

GW emission and EOS (continued)

- ▷ a set of empirical relations linking the mode frequencies to M and R can be inferred from the results of theoretical calculations (Benhar, Ferrari & Gualtieri, 2004)



$$\nu_f = a + b\sqrt{\frac{M}{R^3}}$$

$$a = 0.79 \pm 0.09 \text{ kHz}$$

$$b = 33 \pm 2 \text{ km kHz}$$

Extracting M and R from GW frequencies

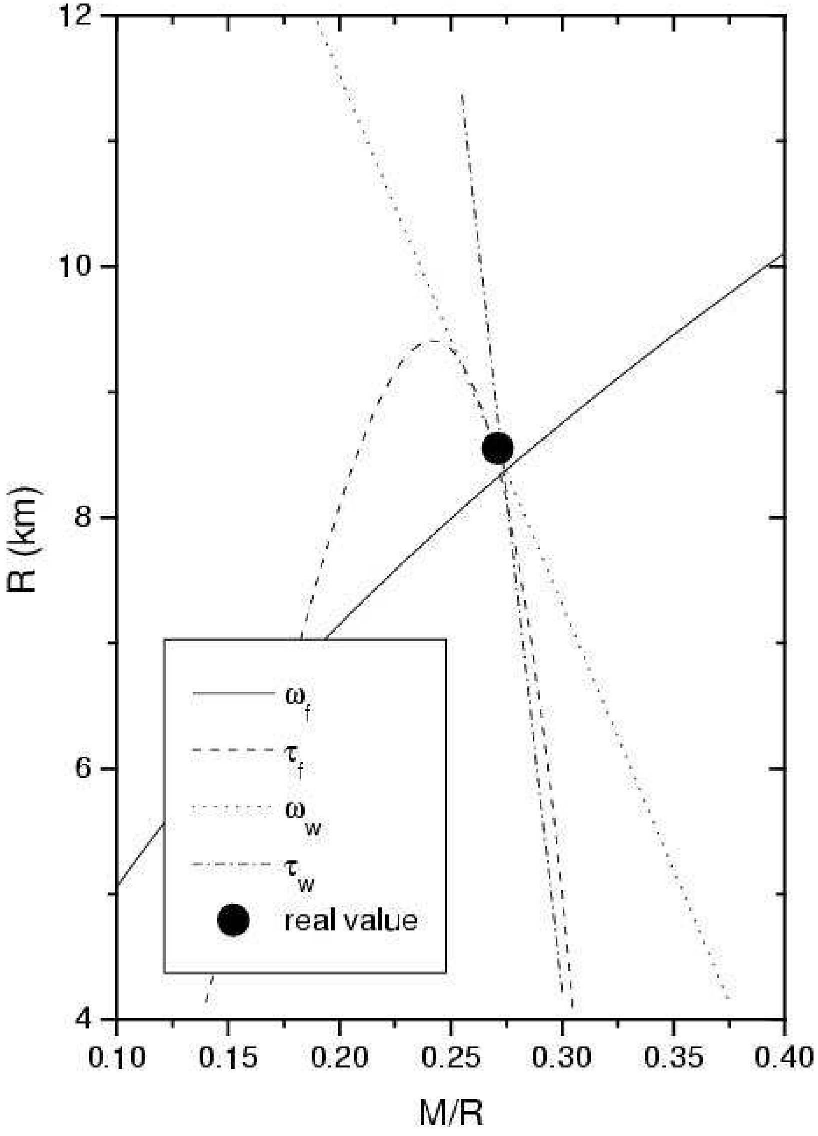
- empirical relations between frequencies and star parameters can also be obtained for the p- and w- modes. For example

$$\nu_w = \frac{1}{K} \left(a + b\frac{M}{R} \right)$$

$$\frac{1}{\tau_w} = 10^{-3}\, M \left[c + d\frac{M}{R} + e\left(\frac{M}{R}\right)^2 \right]$$

- symultaneous detection of GW signals associated with different modes would provide up to five equations for the two unknown R and M

A numerical experiment (Andersson & Kokkotas, 1998)



- select a model polytropic star and compute M and R
- compute frequency and damping time of the f-mode and the 1st w-mode
- plot the four lines corresponding to the empirical relations
- the intersection of the four lines gives the correct M and R with a few percent accuracy

Will GW from neutron stars ever be detected ?

- Assume that the f-mode of a neutron star with $\nu_f = 1.9\,\mathrm{kHz}, \tau_f = 0.184\,\mathrm{s}$ has been excited
- The signal emitted can be modeled as (Ferrari et al, 2003)

$$h(t) = \mathcal{A} e^{(t_{\mathrm{arr}} - t)/\tau_f} \sin\left[2\pi\nu_f\,(t - t_{\mathrm{arr}})\right] \;,$$

and the energy stored into the mode is

$$dE_{\mathrm{mode}} = \frac{\pi}{2}\,\nu^2\,|\,\tilde{h}(\nu)\,|^2\,dS d\nu$$

- Will the VIRGO interferometer be able to detect this signal ?

Detection of GW from neutron stars (continued)

- ▷ VIRGO noise power spectral density ($x = \nu/\nu_0$, $\nu_0 = 500$ Hz)

$$S_n(x) = 10^{-46} \cdot \left\{3.24[(6.23x)^{-5} + 2x^{-1} + 1 + x^2]\right\}\ Hz^{-1},$$

with $x = \nu/\nu_0$ and $\nu_0 = 500$ Hz

- ▷ Signal to noise ratio

$$SNR = 2\left[\int_0^\infty d\nu\, \frac{|\tilde{h}(\nu)|^2}{S_n(\nu)}\right]^{1/2}$$

- ▷ $SNR = 5$ requires $E_{\rm mode} \sim 6 \times 10^{-7}\ M_\odot$ for a source in our galaxy and $\sim 1.3\ M_\odot$ for a source in the VIRGO cluster

Conclusions

- Neutron star structure, reflected by the EOS, affects the frequencies of oscillations leading to GW emission
- Observation of GW emission from neutron stars may provide considerable new insight on the EOS of strongly interacting matter
- While the emitted signal is likely to be out of reach of the existing interferometers, second generation detectors, expected to be more sensitive at frequencies above $1 \div 2$ kHz, may be able to detect f-mode oscillations.
- Development of better theoretical models, particularly of "hybrid" stars, are strongly needed (critical densities for appearance of hyperons and quarks, role of color superconductivity, nature of the phase transitions ...)