Variational approach

Main problem

$$\langle H \rangle = \langle \Psi_{FG} | H | \Psi_{FG} \rangle \quad \text{large}$$

In G-matrix perturbation theory $H \to H_{eff}$ with $v \to G$

$$\langle \Psi_{FG} | H_{eff} | \Psi_{FG} \rangle$$

calculable

Alternative approach

$$|\Psi_{FG}\rangle \to |\Psi_{v}\rangle = F |\Psi_{FG}\rangle$$

$$= \prod_{j>i} f_{ij} |\Psi_{FG}\rangle$$

$$f_{ij} \xrightarrow[r_{ij} \to 0]{} 0$$

Analogy with BBG theory

$$\Psi \approx f \Phi$$

Cluster expansion

$$E_0 = \min_{\{F\}} \frac{\langle \Psi_0 | H | \Psi_0 \rangle}{\langle \Psi_0 | \Psi_0 \rangle}$$

$$= T_F + (\Delta E)_2 + (\Delta E)_3 + \dots$$

Underlying idea: f short ranged

$$\lim_{r_{ij} \to \infty} f_{ij} = 1$$

$$h_{ij} = f_{ij}^2 - 1 \qquad \lim_{r_{ij} \to \infty} h_{ij} = 0$$

Simple case $\qquad \{ [f_{ij}, f_k] = 0$

$$f_{ij} = f(r_{ij}) \quad [v_{ij}, f_{ij}] = 0$$

Potential energy involves $\langle \Phi\, v\, F^2 \Phi \rangle$

$$\prod f^2(r_{ij}) = \prod [1 + h(r_{ij})]$$

$$= 1 + \sum h_{ij} + \sum h_{ij} h_{jk}$$

$$+ \cdots$$

Better explanation

$$\langle \Psi_0 | \sum_{j>i} v_{ij} | \Psi_0 \rangle$$

$$= A(A-1) \langle \Psi_0 | v_{12} | \Psi_0 \rangle$$

$$= A(A-1) \langle \Phi_{FG} | v_{12} F^2 | \Phi_{FG} \rangle$$

$$= A(A-1) \{ \langle \Phi_{FG} | v_{12} [(1 + h_{12})$$

$$(A-2) h_{13} + (A-2) h_{13} h_{23}$$

$$+ \cdots] | \Phi_{FG} \rangle \}$$

$$= \langle v \rangle_2 + \langle v \rangle_3 + \cdots$$

- Convergence of cluster expansion depends on density.

dependent on ...

low order approximation
reasonable at $\rho \gtrsim \rho_0$

- At higher density FHNC summation.
- Form of f determined by Euler-Lagrange equations (conceptually similar to Bethe-Goldstone equation)
- FHNC employed to obtain
  WFF & APR
  equations of state.