Full EOS obtained combining
the EOSs describing different regions

outer crust
inner crust
outer core
inner core

- Neutron stars
- Hybrid stars $\begin{cases} \text{quark core} \\ \text{hyperon core} \end{cases}$

How do you match?

Common features of the
hybrid star EOS
→ stiffness

Matching

Equilibrium conditions

Ex NM & QM

Chemical potentials

$$\mu_B \quad \mu_Q$$

$$P^{NM}(\mu_B^{NM}, \mu_Q^{NM})$$

$$P^{QM}(\mu_B^{QM}, \mu_Q^{QM})$$

$$P^{NM}(\mu_B^{NM}, \mu_B^{QM}) = P^{QM}(\mu_B^{QM}, \mu_Q^{QM})$$

QM occupa una frazione X del volume totale

$$\epsilon = X \epsilon_{QM} + (1-X)\epsilon_{NM}$$

In case of a single

conserved quantity

→ Maxwell construction

In core of neutron star assume that the two phases be electrically neutral independently of one another in their own fraction of volume

$$\left(\frac{\partial \epsilon_{NM}}{\partial n}\right)_{n_A} = \left(\frac{\partial \epsilon_{QM}}{\partial n}\right)_{n_B}$$

$$\epsilon = \mu n - P$$

- chemical potential
  → angular coefficient
- pressure → (minus the) intercept

What now?

Equilibrium properties.

How do they depend on the EOS

Consider the simple case of hydrostatic equilibrium $\vec{\nabla} P = -\rho \vec{\nabla} \phi$

uniform distribution of matter, spherically symmetric

$$\frac{dP(r)}{dr} = -\rho(r) \frac{G M(r)}{r^2}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

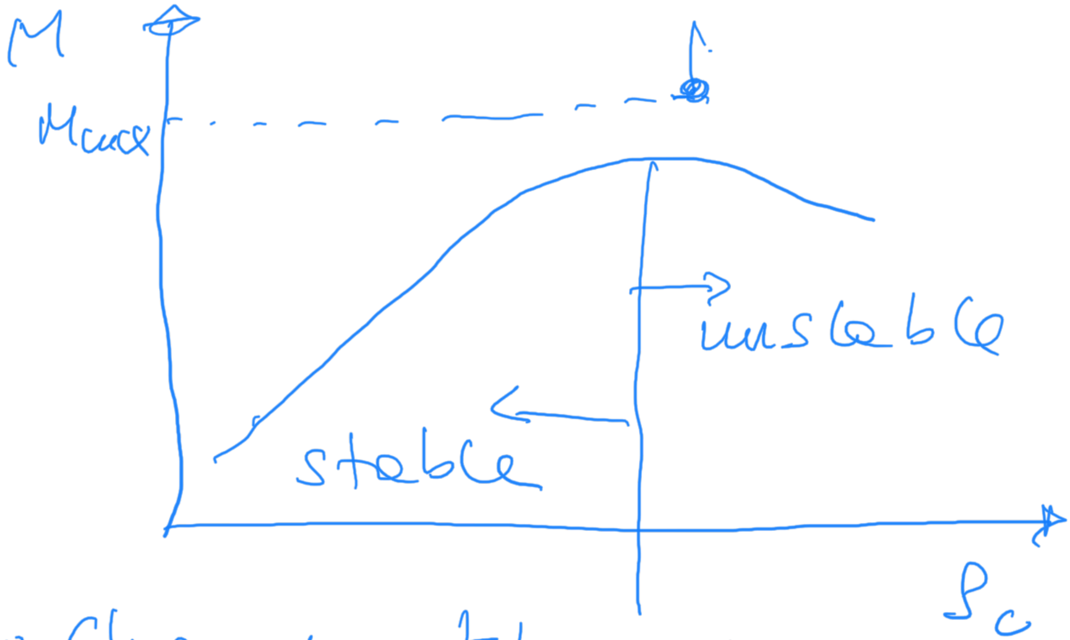
to be solved with the initial condition

at $r=0 \Rightarrow$ central density $\rho(r=0)=\rho_c$

Integrate outward till the point $r=R$ corresponding to $\rho(R)=0$

R radius of the star

Mass
$$M = M(R)$$



Mmax Chandrasekhar mass of white dwarfs

For neutron stars GR important

Continue hydro equilibrium with Einstein equations is

$$G_{\mu\nu} = -8\pi G T_{\mu\nu}$$

static, spherically symmetric spacetime: metric of grav. field

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$= e^{2\nu(r)} dt^2$$

$$- e^{-2\lambda(r)} dr^2 - r^2 d\theta^2$$

$$- r^2 \sin^2\theta \, d\phi^2$$

$$x \equiv (t, r, \theta, \phi)$$

$$T_{\mu\nu} = u_\mu u_\nu (\epsilon + P) - g_{\mu\nu} P$$

$$T = \begin{pmatrix} \epsilon e^{2\nu} & & & \\ & P e^{2\lambda} & & \\ & & P r^2 & \\ & & & P r^2 \sin^2\theta \end{pmatrix}$$

Eq. TOV

$$\frac{dP}{dr} = -\epsilon \frac{G M(r)}{r^2} [1][2][3]$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \epsilon(r)$$

(1), (2) $\to 0$ $\quad \frac{k_F}{m} \to 0$

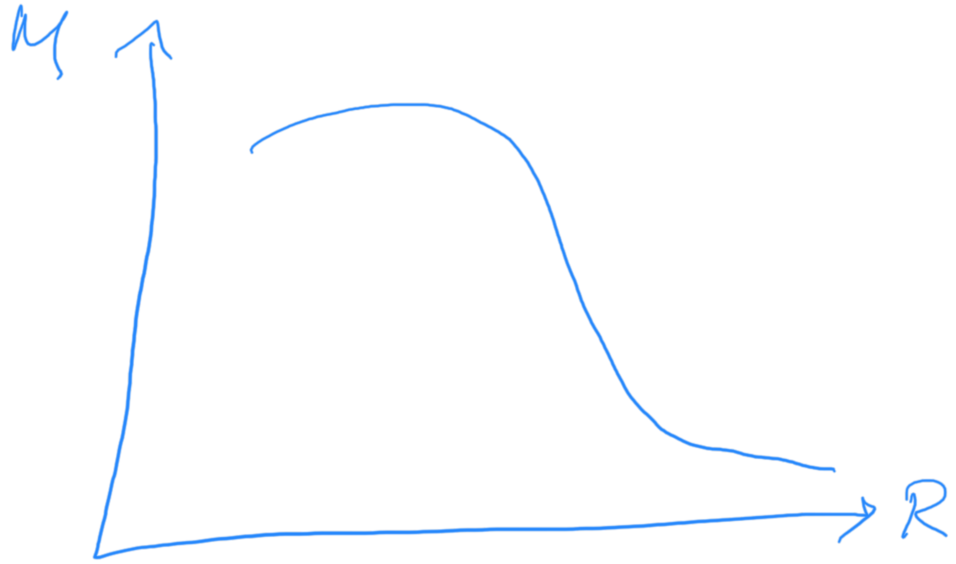
(3) spacetime curvature.

Same procedure as for white dwarfs to generate maximum mass

For any EOS

Maximum mass

Mass Radius relation



Ideally, observations of mass and radius may constrain EOS.

However: only masses are known to very good accuracy

$$\boxed{\text{Red shift } z}$$

binary system emitting X-rays oxygen and iron spectroscopic lines

$$R(1+z) = R\left(1 - 2G\underline{M}\right)^{-1/2}$$

$$\frac{M}{R} = 0{,}153 \frac{M_\odot}{km}$$

$$1{,}4 \leq M/M_\odot \leq 1{,}8$$

$$\Longrightarrow \quad 9 \leq R \leq 12 \text{ km}$$