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Realistic description of neutron-neutron correlation effects on the nuclear matrix element of neutrinoless double β decay

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Preface

In this work we are going to perform a numerical analysis of the effects of neutron-neutron correlation on the nuclear transition matrix element of neutrinoless double beta decay process.

In a double beta decay process a very long-lived nucleus with mass number A and atomic number Z disintegrates spontaneously in a nucleus with the same mass number and atomic number decreased by two units, with emission of two electrons and zero or two anti-neutrinos. In the standard model of particle physics this process is mediated by the weak interaction and it always shows two anti-neutrinos in the final state, but if we consider the neutrino as a “Majorana particle” which means that the same field describes the particle and the antiparticle we can have a final state without neutrinos, in the Feynman diagram of this process, we have a neutrino propagator which became part of the nuclear sector of the transition matrix element, studying it, is possible to extract information on the magnitude of the lightest neutrino mass, which is actually unknown.

The nuclear transition matrix element is a complex object to deal with because of the nature of strong interaction between the nucleons, it is attractive to long distances and highly repulsive for short length. A pure Shell Model approach is incorrect, we need to add this short range correlation effects modifying the nucleons wave functions. The main task of this thesis is to numerically study the effects of this correlations on the neutrinoless double beta decay nuclear matrix element for a ^{48}Ca nucleus which it's going to be used in some new experiment.

After a brief overview on neutrino physics we will study beta decay phenomenology to see how it can be used to increase our knowledge on neutrino masses. Then we move to nuclear physics to study how we could modellize the nucleus in a many bodies physics approach using the correlated wave function formalism. After that we will be able to write down the analytical expression of the nuclear matrix elements whose will be numerically analyzed and finally we will show our results.

Contents

Introduction

Neutrinoless double beta decay ($2\beta0\nu$) is a hypothetical nuclear transition in which two neutrons undergo β-decay simultaneously without emission of neutrinos, if realized in nature this will prove that neutrinos are *Majorana particles* and so, if ν is the field describing the neutrino it will be equivalent to his charge-conjugated field, therefore: $\nu = \nu^c$ such as particle would constitute a new kind of matter because no elementary Majorana particles have been observed so far. Also the observation of $\beta\beta0\nu$ will prove that total lepton number is not a conserved quantum number in physical phenomena and this could be linked to the cosmic asymmetry between matter and antimatter. But the main interest for neutrinoless beta decay phenomenology is in the possibility, as will be shown in the following chapter, to estimate the value of neutrino masses.

In this thesis we are going to numerically study the nuclear transition matrix element for neutrinoless double beta decay in a ^{48}Ca nucleus, in particular we focus on the effects of the neutron-neutron correlation on it.

The first chapter is dedicated to an overview on neutrino physics and summarize our knowledge of this particles, we are going to explain the neutrino oscillations phenomena and why it proves that neutrinos are massive particles, then we'll see a way to add massive neutrinos to the standard model or particles physics (the so called see-saw mechanism). Finally we shown the actual situation of experimental measurement on neutrino properties.

The second chapter talks about beta decay phenomenology, it shows the Lagrangian responsible for nuclear beta decays and how to compute the decay width for single beta decay process, two neutrinos double beta decay and for neutrinoless double beta decay which is the case of our interest. We can see that neutrinoless double beta decay half time is given by an effective mass term, a phase space term which is discussed in some details and the nuclear transition matrix element term which will be the main task of the following chapters.

In the third chapter we focus on nuclear dynamics, we see that it is a complex many body problem due to the nature of the nuclear forces between the nucleons, and that we need some approximation and assumption to solve it. We are going to use the simplest approach is the *nuclear shell model* which is based on a mean field Hartree-Fock Hamiltonian, we assume that the motion of each nucleon isn't influenced by the others, but, this takes to some disagreement with experimental data, and so, we must move to a correlated wave functions formalism.

The analytic form of the nuclear matrix elements that we want to compute is calculated in chapter four. Initially we give the general expression of them using a pure shell model approach making use of 9-j symbols and Talmi-Moshinsky brackets express those matrix element in the center of mass and relative motion frame. The we move to a correlated wave function formalism and we see how the addition of correlations changes the nature of Fermi and Gamow-Teller nuclear transition.

In chapter five we are going to show details of our numerical computation. First of all we choose the nucleus to be used in the calculation, we choose ^{48}Ca because it has a simple shell model structure, then, after comparing shell model results with experimental data we choose the subset of Hilbert states which participates to the decay, this will give us some bound for the quantum numbers involved. After that we are going to show how we obtained the analytical form for the correlation functions. And finally we are going to explain how we implement the numerical computation.

On the final chapter we present the numerical results we obtained during our analysis...

Throughout this Thesis we always use a system of units in which $\hbar = c = 1$ where $\hbar$ is Plank's constant and c is the speed of light.

Chapter 1

Neutrino physics

Neutrinos [1] are electrically neutral, weakly interacting elementary fermion $S = 1/2$. Because they are affected only by the weak interaction, they are able to travel great distances through matter without interact with it. Neutrinos are produced by radioactive decay, or nuclear reactions such as those that take place in stars nuclei, in nuclear reactors, or when cosmic rays hit atmospheric atoms. About 65 billion solar neutrinos per second pass through every square centimeter perpendicular to the direction of the Sun in the region of the Earth. The neutrino existence was postulated first by Wolfgang Pauli in 1930 to explain how beta decay could conserve energy. In 1942 Wang Ganchang first proposed the use of beta-capture to experimentally detect neutrinos, then in 1956 Cowan Reines et. al. published confirmation that they had detected the neutrino, a result that was rewarded almost forty years later with the 1995 Nobel Prize.

In the electroweak interactions standard model, neutrinos are consider as a massless lepton described by a two components spinor with definite chirality (left-handed). But neutrinos oscillations experiment have proved that neutrinos are massive particles (see [2]) and so, they are the lightest massive elementary fermions. We also known that there are only tree

[1]the name was proposed by Enrico Fermi

light[2] active neutrinos families. So, standard model must be extended to include massive neutrino, this can be done in two ways as shown in [1]. If we want neutrinos to be Dirac particles we must add to the standard model Lagrangian a neutrino mass term similar to the ones used for leptons and quarks:

$$\mathcal{L}_D = -m_D \left(\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L\right) \tag{1.1}$$

where ν_R and $\bar{\nu}_L$ are respectively the positive helicity neutrino field and the negative helicity anti-neutrino field that we must add because neutrinos are now massive particles, and $m_D = yv/\sqrt{2}$, with y dimensionless Yukawa coupling coefficient and $v/\sqrt{2}$ is the vacuum expectation value of the neutral Higgs field after electroweak symmetry breaking. Unlike the massless neutrino case, in which we had only the two-component spinor ν_L, now we have four independent components two from ν_L and two from ν_R.

The second way to add neutrino mass terms to the standard model Lagrangian in unique to neutrinos. Has said by Majorana in [5], for neutral particles, one can remove two degrees of freedom by imposing the *Majorana condition*:

$$\nu = \nu^c \tag{1.2}$$

where $\nu^c = C\bar{\nu}^T$ is the Charge-conjugated of the field ν. The Majorana condition implies that

$$\nu_R = (\nu_L)^c \tag{1.3}$$

this result can be obtained by decomposing both left-hand and right-hand side of eq.(1.2) and it proves that the positive chirality component of the Majorana neutrino ν_R depends on its negative chirality counterpart ν_L. We can use this result in the Dirac mass term and obtain the Majorana mass term:

$$\mathcal{L}_L = -\frac{1}{2} m_L \left(\bar{\nu}_L (\nu_L)^c + (\bar{\nu_L})^c \nu_L\right) \tag{1.4}$$

[2] $m_\nu < m_Z/2$

where m_L is a free parameter with dimension of mas. This Lagrangian mass term it's called *negative chirality mass term* and implies the existence of a weak isospin triplet scalar (a Higgs triplet), with neutral component acquiring a non-vanishing vacuum expectation value after electroweak symmetry breaking. If positive chirality fields also exist this is not the only possibility, in this case we may also construct a second Majorana mass term, a *positive chirality mass term*:

$$\mathcal{L}_R = -\frac{1}{2} m_R \left(\bar{\nu}_R (\nu_R)^c + (\bar{\nu_R})^c \nu_R \right) \tag{1.5}$$

In the standard model, right-handed fermion fields are weak isospin singlets, ad a consequence, the mass parameter m_R is not connected to a Higgs vacuum expectation value, and could be arbitrarily high.
All tree mass term convert negative chirality states into positive chirality ones. Chirality is therefore not a conserved quantity in both cases. Majorana mass terms also convert particles into their own antiparticles and therefore they are forbidden for charged particles because of the charge conservation. But it's not over, processes involving Majorana mass terms violate the conservation of standard model total lepton number by two units, which is no more a good quantum number.

And so, the most general mass term will have the following form:

$$\begin{aligned} \mathcal{L}_{mass} &= \mathcal{L}_D + \mathcal{L}_L + \mathcal{L}_R = \\ &= -m_D \left(\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L \right) - \frac{1}{2} m_L \left(\bar{\nu}_L (\nu_L)^c + (\bar{\nu_L})^c \nu_L \right) - \frac{1}{2} m_R \left(\bar{\nu}_R (\nu_R)^c + (\bar{\nu_R})^c \nu_R \right) \end{aligned} \tag{1.6}$$

which can be rewritten as:

$$\mathcal{L}_{mass} = -\frac{1}{2} \begin{pmatrix} \bar{\nu}_L^c & \nu_R \end{pmatrix} \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \bar{\nu}_R^c \end{pmatrix} + h.c. \tag{1.7}$$

We can see, that, if $m_D \neq 0$ the fields ν_L and ν_L^c have not definite mass. To obtain definite mass fields we need to diagonalize the mass matrix by requiring that:

$$U^T \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \tag{1.8}$$

we can parametrize U as a product between a diagonal matrix and a rotation matrix:

$$U = \mathcal{R}(\theta)\rho = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix} \tag{1.9}$$

with: $\rho_k \in \mathcal{C}$ and $|\rho_k|^2 = 1$. $\mathcal{R}$ matrix is obtained by imposing:

$$\mathcal{R}^T \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \mathcal{R} = \begin{pmatrix} m_1' & 0 \\ 0 & m_2' \end{pmatrix} \tag{1.10}$$

And so, we have:

$$\tan(2\theta) = \frac{2m_d}{m_L - m_R} \qquad m_{1,2}' = \frac{1}{2}\left[m_L + m_R \pm \sqrt{(m_L - m_R)^2 - 4m_D^2}\right] \tag{1.11}$$

The phases in ρ matrix are chosen due to have real and positive physical masses.

$$U^T \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} U = \rho^T \mathcal{R}^T \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \mathcal{R}\rho = \begin{pmatrix} \rho_1^2 m_1' & 0 \\ 0 & \rho_2^2 m_2' \end{pmatrix} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \tag{1.12}$$

In eq. 1.6 we have tree different masses which actually are parameters of our theory. In order to be logically consistent with the standard model this parameters must satisfy some relations. In the standard model ν_L is part of an isospin doublet with $I_3 = +1/2$ and so $\mathcal{L}_L$ is an isospin triplet, therefore , is we want to maintain the validity of standard model we have to put: $m_L = 0$. The two remaining mass term are permitted because $\mathcal{L}_R$ is an isospin singlet and $\mathcal{L}_D$ is generated by the Higgs mechanism. Imposing the condition $m_L = 0$ and $|m_D| << m_R$ [3] we obtain:

$$m_1 \approx \frac{m_D^2}{m_R} << |m_D| \quad , \quad m_2 \approx m_R \quad , \quad \tan(\theta) \approx \frac{m_D}{m_R} \ll 1 \quad , \quad \rho_1^2 = -1 \tag{1.13}$$

[3] right-handed neutrinos are not observable in the standard model

this is the so called *see-saw mechanism* for the generation of neutrino masses: to a very massive ν_2 correspond a ν_1 with mass to much smaller than the corresponding lepton. In this conditions we have a mixing angle θ very small and this implies that only the light neutrino participates to the weak interactions, the heavy one is practically sterile.

1.1 Neutrino's mass mixing

Until now, we studied the case of a single neutrinos flavour, but the standard model involves tree different neutrino flavours participating to the weak interactions. Let's consider tree left-handed fields: ν_{eL} , $\nu_{\mu L}$ and $\nu_{\tau L}$ describing flavour neutrinos and the tree corresponding right-handed fields ν_{eR} , $\nu_{\mu R}$ and $\nu_{\tau R}$.[4] The Lagrangian mass term written in the previous section, in the case of tree neutrino families becomes:

$$\mathcal{L}_{mass} = \mathcal{L}_D + \mathcal{L}_L + \mathcal{L}_R \tag{1.14}$$

with:

$$\mathcal{L}_D = -\sum_{\alpha\beta} \bar{\nu}_{\alpha R} M^D_{\alpha\beta} \nu_{\beta L} + h.c. \tag{1.15}$$

$$\mathcal{L}_R = -\frac{1}{2}\sum_{\alpha\beta} \bar{\nu}^c_{\alpha R} M^R_{\alpha\beta} \nu_{\beta R} + h.c. \tag{1.16}$$

$$\mathcal{L}_L = -\frac{1}{2}\sum_{\alpha\beta} \bar{\nu}^c_{\alpha L} M^L_{\alpha\beta} \nu_{\beta L} + h.c. \tag{1.17}$$

$$\tag{1.18}$$

The index α and β runs over the neutrino flavours (e,μ,τ), M^D, M^R and M^L are 3×3 symmetric complex matrix. Regrouping the left-handed fields:

$$N_L = \begin{pmatrix} \nu_L \\ \bar{\nu}^c_R \end{pmatrix} \tag{1.19}$$

[4]The number of right-handed fields cannot be determined in a experimental way because they do no interact with matter. In this work we assume that there are only tree of them

with:

$$\nu_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} \qquad \nu_R^c = \begin{pmatrix} \nu_{eR}^c \\ \nu_{\mu R}^c \\ \nu_{\tau R}^c \end{pmatrix} \tag{1.20}$$

we can rewrite the mass term in this way:

$$\mathcal{L}_{mass} = \bar{N}_L M N_L = \begin{pmatrix} \bar{\nu}_L & \bar{\nu}_R^c \end{pmatrix} \begin{pmatrix} M^L & (M^D)^T \\ M^D & M^L \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} \tag{1.21}$$

where M is a 6×6 matrix which can be diagonalized via a unitary transformation on the field vector:

$$N_L = \mathcal{V}\, n_L \tag{1.22}$$

$\mathcal{V}$ can be determined by imposing the condition:

$$\mathcal{V}^T M \mathcal{V} = \begin{pmatrix} m_1 & 0 & \cdots & 0 \\ 0 & m_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & m_6 \end{pmatrix} \tag{1.23}$$

And so, the Lagrangian mass term becomes:

$$\mathcal{L}_{mass} = -\frac{1}{2} \sum_{k=1}^{6} m_k \bar{\nu}_{kL}^c \nu_{kL} \tag{1.24}$$

The mixing relations can be written like this:

$$\nu_{\alpha L} = \sum_{k=1}^{6} \mathcal{V}_{\alpha k} \nu_{kL} \qquad\qquad \nu_{\beta R}^c = \sum_{k=1}^{6} \mathcal{V}_{\beta k} \nu_{kR} \tag{1.25}$$

we can see that, because sterile and active neutrinos are linear combinations of the same massive fields, this model allows oscillations between them. To parametrize the mixing matrix we use the see-saw mechanism in the tree flavours case:

$$m_L = 0 \quad |m_D| =\ll m_R \quad \Rightarrow \quad M^L = 0 \quad M^D \ll M^R \tag{1.26}$$

then we decompose $\mathcal{V}$ into a product of two unitary matrix (for less than correction of the order M^D/M^R).

$$\mathcal{V} = \mathcal{W}\,\mathcal{U} \tag{1.27}$$

We can expand the product $\mathcal{W}^T M \mathcal{W}$ in power of M^D/M^R to obtain a block diagonal matrix:

$$\mathcal{W}^T M \mathcal{W} = \begin{pmatrix} M_{light} & 0 \\ 0 & M_{heavy} \end{pmatrix} \tag{1.28}$$

where:

$$M_{light} \approx -(M^D)^\dagger \frac{M^D}{M^R} \qquad M_{heavy} \approx M^R \tag{1.29}$$

So, we have a see-saw mechanism for the tree flavours case, the greater are the eigenvalues of M^R the smaller are the one of M_{light}. Because the out of diagonal elements of $\mathcal{W}$ are of the order of M^D/M^R, M_{light} and M_{heavy} are decoupled at low energies, then we can focus on the 3×3 block of $\mathcal{U}$ matrix which diagonalize M_{light}.

$$\mathcal{U}^\dagger M_{light} \mathcal{U} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \tag{1.30}$$

which takes to an effective mixing given by:

$$\nu_{\alpha L} = \sum_{k=1}^{3} \mathcal{U}_{\alpha k} \nu_{kL} \tag{1.31}$$

The unitary matrix $\mathcal{U}$ has nine independent parameter, tree angles and six phases. Tree of this phases can be eliminated by a phase redefinition of the charged lepton fields which are coupled with neutrino fields in the interaction Lagrangian. Two of the remaining phases are factorized in diagonal matrix and are called *Majorana phases* because they appear only if neutrino is a Majorana particle[5], the last phase is then called *Dirac phase*.

According to PDG the parametrization of the mixing matrix is:

$$\mathcal{U} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\phi_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\phi_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix} \tag{1.32}$$

where $c_{ij} = \cos(\theta_{ij})$, $s_{ij} = \sin(\theta_{ij})$ and θ_{ij} are the mixing angles; ϕ_{13} is the Dirac phase and λ_{ij} are the Majorana phases.

[5]in the case of Dirac neutrino they can be deleted with a redefinition of massive neutrino fields

1.2 Neutrino Oscillations

We are now going to see the theoretical explanation of the neutrino flavour oscillations phenomena. Let's consider as example the a decay process such as:

$$A \to B + \bar{\alpha} + \nu_\alpha \tag{1.33}$$

in which the alpha flavour neutrino is produced together with the associated anti-lepton. The state if the emitted neutrino is given by:

$$|\nu_\alpha\rangle \propto \sum_{k=1}^{3} |\nu_k\rangle \langle \nu_k \bar{\alpha}| J^\rho_{CC} |0\rangle J^{A\to B}_\rho \tag{1.34}$$

J^ρ_{CC} is the charged weak current [6] and $J^{A\to B}_\rho$ is the current responsible for the transition $A \to B$. Neglecting the effects of neutrino masses we obtain:

$$\langle \nu_k \bar{\alpha}| J^\rho_{CC} |0\rangle J^{A\to B}_\rho \propto \mathcal{U}*_{\alpha k} \tag{1.35}$$

with an opportune normalization we have:

$$|\nu_\alpha\rangle = \sum_{k=1}^{3} \mathcal{U}^*_{\alpha k} \; |\nu_k\rangle \tag{1.36}$$

This state describe the neutrino where and when is produced. To have the state after it's propagations in vacuum of a distance L in a time T we must apply on it the time evolution operator:

$$|\nu_\alpha(L,T)\rangle = \sum_{k=1}^{3} \mathcal{U}^*_{\alpha k} e^{-iE_kT + i\vec{p}_k\vec{L}} \; |\nu_k\rangle \tag{1.37}$$

where E_k and p_k are energy and momentum of the k type massive neutrino. By inverting eq.1.36 we obtain the expansion of the state $\nu_\alpha(L,T)$ in the base of flavour eigenstates:

$$|\nu_\alpha(L,T)\rangle = \sum_{k=1}^{3} \mathcal{U}^*_{\alpha k} \; e^{-iE_kT + i\vec{p}_k\vec{L}} \; \mathcal{U}_{k\beta} |\nu_\beta\rangle \tag{1.38}$$

[6] $J^\rho_{CC} = -\frac{g}{\sqrt{2}} \sum_\alpha \bar{\alpha}_L \gamma^\mu \mathcal{U}_{\alpha k} \nu_{kL}$

This equation shows that due to the mixing process, a definite flavour state evolves into a superposition of different flavor states. The squared module of the coefficient gives us the probability that a neutrino produced with flavour α it's measured in a flavour β:

$$P_{\nu_\alpha \to \nu_\beta}(L,T) = \left| \sum_{k=1}^{3} \mathcal{U}^*_{\alpha k}\; e^{-iE_kT + i\vec{p}_k\vec{L}}\; \mathcal{U}_{k\beta} \right|^2 \tag{1.39}$$

We cannot experimentally measure the propagation time T and so we have to express P as a function of L only. Due to the ultra relativistic nature of the emitted neutrino we can assume that: $T \simeq L$ and so:

$$E_kT - \vec{p}_k\vec{L} = (E_k - p_k)L = \frac{E_k^2 - p_k^2}{E_k + p_k}L = \frac{m_k^2}{E_k + p_k}L \simeq \frac{m_k^2}{2E}L \tag{1.40}$$

where E is neutrino's energy in the massless limit. Then we obtain:

$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta}(L) &= \left| \sum_{k=1}^{3} \mathcal{U}^*_{\alpha k}\; e^{-i\frac{m_k^2}{2E}L}\; \mathcal{U}_{k\beta} \right|^2 = \\ &= \sum_{k=1}^{3} |\mathcal{U}^*_{\alpha k}|^2 |\mathcal{U}^*_{\beta k}|^2 + 2Re\left\{ \sum_{k>j} \mathcal{U}^*_{\alpha k}\mathcal{U}_{\beta k}\mathcal{U}_{\alpha j}\mathcal{U}^*_{\beta j} e^{-i\frac{\Delta m_{kj}^2}{2E}L} \right\} \end{aligned} \tag{1.41}$$

with: $\Delta m_{kj}^2 = |m_k^2 - m_j^2|$.

in the simple case of only two flavours we have:

$$P_{\nu_\alpha \to \nu_\beta}(L) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E}\right) \tag{1.42}$$

the probability goes like a sin squared that is the reason why we call this phenomena oscillations.

1.3 Experimental results

As shown in the previous section, neutrino oscillations phenomenology can only give us information on the neutrino squared mass differences. Solar and reactor experiments have measured one mass splitting called *solar mass splitting* the other one *atmospheric mass splitting* it's been measured by atmospheric and accelerator-based experiment. The experimental values obtained are:

$$\Delta m^2_{sol} = |m_2^2 - m_1^2| = (7.58^{+0.22}_{-0.26}) \times 10^{-5} eV$$
$$\Delta m^2_{atm} = |m_3^2 - (m_2^2 + m_1^2)/2| = (2.35^{+0.12}_{-0.09}) \times 10^{-3} eV$$

According to [3] the best-fit values and 1σ ranges in the neutrino mixing parameters measured via neutrino oscillations are:

$$|\mathcal{U}_{e3}|^2 = 0.025 \pm 0.07 \qquad |\mathcal{U}_{\mu 3}|^2/(1 - |\mathcal{U}_{e3}|^2) = 0.42^{+0.08}_{-0.03} \qquad |\mathcal{U}_{e2}|^2/(1 - |\mathcal{U}_{e3}|^2) = 0.312^{+0.017}_{-0.016}$$

To complete our knowledge of neutrino masses we need to obtain the neutrino mass ordering and the absolute value of the lightest neutrino mass. The latter can be probed via neutrinoless beta decay searches as it will shown in the following chapters, currently only upper bounds to neutrinos mass scale are known and they are of order $\sim 1eV$. Our current knowledge of neutrinos is summarized in figure:1.1

Studying double beta decay will give as a chance to find the scale of neutrinos masses, because, as will be exposed with more details in the following chapters, the $2\beta 0\nu$ half-life of a nucleus $\mathcal{N}$ is given by:

$$\left[T^{0\nu}_{1/2}\right]^{-1} = G^{\mathcal{N}}_{0\nu} \; \left|\mathcal{M}^{\mathcal{N}}_{0\nu}\right|^2 \; \frac{|m_{2\beta}|^2}{m_e^2}$$

where: $G^{\mathcal{N}}_{0\nu}$ is the phase space factor, $\mathcal{M}^{\mathcal{N}}_{0\nu}$ the nuclear matrix element and $m_{2\beta}$ is the *effective Majorana mass* in $2\beta 0\nu$-decay:

$$m_{2\beta} = \sum_{i=1}^{3} U_{ei}^2 m_i \tag{1.43}$$

Ans so, by measuring double beta decay rate, we can estimate $m_{2\beta}$ and therefore the masses of light neutrinos. Actually the lower bound for $T^{0\nu}_{1/2}$ in reaction: $^{48}Ca \rightarrow ^{48}Ti$ is 1.5×10^{21} years (at 90% C.L.) [check value and reference]

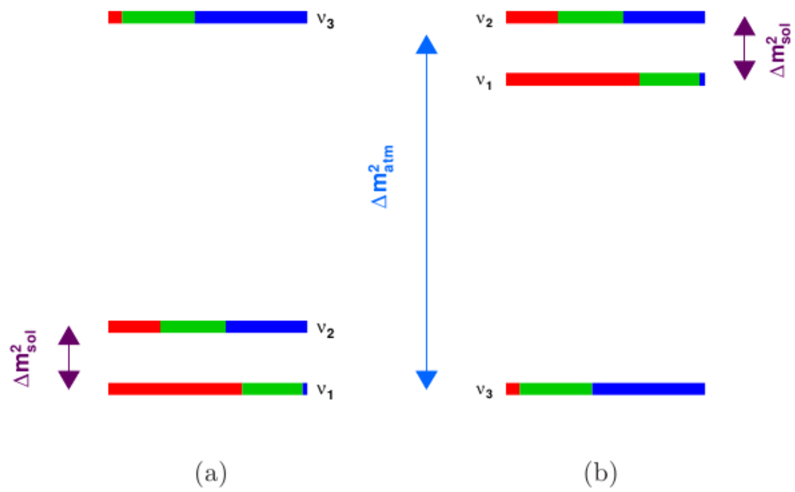

Figure 1.1: Knowledge on neutrino masses and mixing from neutrino oscillation experiments. Panels (a) and (b) show the normal and inverted mass orderings, respectively. Neutrino masses increase from bottom to top. The electron, muon and tau flavour content of each neutrino mass eigenstate is shown via the red, green and blue fractions, respectively.

Chapter 2

Beta decays

In this chapter we are going to familiarize with β decays phenomenology, in particular we are going to see the Lagrangian responsible for this processes and their Feynman diagrams. we also analyze the transition matrix element and compute the leptonic part in the tree case of our interest: the single beta decay, the two neutrinos double beta decay and the neutrinoless beta decay. For all of them we write down the expression for the nuclear transition matrix element and emphasize the differences between the tree cases. The real computation will be performed in the next chapter.

The interaction Lagrangian density in the Fermi electroweak theory responsible for beta decays according to [4] is:

$$\mathcal{L}_\beta = \frac{G}{\sqrt{2}} \left\{ \bar{N}\tau^+\gamma^\mu \left(g_v - g_a\gamma_5\right) N \right\} \left\{ \bar{\psi}_e\gamma_\mu \left(1 - \delta\gamma_5\right) \psi_e \right\} \tag{2.1}$$

where: $G = 1.18 \times 10^{-5}\ GeV^{-2}$ is the Fermi weak coupling constant, $g_v = 1$ $g_a = 1.25$ are the vectorial and axial coupling constant, N is the nucleon field, τ^+ is the isospin raising operator and ψ_e and ψ_ν are the Dirac fields of the electron and neutrino. Such as interaction term violates Charge conjugation and parity symmetry due to the V-A nature of the interaction.

2.1 Single beta decay

Let's start with the single beta decay process, in this case we cannot distinguish between Majorana and Dirac neutrinos, so, for simplicity we speak about Dirac neutrinos.

$$\mathcal{N}\,(A\,;\,Z) \rightarrow \mathcal{N}'\,(A\,;\,Z+1) + e^- + \bar{\nu}_e \tag{2.2}$$

$$\mathcal{N}\,(A\,;\,Z) \rightarrow \mathcal{N}'\,(A\,;\,Z-1) + e^+ + \nu_e \tag{2.3}$$

in this process as we can see from the Feynman diagram in figure 2.1, a nucleus $\mathcal{N}$ with mass number A and atomic number Z decays into a nucleus with the same mass number but atomic number decreased(increased) by one unit and emit an electron(positron) and an antineutrino(neutrino).

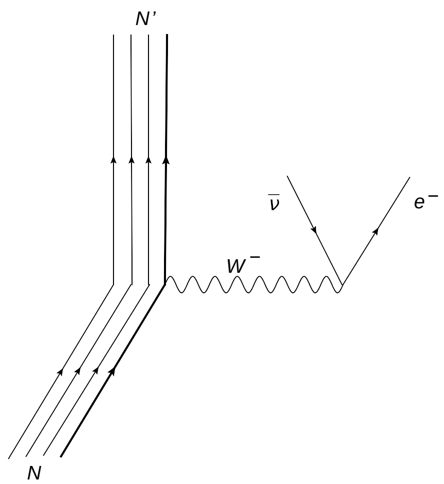

Figure 2.1: Feynman diagram for beta decay

From the relativistic perturbation theory we find the Dyson formula for the S matrix element at the first order. In beta decay case (2.2) we have:

$$S_\beta = -\,i \int d^4x\, \langle\, e, \bar{\nu}, \mathcal{N}' | \mathcal{L}_\beta(x) | \mathcal{N}\,\rangle \tag{2.4}$$

Then summing over the initial nucleus and final particles spin states the squared module of S_β we have:

$$\Gamma_\beta \propto \sum_{S_\mathcal{N} S_{\mathcal{N}'}} \sum_{S_e S_{\bar{\nu}}} |S_\beta|^2 = \frac{G^2}{2} H^{\rho\sigma} L_{\rho\sigma} \tag{2.5}$$

where $L_{\rho\sigma}$ is the leptonic tensor given by: (using the approximation $m_\nu \sim 0$)

$$\begin{aligned} L_{\rho\sigma} &= \sum_{S_e S_{\bar{\nu}}} \left(\bar{u}_e \gamma_\rho \left(1 - \delta\gamma_5\right) v_\nu \bar{v}_\nu \gamma_\sigma \left(1 - \delta\gamma_5\right) u_e\right) = \\ &= Tr\left[\left(\not{P}_e - m\right) \gamma_\rho \left(1 - \delta\gamma_5\right) \not{P}_{\bar{\nu}} \gamma_\sigma \left(1 - \delta\gamma_5\right)\right] = \\ &= 4\left(1 + \delta^2\right)\left(P^e_\rho P^{\bar{\nu}}_\sigma + P^{\bar{\nu}}_\rho P^e_\sigma - \left(P^e \cdot P^{\bar{\nu}}\right) g_{\rho\sigma}\right) - 8i\delta P^\alpha_e P^\beta_{\bar{\nu}} \epsilon_{\alpha\rho\beta\sigma} \end{aligned}$$

and $H^{\rho\sigma}$ is the nuclear tensor:

$$H^{\rho\sigma} = \sum_{S_\mathcal{N} S_{\mathcal{N}'}} \langle\Psi_f|J^\rho|\Psi_i\rangle\langle\Psi_i|J^\sigma|\Psi_f\rangle \tag{2.6}$$

with Ψ_i and Ψ_f wave function of the initial and final state nuclei, J^ρ is the nuclear current calculated in non-relativistic approximation:

$$J^\rho = \sum_{n=1}^{A} \tau_n^+ \left(g_v \delta^{\rho 0} + g_a \delta^{\rho i} \sigma^i\right) \tag{2.7}$$

therefore we have:

$$H^{\rho\sigma} = g_v^2 \; M_F^2 \; \delta^{\rho 0} \delta^{\sigma 0} + g_a^2 \; M_{GT}^2 \; \delta^{\rho i} \delta^{\sigma j} \tag{2.8}$$

where M_F^2 and M_{GT}^2 are respectively the squared module of Fermi and Gamow-Teller nuclear transition matrix element which will be discussed more with more details in the next chapter:

$$M_F^2 = \sum_{S_\mathcal{N} S_{\mathcal{N}'}} ||\langle\Psi_f| \sum_{n=1}^{A} \tau_n^+ |\Psi_i\rangle||^2 \quad M_{GT}^2 = \sum_{S_\mathcal{N} S_{\mathcal{N}'}} ||\langle\Psi_f| \sum_{n=1}^{A} \tau_n^+ \sigma^i |\Psi_i\rangle||^2 \tag{2.9}$$

then, contracting the nuclear and the leptonic tensor:

$$\frac{G^2}{2} H^{\rho\sigma} L_{\rho\sigma} = 2G^2\left(1 + \delta^2\right) E_e E_{\bar{\nu}} \left[g_v^2 \left(1 + \frac{\vec{P}_e \vec{P}_{\bar{\nu}}}{E_e E_{\bar{\nu}}}\right) M_F^2 + g_a^2 \left(1 - \frac{1}{3} \frac{\vec{P}_e \vec{P}_{\bar{\nu}}}{E_e E_{\bar{\nu}}}\right) M_{GT}^2\right] \tag{2.10}$$

from this expression we see that, we need to compute M_F and M_{GT} to evaluate the rate of beta decay process, and this will be the main goal of this thesis.

In the next sections how changes the operator in the nuclear matrix elements if we move from single to 2ν double beta decay and finally to 0ν double beta decay.

2.2 2ν double beta decay

In the standard model, double beta decay can occur in many ways, all of them with neutrinos emission:

$$\mathcal{N}\,(A\,;\,Z) \to \mathcal{N}'\,(A\,;\,Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e \tag{2.11}$$

$$\mathcal{N}\,(A\,;\,Z) \to \mathcal{N}'\,(A\,;\,Z-2) + e^+ + e^+ + \nu_e + \nu_e \tag{2.12}$$

$$\tag{2.13}$$

in $2\beta 2\nu$, the nucleus $\mathcal{N}$ decays into a nucleus with the same mass number but atomic number decreased(increased) by two units and emit two electron(positron) and an antineutrino(neutrino), in figure 2.2 we can see the Feynman diagram of the process.

The double beta decay is a second order process in weak interaction, and so in the case 2.11 for the transition matrix element we have:

$$S_{2\beta 2\nu} = -\int d^4x d^4y \,\langle\, e_1, e_2, \bar{\nu}_1, \bar{\nu}_2, \mathcal{N}' | T\left\{\mathcal{L}_\beta(x)\mathcal{L}_\beta(y)\right\} | \mathcal{N}\,\rangle \tag{2.14}$$

As done previously, summing over the initial nucleus and final particles spin states the squared module of $S_{2\beta 2\nu}$ we have:

$$\Gamma_{2\beta 2\nu} \propto \sum_{S_\mathcal{N} S_{\mathcal{N}'}} \sum_{S_{e1} S_{\bar{\nu}1} S_{e2} S_{\bar{\nu}2}} |S_{2\beta 2\nu}|^2 = \frac{G^4}{4} H^{\mu\nu\rho\sigma} L_{\mu\nu\,(1)} L_{\rho\sigma\,(2)} \tag{2.15}$$

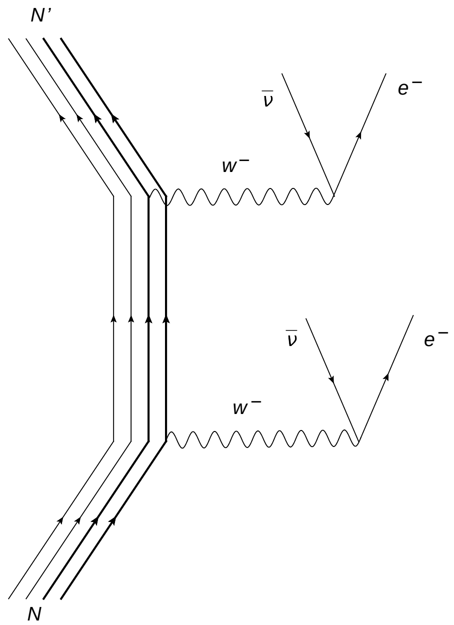

Figure 2.2: Feynman diagram for two neutrinos double beta decay

where the subscripts (1) and (2) indicates the two decaying protons. $L_{\mu\nu}$ is the same of the one seen in the previous section. The tensor: $H^{\mu\nu\rho\sigma}$ contains the new nuclear Fermi and Gamow-Teller transition matrix element:

$$M^2_{F\,2\nu} = \sum_{S_{\mathcal{N}}S_{\mathcal{N}'}} ||\langle\Psi_f|\sum_{i>j}^{A}\tau_i^+\tau_j^+|\Psi_i\rangle||^2 \quad M^2_{GT\,2\nu} = \sum_{S_{\mathcal{N}}S_{\mathcal{N}'}} ||\langle\Psi_f|\sum_{i>j}^{A}\tau_i^+\tau_j^+\sigma_i\sigma_j|\Psi_i\rangle||^2 \quad (2.16)$$

where the index i and j runs over the nucleons, so, now we have to deal with a two bodies operator in the nuclear matrix element.

2.3 0ν double beta decay

As said before neutrinoless double beta decay is possible only if neutrinos are Majorana particles, the field which describes such as kind of particle is:

$$\nu(x) = \int \frac{d^3p}{(2\pi)^3 2E} \sum_n \left(a_n(p)u_n(p)e^{-ipx} + a_n^\dagger(p)v_n(p)e^{ipx}\right) \quad (2.17)$$

neutrinoless beta decay can occur in two ways:

$$\mathcal{N}\,(A\,;\,Z) \rightarrow \mathcal{N}'\,(A\,;\,Z+2) + e^- + e^- \tag{2.18}$$

$$\mathcal{N}\,(A\,;\,Z) \rightarrow \mathcal{N}'\,(A\,;\,Z-2) + e^+ + e^+ \tag{2.19}$$

in this case we have no neutrinos in the final state, the Feynman diagram for the process is in figure 2.3

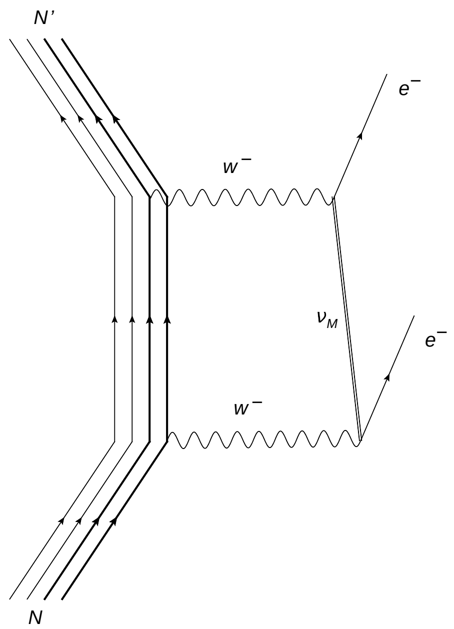

Figure 2.3: Feynman diagram for neutrinoless double beta decay

The transition matrix element for reaction 2.18 is:

$$S_{2\beta 0\nu} = -\int d^4x d^4y \, \langle \, e_1, e_2, \mathcal{N}' | T \left\{ \mathcal{L}_\beta(x) \mathcal{L}_\beta(y) \right\} | \mathcal{N} \, \rangle \tag{2.20}$$

summing over the initial nuclear and final nuclear and leptonic spin states the squared module of $S_{2\beta 0\nu}$ we have:

$$\Gamma_{2\beta 0\nu} \propto \sum_{S_{\mathcal{N}} S_{\mathcal{N}'}} \sum_{S_{e1} S_{e2}} |S_{2\beta 0\nu}|^2 = \frac{G^4}{4} H^{\mu\nu\rho\sigma} L_{\mu\nu\rho\sigma} \tag{2.21}$$

The nuclear part is equal to the case $2\beta 2\nu$ otherwise the leptonic part using Feynman rules for Majorana neutrinos (see [6]) becomes:

$$L_{\mu\nu\rho\sigma} = \sum_{S_{e2}S_{e2}} \left(\bar{u}_{e1}\gamma_\rho \left(1-\delta\gamma_5\right) \sum_i U_{ei} \frac{\not{p}_\nu + m_i}{p^2 - m_i^2} C\gamma_\sigma \left(1-\delta\gamma_5\right) u_{e2} \right) (h.c.) =$$
$$= \sum_i U_{ei} m_i \sum_{S_{e2}S_{e2}} \left(\bar{u}_{e1} \frac{\gamma_\rho(1-\delta\gamma_5)\gamma_\sigma}{p^2} C u_{e2} C \bar{u}_{e2} \frac{\gamma_\mu(1-\delta\gamma_5)\gamma_\nu}{p^2} u_{e1} \right)$$

where C is the Charge-conjugation matrix in the spinor space, P is the Majorana neutrino momentum and we have used the property: $(1-\gamma_5)\not{P}(1-\gamma_5) = 0$. So we can parametrize the leptonic part this way:

$$L_{2\beta 0\nu} = \tilde{H}(p)\, m_{2\beta}\, G_{0\nu} \tag{2.22}$$

$G_{0\nu}$ is a phase space factor and $H(p)$ is the *neutrino potential* which will be transferred into the nuclear part and studied numerically in the next chapter.

Finally putting the nuclear and leptonic part together we obtain the expression for

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}^{\mathcal{N}} \, \left|\mathcal{M}_{0\nu}^{\mathcal{N}}\right|^2 \, \frac{\left|m_{2\beta}\right|^2}{m_e^2} \tag{2.23}$$

where $G_{0\nu}^{\mathcal{N}}$ is the phase space factor given by:[8]

$$G_{0\nu}^{\mathcal{N}} = \frac{a_{0\nu}}{m_e^2 \, \ln 2} \int d\Omega_{0\nu} F_0(Z, E_1) F_0(Z, E_2) \tag{2.24}$$

with:

$$a_{0\nu} = \frac{(G g_A)^4 m_e^9}{64\pi^2} \tag{2.25}$$

$$d\Omega_{0\nu} = m_e^{-5}\, p_1 p_2 \, E_1 E_2 \, \delta\left(E_1 + E_2 + E_f - E_i\right) \, dE_1 \, dE_2 \, d(p_1 \cdot p_2) \tag{2.26}$$

E_f and E_i indicates the energy of the final and initial state and the Fermi function F_0 which takes into account that the two emitted electrons are influenced by the other atomic electrons, is given by:

$$F_0(Z, E) = \frac{\mp 2\pi Z e^2/v}{1 - e^{\pm 2\pi Z e^2/v}} \qquad for \quad \beta^\pm \quad decays \tag{2.27}$$

where v is the velocity of the electron far of the nucleus, this functions enhances the probability of β^- emission and decreases that of β^+ especially at low energies.

$\mathcal{M}_{0\nu}^{\mathcal{N}}$ is the nuclear transition matrix element and it is composed of a Fermi and a Gamow-Teller transition part:

$$\mathcal{M}_{0\nu}^{\mathcal{N}} = M_{0\nu}^{GT} - \left(\frac{g_v}{g_a}\right)^2 M_{0\nu}^{F} \tag{2.28}$$

where:

$$M_{0\nu}^{F} = \langle\Psi_f|\sum_{i>j}^{A}\tau_i^+\tau_j^+ H(r)|\Psi_i\rangle \quad M_{0\nu}^{GT} = \langle\Psi_f|\sum_{i>j}^{A}\tau_i^+\tau_j^+\sigma_i\sigma_j H(r)|\Psi_i\rangle \tag{2.29}$$

the numerical computation of this two objects will be the main topic of the thesis, and will be analyzed in the following chapters.

Chapter 3

Elements of nuclear dynamics

What we want to do now is to construct the nuclear wave functions that we have to use in our numerical computation. We'll do that basing on the *nuclear shell model* and so we must start with an overview of it following [7]
Everybody knows that nuclei are composed by protons and neutron kept together by the strong interaction, the length scale of the strong nuclear interaction e so the radius of the nucleus are of the order of few Fermis ($1fm = 10^{-15}m$). The nuclear shell model attempt to solve the quantum mechanic problem of the motion of one nucleon in a nucleus comparing it with the motion of an electron in the hydrogen atom (the difference is in the length scale $\sim 10^{-10}m$)

Single-particle potential eigenstates are characterized by their energies and quantum numbers, so, the properties of a nucleus with Z protons and A-Z neutrons are determined by filling the lowest single-particle energy levels allowed by the Pauli exclusion principle which allows only one proton or neutron to occupy a state with a given set of quantum numbers.

The shell model in its simplest form is able to successfully predict the properties of nuclei which are one nucleon removed or added to the one of the *magic number*. In the shell model, magic numbers nucleus are nuclei whose nucleons completely fill the external shell, the seven most used magic numbers are 2, 8, 20, 28, 50, 82, 126 . The shell model can also be extended to include the more complex configurations that arise for the nuclei with nucleon numbers that are in between the magic numbers, current theoretical investigations using the shell model focus on these complex configurations.

In this chapter after a brief overview on nuclear forces we are going to study the many bodies nuclear problem and try to solve it using a "mean field" approach (the so called nuclear shell model) and finally we'll see how to correct it to have a more realistic model for nuclear dynamics.

3.1 Nuclear Forces

The properties of strong interaction between nucleon due to experimental results can be summarized as follow:

- *Short range repulsive core:* the fact that density in the interior atomic nuclei is nearly constant and independent of the mass number A, tells us that nucleons cannot be packed together too tightly. And so, at short distances the nucleon-nucleon interaction must be repulsive. Being R_c the radius of the repulsive core we have:

$$V(r) > 0 \quad if \quad |r| < R_c \tag{3.1}$$

  V is a non relativistic potential depending on the distances between the two nucleons.

- *Finite range interaction:* the nuclear binding energy per nucleon is practically the same for all nuclei with $A \geq 20$ suggest that the interaction as a finite range R_0 i.e.

$$V(r) = 0 \quad if \quad |r| > R_c \tag{3.2}$$

- *Isotopic invariance:* the spectra of the so called *mirror nuclei*[1] exhibit similarities: the energy of the levels with the same parity and angular momentum are the same (up to small electromagnetic corrections) this means that nuclear forces are charge symmetric.

This isospin invariance implies that the interaction between two nucleons separated by distance r and having total spin S depend on their total isospin T but not on its projection M_T.

3.2 Many bodies nuclear problem

Now that we known the main characteristic of the nuclear forces, we can try to build up the nuclear Hamiltonian as a sum of interaction terms between a given number of nucleons:

$$H_{\mathcal{N}} = \sum_i T_i + \sum_{ij} v_{ij} + \sum_{ijk} v_{ijk} + \cdots \tag{3.3}$$

where the index i,j,k... runs over the nucleons coordinates. We can simplify the problem making the assumption that only the nucleon-nucleon terms are relevant and so the Hamiltonian reduces to:

$$H_{\mathcal{N}} = \sum_i T_i + \sum_{ij} v_{ij} \tag{3.4}$$

the resolution of the Schroedinger equation for such as a nuclear Hamiltonian is still problematic, we need to make another approximation: we have to make separable that our Hamiltonian in such a way to have an Independent Schroedinger equation for every nucleon. In order to do that, we substitute the coupling interaction with a mean field term:

$$U_i = \sum_j v_{ij} \tag{3.5}$$

[1] pairs of nuclei having the same A and Z differing by one unit, this implies that the number of protons in a nucleus is the same as the number of neutron in its mirror companion (for example ${}^{1}5_{7}N$ ${}^{1}5_{8}O$)

This is a strong approximation and (as will be discussed in the following sections) takes to a great disagreement with the experimental results, to avoid this will use the correlated wave function formalism.Finally our nuclear Hamiltonian becomes:

$$H_{\mathcal{N}} = \sum_{i}^{A} H_i = \sum_{i} (T_i + v_{ij}) \tag{3.6}$$

We can now solve the Schroedinger equation for every H_i and obtain the single particle wave function for every nucleon:

$$H_i \phi_i = (T_i + v_i(r))\, \phi_i = E_i \phi_i \tag{3.7}$$

and finally construct the nuclear wave function as a Slater determinant of the 'ϕ_i:

$$\Psi_{\mathcal{N}} = \det \{\phi_i\} \tag{3.8}$$

3.3 Mean field approach

As said in the previous section we want to build up a model for nuclear dynamics based on a main field approach,in this section we are going to build up the single-particle Hamiltonian that will provide us the shell model states to be used into our computation.

The Schroedinger equation for a particle of mass m in a spin-independent central potential $V_0(r)$ is:

$$H_0|\alpha\rangle = (T + V_0(r))\,|\alpha\rangle = E_\alpha|\alpha\rangle \tag{3.9}$$

T is the kinetic energy operator:

$$T = -\frac{1}{2m}\nabla^2 = -\frac{1}{2m}\left\{\frac{1}{r}\frac{d^2}{dr^2} - \frac{\vec{l}^2}{r^2}\right\} \tag{3.10}$$

for the central potential there are many possibility, the most used of them are: the Saxon-Woods and the 3d isotropic harmonic oscillator which is the one we are going to use because the Saxon-Wood potential give us wave function that cannot be separated into relative and center of mass part, and we need to do this operation because of the two-body nature of the processes that we want to study. so we have:

$$V_0(r) = \frac{1}{2} m\omega r^2 \tag{3.11}$$

Solving the Schroedinger equation we obtain:

$$\phi_{k,l} = R_{kl}(r) Y_{l,m}(\theta,\phi) = N_{k,l}\; r^l e^{-\nu r^2}\; L_k^{l+\frac{1}{2}}(2\nu r^2)\; Y_{l,m}(\theta,\phi) \tag{3.12}$$

where $\nu = m\omega/2$, $Y_{l,m}(\theta,\phi)$ are the spherical harmonic function and $N_{k,l}$ is the normalization factor:

$$N_{k,l} = \sqrt{\sqrt{\frac{2\nu^3}{\pi}}\,\frac{2^{k+2l+3}\; k!\; \nu^l}{(2k+2l+1)!!}} \tag{3.13}$$

$L_k^{l+\frac{1}{2}}(x)$ is the generalized Laguerre polynomial:

$$L_k^{l+\frac{1}{2}}(x) = \sum_{i=0}^{k} \frac{-x^i}{i!} \binom{k+l+\frac{1}{2}}{k-i} \tag{3.14}$$

the energy spectrum is given by:

$$E_{k,l} = \omega\,(2k + l + \frac{3}{2}) \tag{3.15}$$

To obtain magic numbers, a spin-orbit potential must be added:

$$V_{SO} = 2\lambda\, \vec{l} \cdot \vec{s} \tag{3.16}$$

with this additional term the orbital and intrinsic angular momentum must be coupled to a definite total angular momentum $\vec{j} = \vec{l} + \vec{s}$. Spin-orbit potential eigenstates are determined by the total angular momentum quantum number $j = l \pm 1/2$ and the quantum number m_j associated with the z components of j. The energy levels obtained are:

$$E_{k,l} = \omega\,(2k + l + \frac{3}{2}) + \lambda\, l \qquad for\ j = l + \frac{1}{2} \tag{3.17}$$

$$E_{k,l} = \omega\,(2k + l + \frac{3}{2}) + \lambda\,(l+1) \qquad for\ j = l - \frac{1}{2} \tag{3.18}$$

And the wave functions are:

$$\phi_{k,l} = N_{k,l}\, r^l e^{-\nu r^2}\; L_k^{l+\frac{1}{2}}(2\nu r^2)\; \left[Y_{l,m_l}(\theta,\phi)\otimes\chi_{s,m_s}\right]_{j,m_j} \tag{3.19}$$

where χ_{s,m_s} is the spin wave function and the symbol $\otimes$ denotes the Clebsh-Gordan product:

$$\left[Y_{l,m_l}(\theta,\phi)\otimes\chi_{s,m_s}\right]_{j,m_j} = \sum_{m_l,m_s}\langle l,m_l,s,m_s|j,m_j\rangle Y_{l,m_l}(\theta,\phi)\otimes\chi_{s,m_s} \tag{3.20}$$

In table 3.1 we summarize some relevant information about the orbitals we use in the rest of the thesis. We are now able to compute nuclear densities using the wave function that we have just found:

$$\rho(r) = \sum_{i=1}^{A}|\phi_i|^2 \tag{3.21}$$

and compare it with the one obtained from experiment. What we find is that the states density obtained with pure shell model states is very different from the experimental one especially for the short range region, this difference is probably caused by the short distance nature of nucleon-nucleon strong interaction. In the next section we are going to try to solve this problem using correlations formalism which will provide us the definitive form of the wave functions that we use in the numerical computation.

3.4 Correlated wave function formalism

The shell model shows evidence of the intrinsic limitations of its applicability: as said in [19] Electron scattering experimens aimed at assessing the limits of applicability of the nuclear shell model (reviews of this kind of experiment can be found in [20] [21] and [22]) They are, mainly based on measurement of the cross section of the proton knock out process:

$$e + \mathcal{N}_A \to e' + \mathcal{N}_{A-1} \tag{3.22}$$

The most striking feature emerging from the analysis of this process is that, while the spectroscopic lines corresponding to knock out from shell model states are clearly seen, the corresponding strenghts are consistenltly and sizably lower than expected, regardless of the nuclear mass number as shown infigure 3.1 which is a recent compilation of the strengths of the valence shell model orbits of a number of nuclei, ranging from carbon to lead, measured by both electron- and hadron-induced proton knock out [23]. It clearly appears that all the observed strengths are largely below the shell model prediction. This shows according to "Urbana" and "Argonne" models that a significant fraction of the target nucleons do not behave as indipendent particles thus providing one of the cleanest signatures of correlation effects which are manifestation of the strongly repulsive core of nucleon nucleon interaction, this reduces the possibility that two nucleons can approach each other, and this modifies the shell model picture, in which, by definition the motion of each nucleon does not depend on the presence of the others.Strong nucleon-nucleon interactions give rise to virtual scattering processes leading to the exitation of the participating nucleons to states of energy larger than the Fermi energy, thus depleting the shell model states whithin the Fermi sea. To take into account of this phenomenon we have to

$N = 2k + l$	k	l	Orbital name	Energy	N_j	$\sum_j N_j$
0	0	0	$1s_{\frac{1}{2}}$	$3/2\ \omega$	2	2
1	0	1	$1p_{\frac{3}{2}}$	$5/2\ \omega - \lambda$	4	
1	0	1	$1p_{\frac{1}{2}}$	$5/2\ \omega + 2\lambda$	2	8
2	0	2	$1d_{\frac{5}{2}}$	$7/2\ \omega - 2\lambda$	6	
2	1	0	$2s_{\frac{1}{2}}$	$7/2\ \omega$	2	
2	0	2	$1d_{\frac{3}{2}}$	$7/2\ \omega + 3\lambda$	4	20
3	0	3	$1f_{\frac{7}{2}}$	$9/2\ \omega - 3\lambda$	8	28

Table 3.1: Harmonic oscillator plus spin-orbit orbitals, N_j is the number of nucleons that can be located into the orbital

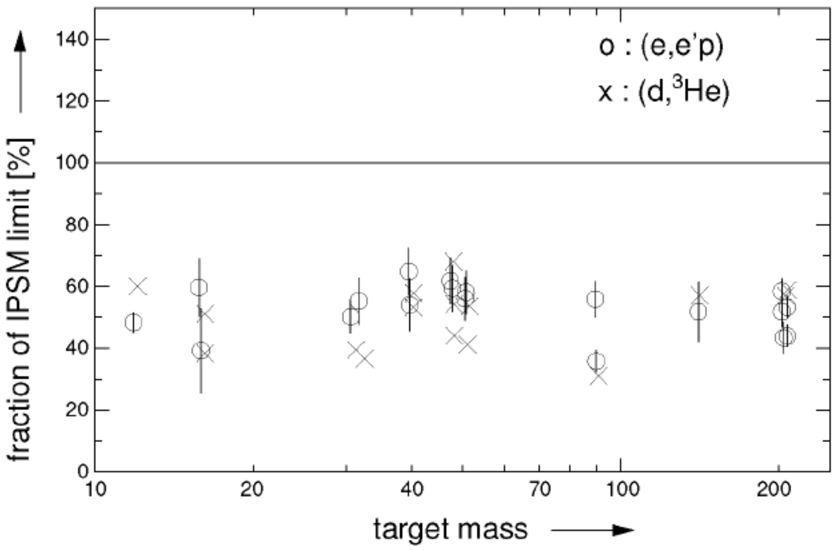

Figure 3.1: Integrated strengths of the valence shell model states, meassured in electron- (open circles)and hadron-induced (crosses) proton knock out experiments, as a function of the target mass number(taken from [23]). The solid horizontal line rep-resents the shell model prediction.

use the so called *correlated wave function formalism.* Let's see how this formalism work[9].

We solve the many-body Schroedinger equation by using the variational principle

$$\delta E[\Psi] = \delta \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} = 0 \tag{3.23}$$

The search for the minimum of the energy functional is done within a subspace of the full Hilbert space spanned by the A-body wave functions, which can be expressed as:

$$\tilde{\Psi}(A) = \mathcal{F}(1, ..., A)\Psi(1, ..., A) \tag{3.24}$$

where $\mathcal{F}(1, ..., A)$ is a many-body correlation operator, and $\Psi(1, ..., A)$ is a Slater determinant composed of single particles wave functions $\Phi_\alpha(r_i)$. In our computation we used a subset of two-body interaction of Argonne and Urbana type:

$$\mathcal{F} = \mathcal{S}\left(\prod_{i<j=1}^{A} F_{ij}\right) \tag{3.25}$$

$\mathcal{S}$ is a symmetrizing operator and F_{ij} is expressed in terms of two-body correlation functions f_p as:

$$F_{ij} = \sum_{p=1}^{6} f_p(r_{ij})\, \mathcal{O}_{ij}^p \tag{3.26}$$

the six operators $\mathcal{O}_{ij}^p$ are defined as:

$$\mathcal{O}_{ij}^p = [1, \sigma_i \cdot \sigma_j, S_{ij}] \otimes [1, \tau_i \cdot \tau_j] \tag{3.27}$$

with σ_i and τ_i spin and isospin Pauli operators and S_{ij} is the tensor operator:

$$S_{ij} = (3\hat{r}_{ij} \cdot \sigma_i \hat{r}_{ij} \cdot \sigma_j - \sigma_i \cdot \sigma_j) \tag{3.28}$$

In our numerical computation we are going to ignore the tensor operator, so we have only four operator, the details of the calculation are described in the following chapter.

Chapter 4

Computation of nuclear matrix element

We have seen in the previous chapter that nuclear halflife for neutrinoless double beta decay is given by eq.2.23, now we are going to calculate the analytical expression for the nuclear matrix elements for Fermi and Gamow-Teller transitions. They can be write in this general form:

$$M_{0\nu}^{\alpha} = \langle \Psi_F, \mathcal{J}^{\pi} | \sum_{j>i} \tau_i^+ \tau_j^+ O^{\alpha}(r_{ij}) | \Psi_I, \mathcal{J}^{\pi} \rangle \tag{4.1}$$

where $\mathcal{J}$ and π are the total angular momentum and parity of the initial and final nuclei, and $O^{\alpha}(r_{ij})$ is an operator which takes into account of the neutrino potential and the nature of transition (Fermi or Gamow-Teller):

$$O^F(r_{ij}) = \mathcal{I} \cdot H(r_{ij}) \qquad O^{GT}(r_{ij}) = (\vec{\sigma}_i \cdot \vec{\sigma}_j) \cdot H(r_{ij}) \tag{4.2}$$

$r_{ij} = |r_i - r_j|$ is the module of the distance between two nucleons and $H(r_{ij})$ is the neutrino potential given by:

$$H(r) = \frac{2R_N}{\pi} \int_0^{+\infty} \frac{\sin(qr)}{r(q + \langle E \rangle)} \tag{4.3}$$

where $R_N = 1.2 \times A^{\frac{1}{3}}$ is the nuclear range and $\langle E \rangle$ is is the average energy of the virtual intermediate states used in the closure approximation. In the next section we perform the computation of $M_{0\nu}^{\alpha}$ in a pure shell model picture, after that we are going to insert the correlations effects.

Mancano: approssimazione di chiusura e come propagatore a diventare in quel modo?

4.1 Pure shell model

In this section we are going to compute the nuclear matrix element in a pure shell model approach. We assume that two of the neutrons of the initial state nucleus decays and all the other nucleons are spectator, so we can write the matrix element as a sum over the total angular momentum of the neutron and protons couples of antisymmetrized matrix elements with weight given by shell model calculations:

$$M^{\alpha}_{0\nu} = \sum_{j_1' j_2' j_1 j_2 \mathcal{J}^\pi} K_{sm}(j_1, j_2, j_1', j_2', J, \mathcal{J}^\pi) \langle k_1' l_1' j_1', k_2' l_2' j_2', \mathcal{J}^\pi, T_f | \tau_1^+ \tau_2^+ O^\alpha(r_{12}) | k_1 l_1 j_1, k_2 l_2 j_2, \mathcal{J}^\pi, T_i \rangle_A \quad (4.4)$$

where the index 1 and 2 denotes the quantum numbers of the two decaying neutrons and 1' and 2' are for the two final protons, T_i and T_f are the total isospin of the two particles initial and final state which is 1 in both cases and $K_{sm}(j_1, j_2, j_1', j_2', J, \mathcal{J}^\pi)$ is the shell model coefficient which takes into account that the spectator nucleons reorganize themselves in function of the angular momentum of the nucleons participating into to the decay to have total nuclear angular momentum $\mathcal{J}$.

In order to carry out the calculation, the two-body matrix element in (4.4) must be decomposed into products of reduced matrix element of operator acting in spin and coordinate space. In addiction the coordinate space matrix element must be decomposed into the contributes arising from the center of mass and relative motion terms. And the states appearing in (4.4) can be rewritten in the following form:

$$|k_1 l_1 j_1, k_2 l_2 j_2, \mathcal{J}^\pi\rangle = \sum_{S,\Lambda} \langle l_1, \frac{1}{2}, j_1; l_2, \frac{1}{2}, j_2 | \frac{1}{2}, \frac{1}{2}, S, l_1, l_2, \Lambda\rangle \quad |l_1, l_2, \Lambda, \frac{1}{2}, \frac{1}{2}, S; J; \mathcal{J}^\pi\rangle$$
$$\times \sum_{klKL} \langle k, l, K, L | k_1, l_1, k_2, l_2\rangle_\Lambda \quad |k, l\rangle\ |K, L\rangle \tag{4.5}$$

where:

$$\langle l_1, \frac{1}{2}, j_1; l_2, \frac{1}{2}, j_2 | \frac{1}{2}, \frac{1}{2}, S, l_1, l_2, L\rangle = [(2j_1+1)(2j_2+1)(2\Lambda+1)(2S+1)]^{\frac{1}{2}} \times \begin{pmatrix} l_1 & \frac{1}{2} & j_1 \\ l_2 & \frac{1}{2} & j_2 \\ \Lambda & S & J \end{pmatrix} \tag{4.6}$$

Λ , S and J are respectively orbital angular momentum, spin and total angular momentum of the nucleon couple. The last factor is called *9-j symbol* and $\langle k, l, K, L|k_1, l_1, k_2, l_2\rangle_L$ is the coefficient of the transformation from the (r_1, r_2) representation to the $(r = |\vec{r_1} - \vec{r_2}|, R = |\vec{r_1} + \vec{r_2}|/2)$ representation and it is called *Talmi–Moshinsky brakets*:

$$\langle r_1|k_1, l_1\rangle\ \langle r_2|k_2, l_2\rangle = \sum_{klKL} \langle k, l, K, L|k_1, l_1, k_2, l_2\rangle_L ambda\ \langle r|k, l\rangle\ \langle R|K, L\rangle \tag{4.7}$$

And finally the matrix elements are given by

$$\langle k_1' l_1' j_1', k_2' l_2' j_2', \mathcal{J}^\pi | \tau_1^+ \tau_2^+ O^\alpha(r_{12}) | k_1 l_1 j_1, k_2 l_2 j_2, \mathcal{J}^\pi\rangle_A =$$
$$= \sum_{S,\Lambda} \langle l_1, \frac{1}{2}, j_1; l_2, \frac{1}{2}, j_2 | \frac{1}{2}, \frac{1}{2}, S, l_1, l_2, \Lambda\rangle_J \quad \langle l_1', \frac{1}{2}, j_1'; l_2', \frac{1}{2}, j_2' | \frac{1}{2}, \frac{1}{2}, S, l_1', l_2', \Lambda\rangle_J$$
$$\times \frac{1}{\sqrt{2S+1}}\ \langle \frac{1}{2}, \frac{1}{2}, S | \hat{O}_{12}^\alpha | \frac{1}{2}, \frac{1}{2}, S\rangle$$
$$\times \sum_{k,k',l,l'} \sum_{K,L,K',L'} \langle k, l, K, L|k_1, l_1, k_2, l_2\rangle \quad \langle k', l', K', L'|k_1', l_1', k_2', l_2'\rangle \times \langle k', l'|H(r)|k, l\rangle \tag{4.8}$$

where the reduced matrix element of the relevant operator are:

$$\langle \frac{1}{2}, \frac{1}{2}, S|\ \mathcal{I}\ | \frac{1}{2}, \frac{1}{2}, S\rangle = \sqrt{2S+1} \tag{4.9}$$
$$\langle \frac{1}{2}, \frac{1}{2}, S|\ (\vec{\sigma}_1 \cdot \vec{\sigma}_2)\ | \frac{1}{2}, \frac{1}{2}, S\rangle = \sqrt{2S+1}\ \ [2S(S+1) - 3] \tag{4.10}$$

and the radial relative motion matrix element is given by:

$$\langle k', l'|H(r)|k, l\rangle = \int_0^\infty r^2 dr \; R_{kl}(r) \; H(r) \; R_{k'l'}(r) \tag{4.11}$$

as said before this is what we to compute if we don't take into account the correlations. Other details will be given in the next chapter when we will make the choice of the nucleus and the subset of Hilbert space states to use in the computation.

4.2 Including correlation

Now we are able to include the correlations studied in the third chapter in the matrix element calculated in the previous section. We are going to use only four of the six operators in eq. 3.27 and so, in our case eq. 3.26 reduces to:

$$F_{12} = f_c(r)\,\mathcal{I} + f_\sigma(r)\,(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + f_\tau(r)\,(\vec{\tau}_1 \cdot \vec{\tau}_2) + f_{\sigma\tau}(r)\,(\vec{\sigma}_1 \cdot \vec{\sigma}_2)\,(\vec{\tau}_1 \cdot \vec{\tau}_2) \tag{4.12}$$

in our case (remembering that $T = 1$ for initial and final state) we have:

$$(\vec{\tau}_1 \cdot \vec{\tau}_2) = \frac{1}{2}\,[4T(T+1) - 6] = 1 \tag{4.13}$$

Eq.4.12 becomes:

$$F_{12} = (f_c(r) + f_\tau(r))\;\mathcal{I} + (f_\sigma(r) + f_{\sigma\tau}(r))\,(\vec{\sigma}_1 \cdot \vec{\sigma}_2) \tag{4.14}$$

In correlated wave function formalism we substitute the shell model wave function with a correlated one:

$$\phi_\alpha(r_i) \rightarrow \mathcal{F}\phi_\alpha(r_i) \tag{4.15}$$

This is, for our purpose equivalent to substitute the transition operators $O^\alpha(r_{12})$ with their correlate version:

$$O^\alpha(r) \rightarrow \tilde{O}^\alpha(r) = F_{12}\,\hat{O}^\alpha(r)\,F_{12}\,H(r) \tag{4.16}$$

So, we need to compute F_{12}^2, using the property: $(\vec{\sigma}_1 \cdot \vec{\sigma}_2)^2 = 3 - 2(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$ we obtain:

$$F_{12}^2 = \left[(f_c + f_\tau)^2 + 3(f_\sigma + f_{\sigma\tau})^2\right] \mathcal{I} + 2(f_\sigma + f_{\sigma\tau})\left[(f_c + f_\tau) - (f_\sigma + f_{\sigma\tau})\right](\vec{\sigma}_1 \cdot \vec{\sigma}_2) \tag{4.17}$$

In our analysis we will study two cases in the first case we have only $(f_c + f_\tau) \neq 0$ therefore for a Fermi transitions we have:

$$\tilde{O}^F(r) = F_{12} \quad \hat{O}^F \quad F_{12}\,; H(r) = F_{12}^2 \;\; \mathcal{I} \;\; H(r) = (f_c + f_\tau)^2 + \mathcal{I} \;\; H(r) \tag{4.18}$$

and for a Gamow-Teller one:

$$\tilde{O}^{GT}(r) = F_{12}\,\hat{O}^{GT}\,F_{12}\,H(r) = F_{12}^2\,(\vec{\sigma}_1 \cdot \vec{\sigma}_2)\;H(r) = (f_c + f_\tau)^2(\vec{\sigma}_1 \cdot \vec{\sigma}_2)\,H(r) \tag{4.19}$$

In the second case we also have $(f_\sigma + f_{\sigma\tau}) \neq 0$ and so

$$\begin{aligned}\tilde{O}^F(r) = F_{12}\,\hat{O}^F\,F_{12}\,H(r) = F_{12}^2 \mathcal{I}\,H(r) = \\ = \left[(f_c + f_\tau)^2 + 3(f_\sigma + f_{\sigma\tau})^2\right] \mathcal{I}\;H(r) + \\ 2(f_\sigma + f_{\sigma\tau})\left[(f_c + f_\tau) - (f_\sigma + f_{\sigma\tau})\right](\vec{\sigma}_1 \cdot \vec{\sigma}_2)\,H(r)\end{aligned} \tag{4.20}$$

and

$$\begin{aligned}\tilde{O}^{GT}(r) = F_{12}\,\hat{O}^{GT}\,F_{12}\,H(r) = F_{12}^2\;(\vec{\sigma}_1 \cdot \vec{\sigma}_2)\;H(r) = \\ = 6(f_\sigma + f_{\sigma\tau})\left[(f_c + f_\tau) - (f_\sigma + f_{\sigma\tau})\right]\mathcal{I}\;H(r) + \\ \left[(f_c + f_\tau)^2 + 7(f_\sigma + f_{\sigma\tau})^2 - 4(f_c + f_\tau)(f_\sigma + f_{\sigma\tau})\right](\vec{\sigma}_1 \cdot \vec{\sigma}_2)\,H(r)\end{aligned} \tag{4.21}$$

what we can see is that in the second case the Fermi transitions acquire a "Gamow-Teller part" and vice versa.

Using the correlated operator instead of the standard operator in the calculation of the previous chapter we obtain the analytical form of the nuclear matrix elements that we are going to compute numerically in the following chapter.

Chapter 5

Numerical Results

We are now able to perform our calculation, we still need to choose some final details such as the subset of Hilbert space states involved into the process or the parameters for the harmonic oscillators wave functions and for the neutrino propagator, constraints on the sum over the quantum numbers and obviously the decaying nucleus.

First of all, we need to choose a nucleus in which neutrinoless double beta decay can occur, also, it's properties must be well reproduced by the shell model, because want to study effects which are not taken into account by the shell model and so, we need a nucleus with the simplest shell model structure to minimize complications connected to the approximations used in it, as said in the third chaper, nuclear shell model is able to well reproduce the properties of *magic nuclei* the best results is obtained using a *double magic nucleus*: a nucleus with a magic number of protons and neutrons.

Among the known nucleus with this properties we choose ${}^{48}Ca$ ($\mathcal{J}^{\pi} = 0^{+}$) the shell model structure of ${}^{48}Ca$ is quite simple: have 48 nucleons 20 of them are protons and the other 28 are neutrons Both of them are magic numbers and so, as shown in table 3.1 they complete their shells: the 28 neutrons fill all the levels of the orbitals: $1s_{\frac{1}{2}}$, $1p_{\frac{3}{2}}$, $1p_{\frac{1}{2}}$, $1d_{\frac{5}{2}}$, $2s_{\frac{1}{2}}$, $1d_{\frac{3}{2}}$ and $1f_{\frac{7}{2}}$; otherwise the 20 protons fill up the levels: $1s_{\frac{1}{2}}$, $1p_{\frac{3}{2}}$, $1p_{\frac{1}{2}}$, $1d_{\frac{5}{2}}$

, $2s_{\frac{1}{2}}$ and $1d_{\frac{3}{2}}$. Using the value for the harmonic oscillator wave function parameter for ^{48}Ca nuclei given in [16]: $\nu = 0.126 fm^{-2}$ we can numerically compute the state density (eq.3.21) for ^{48}Ca plotted in Figure: 5.1. We can see that, as said before harmonic os-

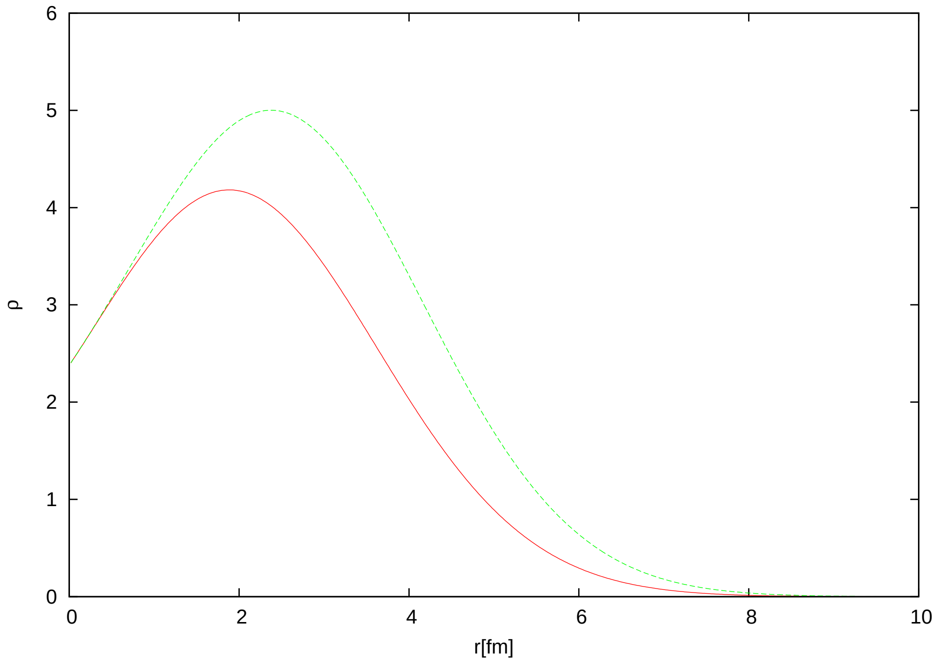

Figure 5.1: Nucleon densities for ^{48}Ca: protons in red and neutrons in green

cillator wave functions do not reproduce experimental densities in low r range, otherwise for the external levels we have a good match.

Manca: plot o referenza per confronto con dati sperimentali

5.1 Hilbert space

So our model reproduce nuclear properties of ^{48}Ca only for the external levels, to avoid this problem and to simplify our calculation we chose to limit the Hilbert space of two the decaying neutrons to the external shell. So our nuclear transition is given by two neutrons from the $1f_{\frac{7}{2}}$ which become two protons in their $1f_{\frac{7}{2}}$ shell. this implies referring to eq. 4.8 that:

$$k_1 = k_1' = k_2 = k_2' = 0 \tag{5.1}$$

$$l_1 = l_1' = l_2 = l_2' = 3 \tag{5.2}$$

$$j_1 = j_1' = j_2 = j_2' = \frac{7}{2} \tag{5.3}$$

Because the transition operator act only on the relative wave functions we have that the quantum numbers relative to the center of mass motion are conserved and so:

$$K = K' \tag{5.4}$$

$$L = L' \tag{5.5}$$

$$\tag{5.6}$$

It depends only on relative distance between the couple of nucleon then:

$$l = l' \tag{5.7}$$

$$\tag{5.8}$$

So the matrix element in eq.4.8 becomes:

$$\begin{aligned}
&\langle k_1' l_1' j_1', k_2' l_2' j_2', 0^+ | \tau_1^+ \tau_2^+ O^\alpha(r_{12}) | k_1 l_1 j_1, k_2 l_2 j_2, 0^+ \rangle_A = \\
&= \sum_{S,L} |\langle l_1, \frac{1}{2}, j_1; l_2, \frac{1}{2}, j_2 | \frac{1}{2}, \frac{1}{2}, S, l_1, l_2, \Lambda \rangle_J|^2 \quad \times \frac{1}{\sqrt{2S+1}} \langle \frac{1}{2}, \frac{1}{2}, S | \hat{O}_{12}^\alpha | \frac{1}{2}, \frac{1}{2}, S \rangle \\
&\quad \times \sum_{k,k',l} \sum_{K,L} \langle k, l, K, L | k_1, l_1, k_2, l_2 \rangle \quad \langle k', l, K, L | k_1, l_1, k_2, l_2 \rangle \times \langle k', l | H(r) | k, l \rangle
\end{aligned} \tag{5.9}$$

The conservation of energy is guaranteed by the following relation:

$$2k_1 + l_1 + 2k_2 + l_2 = 2k_1' + l_1' + 2k_2' + l_2' = 6 \tag{5.10}$$

and so:

$$2k + l + 2K + L = 2k' + l' + 2K' + L' = 6 \tag{5.11}$$

which implies, using eq.5.4 , 5.5 and 5.7 that

$$k = k' \tag{5.12}$$

This, simplify again the analytic form of the nuclear matrix element:

$$\begin{aligned}
&\langle k_1' l_1' j_1', k_2' l_2' j_2', 0^+ | \tau_1^+ \tau_2^+ O^\alpha(r_{12}) | k_1 l_1 j_1, k_2 l_2 j_2, 0^+ \rangle_A = \\
&= \sum_{S,L} |\langle l_1, \frac{1}{2}, j_1; l_2, \frac{1}{2}, j_2 | \frac{1}{2}, \frac{1}{2}, S, l_1, l_2, \Lambda \rangle_J|^2 \quad \times \frac{1}{\sqrt{2S+1}} \langle \frac{1}{2}, \frac{1}{2}, S | \hat{O}^\alpha_{12} | \frac{1}{2}, \frac{1}{2}, S \rangle \\
&\times \sum_{k,l} \sum_{K,L} |\langle k, l, K, L | k_1, l_1, k_2, l_2 \rangle|^2 \quad \langle k, l | H(r) | k, l \rangle
\end{aligned} \tag{5.13}$$

What is left to do now is to fix the bounds on the sums over the quantum numbers k,l,K,L,Λ,S and J. From the conservation of energy relation (eq.5.11) we can obtain some of those bounds:

$$\sum_{k,l} \sum_{K,L} \Rightarrow \sum_{L=0}^{6} \sum_{l=0}^{6-l} \sum_{K=0}^{\frac{1}{2}(6-L-l)} \tag{5.14}$$

after summing over L,l and K, the value of k in fixed by the conservation of energy. S is the total spin of a couple of two fermions with $s = \frac{1}{2}$ so, the sum over S runs from 0 to 1. Λ is the composition of orbital angular momenta of the two decaying nucleons whose have $l_1 = l_2 = 3$ so the sum over it can run from 0 to 6, but, to conserve parity we can take only even values of Λ, so $\Lambda = 0, 2, 4, 6$. J is the composition of total angular momenta of the nucleons couple whose have $j_1 = j_2 = \frac{7}{2}$ so the sum over it can run from 0 to 7, but we have to take only the even value because we are evaluating an antisymmetric matrix element, so $J = 0, 2, 4, 6$ Using the definition of 9-j symbols in eq.4.6 we find out the final expression for the nuclear matrix element:

$$M^{\alpha}_{0\nu} = \sum_{J=0,2,4,6} K_{sm}(J,0^{+}) \sum_{\Lambda=0,2,4,6} \sum_{S=0}^{1} 64\,(2\Lambda+1)\,(2S+1) \begin{pmatrix} 3 & \frac{1}{2} & \frac{7}{2} \\ 3 & \frac{1}{2} & \frac{7}{2} \\ \Lambda & S & J \end{pmatrix}^{2}$$
$$\times \frac{1}{\sqrt{2S+1}} \langle \frac{1}{2}, \frac{1}{2}, S|\hat{O}^{\alpha}_{12}|\frac{1}{2}, \frac{1}{2}, S\rangle$$
$$\sum_{L=0}^{6} \sum_{l=0}^{6-l} \sum_{K=0}^{\frac{1}{2}(6-L-l)} |\langle k,l,K,L|k_1,l_1,k_2,l_2\rangle|^{2} \quad \langle k,l|H(r)|k,l\rangle \tag{5.15}$$

The $K_{sm}(J,0^{+})$ coefficient can be found in [16] and they are show in table 5.1 Now we

	0	2	4	6
$K_{sm}(J,0^{+})$	1.214	−0.572	0.021	0.000

Table 5.1: $K_{sm}(J,0^{+})$ shell model coefficients

are able to perform our calculation in a pure shell model picture.

Mancano: parametri potenziale neutrino con referenze

5.2 Correlation functions

To extend our computation to the correlated wave function formalism we need to find the analytical form of the correlating function given in eq. 3.26, from eq.4.17 we see that for our numerical analysis we need the analytic form of the function f_c+f_τ and $f_\sigma+f_{\sigma\tau}$ we have used their numerical form given in ??? and fitted them using Gnu-plot and obtained:

$$f_1(r) = f_c + f_\tau = a - be^{-cr^2} \tag{5.16}$$

$$f_2(r) = f_\sigma + f_{\sigma\tau} = de^{-cr^2}(1 + ar + br^2) \tag{5.17}$$

The parameters are given in table 5.2 and their graphics are in figure 5.2 and 5.3.

Manca: referenza per funzioni fittate

$f_1(r)$	Value	Error	$f_2(r)$	Value	Error
a	1.00288	± 0.00014	a	2.92003	± 0.03582
b	0.75298	± 0.00056	b	-5.96629	± 0.05061
c	2.74232	± 0.00489	c	1.39292	± 0.00312
			d	0.04159	± 0.00016
χ^2/dof	4.04908×10^{-5}		χ^2/dof	1.14111×10^{-6}	

Table 5.2: Fit parameter for the correlation functions

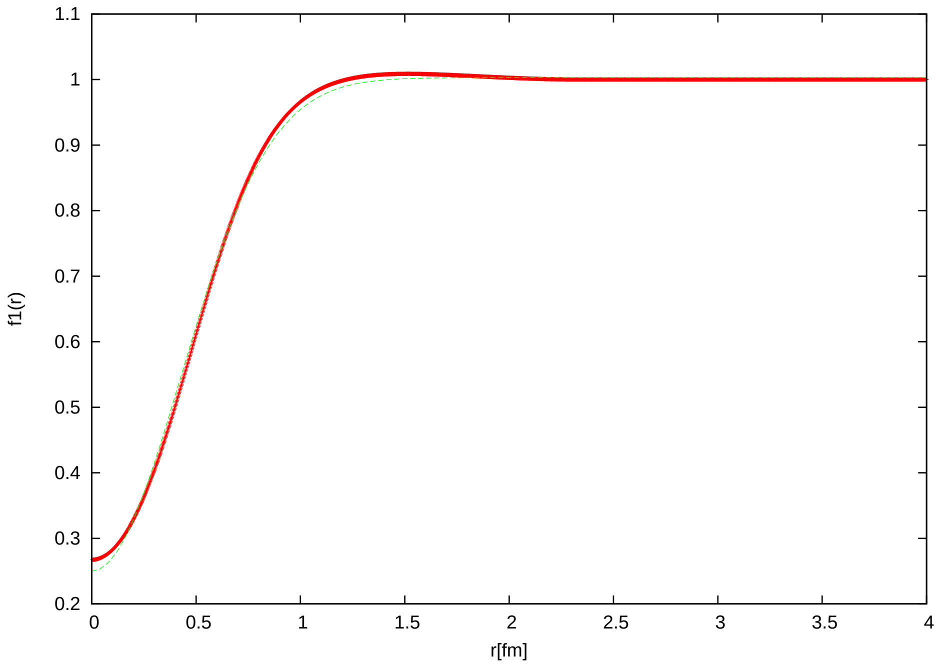

Figure 5.2: Fit for f_1: numerical data in red, fit function in green

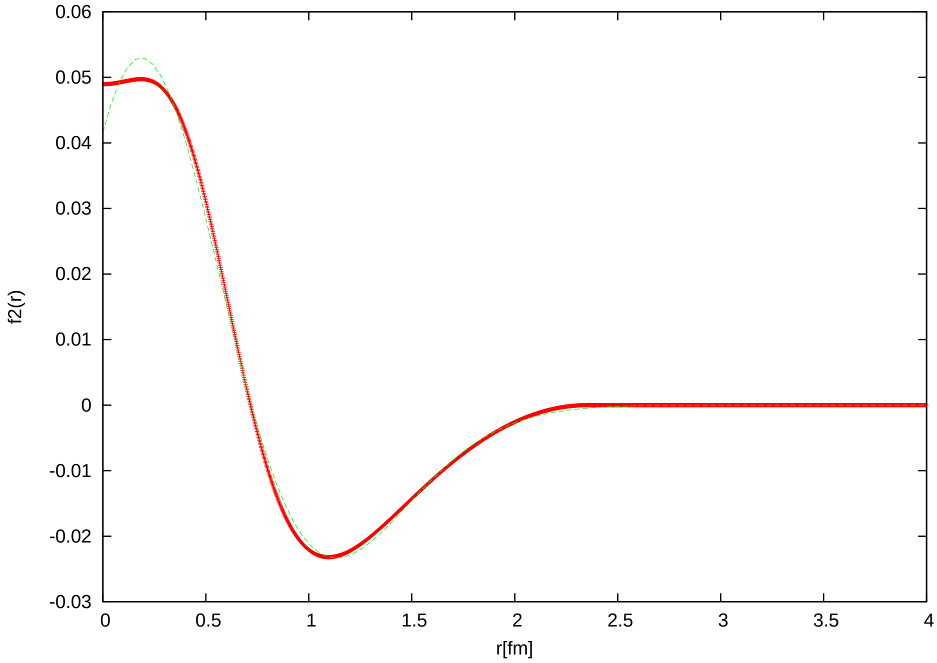

Figure 5.3: Fit for f_2: numerical data in red, fit function in green

5.3 Numerical computation

Now we have all the instruments to make our numerical computation using the definitions in eqs.5.16 , 5.17 and taking eq. 4.18 and 4.19 inside eq.5.15 we can write down the expressions for Fermi and Gamow-Teller nuclear transitions matrix elements in the case of $f_1 \neq 0$:

$$M_{0\nu}^F = \sum_{J=0,2,4,6} K_{sm}(J,0^+) \sum_{\Lambda=0,2,4,6} \sum_{S=0}^{1} 64\,(2\Lambda+1)\,(2S+1) \begin{pmatrix} 3 & \frac{1}{2} & \frac{7}{2} \\ 3 & \frac{1}{2} & \frac{7}{2} \\ \Lambda & S & J \end{pmatrix}^2$$
$$\sum_{L=0}^{6} \sum_{l=0}^{6-l} \sum_{K=0}^{\frac{1}{2}(6-L-l)} |\langle k,l,K,L|k_1,l_1,k_2,l_2\rangle|^2 \quad \langle k,l|f_1(r)^2\, H(r)|k,l\rangle \tag{5.18}$$

$$M_{0\nu}^{GT} = \sum_{J=0,2,4,6} K_{sm}(J,0^+) \sum_{\Lambda=0,2,4,6} \sum_{S=0}^{1} 64\,(2\Lambda+1)\,(2S+1) \begin{pmatrix} 3 & \frac{1}{2} & \frac{7}{2} \\ 3 & \frac{1}{2} & \frac{7}{2} \\ \Lambda & S & J \end{pmatrix}^2$$
$$[2S(S+1)-3] \sum_{L=0}^{6}\sum_{l=0}^{6-l}\sum_{K=0}^{\frac{1}{2}(6-L-l)} |\langle k,l,K,L|k_1,l_1,k_2,l_2\rangle|^2 \quad \langle k,l|f_1(r)^2\, H(r)|k,l\rangle \quad (5.19)$$

we also used eq. 4.9 and 4.10. Otherwise, in the case in which $f_1 \neq 0$ and $f_2 \neq 0$ using eq. 4.20 and 4.21 we have

$$M_{0\nu}^{F} = \sum_{J=0,2,4,6} K_{sm}(J,0^+) \sum_{\Lambda=0,2,4,6} \sum_{S=0}^{1} 64\,(2\Lambda+1)\,(2S+1) \begin{pmatrix} 3 & \frac{1}{2} & \frac{7}{2} \\ 3 & \frac{1}{2} & \frac{7}{2} \\ \Lambda & S & J \end{pmatrix}^2$$
$$\left\{ \sum_{L=0}^{6}\sum_{l=0}^{6-l}\sum_{K=0}^{\frac{1}{2}(6-L-l)} |\langle k,l,K,L|k_1,l_1,k_2,l_2\rangle|^2 \quad \langle k,l|\left(f_1(r)^2+3f_2(r)^2\right) H(r)|k,l\rangle + \right.$$
$$\left. [2S(S+1)-3] \sum_{L=0}^{6}\sum_{l=0}^{6-l}\sum_{K=0}^{\frac{1}{2}(6-L-l)} |\langle k,l,K,L|k_1,l_1,k_2,l_2\rangle|^2 \quad \langle k,l|2f_1(r)\left(f_1(r)-f_2(r)\right) H(r)|k,l\rangle \right\} \quad (5.20)$$

$$M_{0\nu}^{GT} = \sum_{J=0,2,4,6} K_{sm}(J,0^+) \sum_{\Lambda=0,2,4,6} \sum_{S=0}^{1} 64\,(2\Lambda+1)\,(2S+1) \begin{pmatrix} 3 & \frac{1}{2} & \frac{7}{2} \\ 3 & \frac{1}{2} & \frac{7}{2} \\ \Lambda & S & J \end{pmatrix}^2$$
$$\left\{ \sum_{L=0}^{6}\sum_{l=0}^{6-l}\sum_{K=0}^{\frac{1}{2}(6-L-l)} |\langle k,l,K,L|k_1,l_1,k_2,l_2\rangle|^2 \quad \langle k,l|6f_2(r)\left(f_1(r)-f_2(r)\right) H(r)|k,l\rangle + [2S(S+1)-3] \right.$$
$$\left. \sum_{L=0}^{6}\sum_{l=0}^{6-l}\sum_{K=0}^{\frac{1}{2}(6-L-l)} |\langle k,l,K,L|k_1,l_1,k_2,l_2\rangle|^2 \quad \langle k,l|\left(f_1(r)^2+7f_2(r)^2-4f_1(r)f_2(r)\right) H(r)|k,l\rangle \right\} \quad (5.21)$$

To run our numerical calculation we used FORTRAN 77 based programs, let's how we compute each of the components of this matrix elements:

Transition amplitudes: Objects like:

$$\langle k,l|g(r)|k,l\rangle = \int_0^\infty R_{kl}^2(r)g(r)r^2dr \quad (5.22)$$

to integrate it numerically we have to cut-off the integration

$$\langle k,l|g(r)|k,l\rangle = \int_0^{R_{max}} R_{kl}^2(r)g(r)r^2dr \tag{5.23}$$

with $R_{max} = 20fm$ and then discretize the sum:

$$\langle k,l|g(r)|k,l\rangle = \sum_{i=1}^{N} R_{kl}^2(i\,DR)\;g(i\,DR)\;(i\,DR)^2\;DR \tag{5.24}$$

where $N = 1000$ is the number of steps of the integration, and $DR = R_{max}/N$ is the width of the steps.

Talmi-Moshinsky brackets: They are the coefficient of the transformation between the (r_1, r_2) representation to the $(r = |\vec{r_1} - \vec{r_2}|, R = |\vec{r_1} + \vec{r_2}|/2)$ representation:

$$\langle k,l,K,L|k_1,l_1,k_2,l_2\rangle \tag{5.25}$$

In our program we used the FORTRAN 77 function **TMB** given in [18].

9-j symbols: The are needed for coupling of four angular momenta [1] they are symbolically written as:

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \\ j_7 & j_8 & j_9 \end{pmatrix} \tag{5.26}$$

to compute them, in our work we used the the program **W9J** also written in FORTRAN 77. Details on the program are given in [17]

5.4 Results

[1] l_1 , l_2 , j_1 , j_2

Conclusions

In this thesis...

Appendix A

Correlated two particles states

The correlation operator of eq. 3.25 is defined in such a way that, if any subset of the particles, say $i_1, \cdots i_p$, is removed far from the remaining $i_{p+1}, \cdots i_N$, it factorizes according to

$$\mathcal{F}(1, \cdots, N) \to \mathcal{F}_p(i_1, \cdots i_p)\mathcal{F}_{N-p}(i_{p+1}, \cdots i_N) \tag{A.1}$$

The above property is the basis of the *cluster expansion formalism*, that allows one to write the matrix element of a many-body operator between correlated states as a sum, whose terms correspond to contributions arising from isolated subsystems (clusters) involving an increasing number of particles.

Let us consider for example the expectation value of the Hamiltonian in the correlated ground state, the starting point is the definition of the generalized normalization integral:

$$I(\beta) = \langle 0| \exp[\beta(H - T_0)]|0\rangle \tag{A.2}$$

T_0 is the shell model ground state energy. Using the previous equation we can rewrite the expectation value of the Hamiltonian in the form:

$$\langle H \rangle = \frac{\langle 0|H|0\rangle}{\langle 0|0\rangle} = T_0 + \frac{\partial}{\partial \beta} \ln I(\beta) \mid_{\beta=0} \tag{A.3}$$

Exploiting the cluster property of $\mathcal{F}$ one can rewrite the expectation value of the Hamiltonian in the form:

$$\langle H\rangle = T_0 + (\Delta E)_2 + (\Delta E)_3 + \cdots (\Delta E)_N \tag{A.4}$$

because we are interest in coupling correlations between nucleons we need only $(\Delta E)_2$, it can be express as a function of F_{ij} (eq. 3.26)

$$(\Delta E)_2 = \sum_{i<j} \langle ij| \frac{1}{2}\left[F_{12}; \left[\frac{1}{m}\nabla_r^2; F_{12}\right]\right] + F_{12}v_{12}F_{12}|ij - ji\rangle \tag{A.5}$$

where v_{12} is the potential between the two nucleons $|ij\langle$ is the two particle states and $|ij - ji\rangle$ is it's antisymmetrized version. By minimization of $(\Delta E)_2$ [1] one can find Euler-Lagrange equations for the correlation functions which can be numerically integrated.

Spiegazione del perchè funziona nella materia continua ma va bene anche per il nucleo

[1]more details on this procedure are given in [15]

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