GAUGE THEORIES, SPIN GLASSES AND REAL GLASSES

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ABSTRACT

In this talk I will show that usual spin glasses are a peculiar kind of Abelian gauge theory. I will shortly review the techniques used to study them. At the end I will consider more general models (e.g. spin glasses based on non Abelian gauge group) and I will discuss the relevance of these models to real glasses. Finally I will derive from first principles a generalised Vogel-Fulcher law for the divergence of the characteristic time near the glass transition.

1. Introduction

The aim of my talk is to show how some of the subjects on which Oscar Klein worked (in particular I refer to gauge theories) have been useful in apparently far away subjects, i.e. spin glasses and real glasses. Indeed some models of spin glasses are a gauge theories of a rather peculiar type.

I will start (in the second section) by briefly recalling the definition of a spin glass and presenting an idealised model. Later (in the third section) I will show that this model is in reality a gauge theory corresponding to the Z_2 group. Next (in the forth section) I will review the basic ingredients of the replica method which are used to study these models. I will later (in the fifth section) consider Abelian and non Abelian generalisations and I will derive from first principles a generalised Vogel-Fulcher law for the divergence of the characteristic time near the glass transition. In the last section I will argue that these non Abelian models may share many properties with real glasses.

2. Spin Glasses

I introduce here a very simple model of spin glasses[1,2]. I consider a material where there are three kinds of atoms: M, A and B. M is a magnetic atom (it has a non zero magnetic moment) while A and B are magnetically inert.

I suppose that at low temperature the system crystallises in such a way that the M-atoms stay on a regular lattice and the A and B atoms stay on the links of the same lattice. The position of the magnetically inert atoms is supposed to be random. In other words we consider an $M\ A_x\ B_{100-x}$ alloy; the case $x = 50$ corresponds to an equal proportion of A and B.

Let us assume that the magnetic interaction among the M-atoms is of the nearest neighbour type and that it is mediated by the non magnetic atoms. The interaction among two M-atoms is ferromagnetic if the link is occupied by an A atom, while it is antiferromagnetic if the link is occupied by a B atom. We also assume for simplicity that the strengths of the ferromagnetic and of the antiferromagnetic interactions are equal.

Usually the magnetic interaction is relevant only at temperatures much lower than the melting temperature and it may be neglected during the formation of the alloy. At low temperature the atoms may only oscillate around their equilibrium positions. If the temperature is decreased fast enough the position of the atoms in not influenced by the magnetic interaction. We can describe this situation by saying that we are in presence of a *quenched* disorder.

If we assume that the spin are Ising variables, the corresponding Hamiltonian, in presence of a magnetic field h, is

$$H_U[\sigma] \equiv -\sum_{i,k} \sigma_i U_{i,k} \sigma_k - h \sum \sigma_i \ . \tag{1}$$

The variables σ are defined on the sites of the lattice and they take the values ± 1. The variables $U_{i,k}$ are defined on the links of the lattice, i.e. when i and k are nearest neighbours; they also take the values ± 1.

The variables U are random independent variables. For each choice of the U we can define a statistical expectation value:

$$\langle g(\sigma) \rangle_U = \frac{\sum_\sigma \exp(-\beta H_U[\sigma]) g(\sigma)}{\sum_\sigma \exp(-\beta H_U[\sigma])}. \tag{2}$$

We are interested in computing the statistical expectation values averaged over the probability distribution of the samples, in other words the quantity

$$\langle g(\sigma) \rangle \equiv \overline{\langle g(\sigma) \rangle_U} \equiv \int dP(U) \langle g(\sigma) \rangle_U \ , \tag{3}$$

where we denote by an horizontal bar the average over the U and $P(U)$ is the probability distribution of the variables U. We have already remarked that the probability distribution of the variables U does not depend from their interaction with the σ: for each physical realization of the system they are fixed (quenched) variables.

In the infinite volume limit the expectation value of intensive quantities does not depend on the realisation of the couplings U (i.e. all the samples have essentially the same properties) and the sample to sample fluctuations vanish in this limit.

A very interesting quantity to evaluate is the magnetic susceptibility. A naive computation would give the following formula

$$\chi = \beta(1 - \overline{\langle \sigma_i \rangle_U^2}) = \beta(1 - \overline{m(i)_U^2}) = \beta(1 - q_{EA}) \ , \tag{4}$$

where $m(i)_U \equiv \langle \sigma_i \rangle_U$ is the site dependent spontaneous magnetisation and $q_{EA} \equiv \overline{m(i)_U^2}$ is the so called Edward Anderson order parameter[1]. We shall see later how this formula for the susceptibility is modified by a more sophisticated treatment.

Physical intuition tell us that at high temperature at zero magnetic field there is no spontaneous magnetization and consequently $q_{EA} = 0$. At low temperature each sample should develop its own spontaneous magnetization, and consequently $q_{EA} \neq 0$ at low temperature. The non vanishing of q_{EA} should therefore mark the spin glass transition.

Before studying this model further it is convenient to analise its symmetries and in particular the consequences of gauge invariance.

3. Gauge invariance

Let us consider two different systems such that the couplings of the two systems are equal in all links not connected to a given point i and they differ by a sign only for the links connected to that point. These two sets of couplings have the same probability to be realised (all sets of couplings have the same probability). They have also the same free energy because the Hamiltonian does not change (at zero magnetic field) if simultaneously we change sign to the spin variable sitting at the point.

In other words both the measures (on U and σ) and the Hamiltonian are invariant with respect to the local gauge transformation[3]:

$$\sigma_i \to -\sigma_i \ ; \qquad U_{i,k} \to U_{i,k} \ . \tag{5}$$

The set of all possible realizations of the system at zero magnetic field is gauge invariant under the gauge group Z_2. The couplings and the spins play respectively the role of the gauge connection and of the matter field. At non-zero magnetic field the gauge invariance is explicitly broken. The Hamiltonian at zero magnetic field is (apart from a constant) the square of the covariant lattice-gradient of the σ variables. It can be written as

$$\sum_i \sum_\mu (\sigma_i - U_{i,i+\mu}\sigma_{i+\mu})^2 \ . \tag{6}$$

The relevant quantities are gauge invariant. If two realizations of the couplings U and U' differ by a gauge transformation, their thermodynamic properties are the same. It is important to concentrate our attention on gauge invariant quantities.

Let us study in more details how the thermodynamical quantities depend on the choice of the couplings. A quantity which is often used to characterise the gauge fields is the Wilson loop. We can associate to each closed circuit on the lattice the ordered product of all the links of the circuit. We thus define:

$$W(C) \equiv \prod_{(i,k)\in C} U_{i,k} \ . \tag{7}$$

This quantity may take the values ± 1. If $W(C) = -1$, the loop is said to be *frustrated*[3]: along that loop it is not possible to find a configuration of spins such that

$$U_{i,k}\sigma_i\sigma_k = 1 \quad \forall \ (i,k) \in C. \tag{8}$$

If there are frustrated loops (i.e. if there is no gauge transformation which brings all couplings U to 1) it is not possible to find a configuration of the spins such that all terms in the Hamiltonian are positive. Some defects (i.e. links for which the contribution to the energy is negative) must be present.

At low temperature the equilibrium probability distribution is concentrated on those spin configurations which have the minimal energy. It is interesting to study also those configurations which are local minima of the Hamiltonian in the sense that the Hamiltonian increases when we flip a spin. These local minima are very important in the dynamics outside equilibrium because at low temperatures the system may be trapped for a very long time in these minima.

If we study the structure of local and global minima with great care, we discover that frustration implies the presence of defects which can be put in many ways on the lattice. The ground state is degenerate. The number of local and not global minima is also large.

This phenomenon is well known in gauge field. On the lattice the choice of the Landau gauge corresponds to find the maximum of

$$\sum_{i,k} \mathrm{Tr} g_i^{\star} U_{i,k} g_k, \tag{9}$$

where g is the gauge transform which brings the gauge fields in the Landau gauge[4]. This problem is equivalent (in the our case) to find the minimum of the Hamiltonian eq. (1). Gribov ambiguity tell us that in the general case there are many possibility of choosing the gauge. This result implies the existence of many minima of the Hamiltonian eq. (1).[*]

4. The Replica Method

We face now the problem of evaluating the quantities which appear in equation (2). The first proposal would be to sum over the variables U and to remain with an effective interaction for the variables σ. This approach clashes with fact that both the numerator and the denominator of eq. (2) depend on the variables U and the sum over the U is not easy.

This difficulty may be avoided by introducing n identical copies (or replicas) of the same system [1]. We define

$$\langle g(\sigma) \rangle_n = \frac{\sum_\sigma \sum_U \exp(-\beta H_n[\sigma, U]) g(\sigma^1)}{\sum_\sigma \sum_U \exp(-\beta H_n[\sigma, U])}, \tag{10}$$

where the spins σ_i^a carry an other index (a) which ranges from one to n. The new Hamiltonian is the sum of n identical Hamiltonians

$$H_n(\sigma, U) = \sum_{a=1,n} H_U[\sigma^a]. \tag{11}$$

[*]In the continuum Gribov ambiguity is normally present in non Abelian gauge theories. It is also present in Abelian theories when we allow configurations which are singular in the continuum limit, e.g. like magnetic monopoles.

It easy to check that

$$\langle g(\sigma)\rangle_n = \frac{\sum_U \langle g(\sigma)\rangle_U (Z_U)^n}{\sum_U (Z_U)^n}, \tag{12}$$

where the U-dependent partition function is defined as

$$Z_U = \sum_\sigma \exp(-\beta H_U[\sigma]). \tag{13}$$

We finally find that

$$\langle g(\sigma)\rangle = \langle g(\sigma)\rangle_n|_{n=0} \ . \tag{14}$$

In this way one finds that properties of the matter fields averaged over the disordered (quenched) gauge fields can be computed by considering the gauge fields interacting with n copies of the matter field and computing the expectation values in the limit $n \to 0$. The argument is familiar to those who work in numerical simulation of latttice gauge theories, where n plays the role of the number of quark flavours in the sea.[†]

The average of the U fields can be done and one remains with an effective interaction for the σ variables. This effective interaction must be written in terms of the gauge invariant combinations. In this case the most appropriate variables are

$$Q_i^{ab} = \sigma_i^a \sigma_i^b. \tag{15}$$

The diagonal terms of the matrix Q are identically equal to 1, so that we can consider only the off-diagonal term. All computations must be done for generic values of n and we must send n to zero at the end.

As usually we can expand the effective interaction in powers of Q, being careful to preserve the various symmetries of the problem, i.e.:

- The group of permutations of the n replicas $S(n)$.
- The spin reversal symmetry for each replica, i.e. n times the direct product of the Z_2 global group, where each Z_2 group acts on a different replica.[‡]

In the continuum limit in D dimensions the simplest form for the effective free energy, in which all important terms are contained, is the following :

$$F[Q] = \int d^D x \left(\frac{1}{2} \sum_\mu \mathrm{Tr}(\partial_\mu Q(x))^2 + W(Q(x)) \right), \tag{16}$$

where the function $W(Q)$is given by

$$W(Q) = \tau \mathrm{Tr}(Q^2) + g_3 \mathrm{Tr}(Q^3) + g_4 \mathrm{Tr}(Q^4) + y \sum_{ab} Q_{ab}^4 \ . \tag{17}$$

[†]It is evident why I proposed the name *quenched* for the approximation of neglecting quark loops in QCD.

[‡]This symmetry is present only at zero magnetic field.

The usual strategy consists in computing the minimum of F and constructing the usual perturbative expansion for the small fluctuations around this minimum (the so called loop expansion). As we shall see even the first step is not very simple.

Let us assume that the $F[Q]$ is minimized by a function $Q(x)$ which does not depend from x. We can this set $Q(x) = Q$ and look for the minimum of $W(Q)$.

For $\tau > 0$ we find that there is only a minimum at $Q = 0$. When τ is negative we easily find that there is a stationary points at

$$Q_{ab} = q, \quad \forall\ a\ b. \tag{18}$$

This point is invariant under the replica group $S(n)$. In order to decide if this stationary point is a minimum of W, we have to compute the small fluctuations around this point. The Hessian

$$\mathcal{H}_{ab,cd} = \frac{\partial W}{\partial Q_{ab} Q_{cd}}\ , \tag{19}$$

must have non negative eigenvalues (in a field theory language there should be no negative squared masses).

If we consider the case where $y = 0$, the $S(n)$ symmetry is promoted to an $O(n)$ symmetry (i.e. there is an accidental symmetry). This $O(n)$ symmetry is spontaneously broken by the choice in eq. (18). Consequently the Hessian has zero eigenvalues (i.e. there are Goldstone Bosons).

If y is different from zero, the $O(n)$ symmetry is explicitly broken and the Goldstone Bosons acquire a mass squared proportional to y. A detail computation shows that in the general case y is negative and the Goldstone Bosons acquire a negative mass squared (i.e. the Hessian has negative eigenvalues).

This instability implies that the proposed stationary point is not a local minimum and one has to look for a minimum which will be no more invariant under the S_n group. Such a minimum can be constructed [5,6]; we have to introduce an infinite sequence of steps breakings the $S(n)$ symmetry. After a rather long computation one finally finds a matrix Q to which one can associate a function $q(u)$, where u belongs to the interval $0 - 1$.

There is a simple physical interpretation of this symmetry breaking. The presence of frustration implies that for each realization of the couplings there are different equilibrium states with different local magnetisation (i.e. different *vacua*). We indicate by w^α the probability of finding the system in the state labelled by α and by m_i^α the local magnetization (i.e. the expectation value of σ_i in the state α). The overlap among different states may be defined as

$$q^{\alpha\gamma} = \frac{\sum_{i=1,N} m_i^\alpha m_i^\gamma}{N}. \tag{20}$$

After some computations one finds that

$$\overline{\sum_{\alpha\ \gamma} w^\alpha w^\gamma f(q^{\alpha\gamma})} \equiv \int dq P(q) f(q) = \int_0^1 du f(q(u)). \tag{21}$$

The probability of having two states with given overlap q is thus controlled by the function $q(u)$.

This approach is able to explain in a qualitative and sometimes a quantitative way many of the properties of real spin glass. When the replica symmetry is broken, a change in the external magnetic field changes the structure of equilibrium states and one must be quite carefully in the definition of the susceptibility. One finds that there are two susceptibilities:

- The linear response susceptibility which quantifies the response to a small variation of the magnetic field measured on a short time scale such that no global rearrangement of spins is possible. This susceptibility is given by $\chi_{LR} = \beta(1-q_{EA}$.
- The thermodynamic susceptibility which quantifies the response to a small variation of the magnetic field on a very long time scale such that global rearrangements of spins are possible. This susceptibility is larger[§] and it is given by $\chi_{eq} = \beta(1 - \int duq(x))$.

The difference between these two susceptibilities is one of the most characteristic phenomena experimentally observed in spin glasses and it is well explained by this theory[2].

At this stage of the theory, where fluctuations are neglected, is not clear which of the theoretical predictions does survive when the effect of the fluctuations is included. In order to have more precise theoretical prediction for real three dimensional systems it is necessary to go beyond the mean field approximation. This has been the subject of intensive studies. Unfortunately the computation of the corrections to the mean field theory are rather involved. The correlation functions at zero loops are rather complicated [7] and the one loop corrections to the correlation functions have not yet fully computed.

Numerical simulations in 4 dimensions [8,9] strongly suggest the correctness of the broken replica picture. No anomaly is observed. Numerical simulations in 3 have not produced a compleately clear picture, although some progresses have recently been done [10,11].

When we will master the loops corrections, we will be hopefully able to set up a renormalization group study of the properties of the system in the low temperature region and in particular to compute the value of the lowest critical dimension, i.e. the dimension where the structure of replica symmetry breaking scheme is no more consistent and the predictions of the mean field theory do not apply anymore.[¶]

5. Other Gauge Groups

There is no need to consider only the Z_2 group. The generalisations to other groups are very interesting. In particular the case of the $U(1)$ group is relevant for

[§] One finds that $q_{EA} = \max(q(u)))$.

[¶] For the $O(n)$ symmetry the lowest critical dimension is 2, for the Z_2 symmetry it is 1.

the study of irreversibility in high T_c superconductors [12]. Here we consider a different gauge group, non-Abelian, for which new results can be found.

In this model each spin can take p values[13]. The gauge field $U_{i,k}$ is an element of the permutation group of p elements[14]. The Hamiltonian is

$$H_U[\sigma] = \sum_{i,k} \delta_{\sigma_i U_{i,k} \sigma_k} \ . \tag{22}$$

The gauge group is the group of permutation of p objects ($S(p)$). If $p = 2$ we recover the previous case ($Z_2 = S(2)$).

Due to the non Abelian nature of the gauge group there is no global $S(p)$ symmetry at fixed U. As an effect of this decrease in the symmetry the effective free energy contains an extra cubic term

$$W(Q) = \tau \mathrm{Tr}(Q^2) + g_3 \mathrm{Tr}(Q^3) + z \sum_{ab} Q_{ab}^3 + O(Q^4), \tag{23}$$

which is forbidden in the Abelian case at zero magnetic field.

The value of z is crucial. If z is small with respect to g_3 ($z = 0$ at $p = 2$) we find that the value of Q at the saddle point is a continuous function of the temperature. For large values of z (in mean field we need $z > 4$) we fins that the value of Q jumps discontinuously at the phase transition T_c.

A dynamical computation shows that in these conditions there are metastable states above the critical temperature, which correspond to vacua having high free energy of the true ground states. The system may be trapped in these false vacua, and it may remain there for a quite long time [15,16].

If we consider only point independent minima of the effective Hamiltonian we find that there is a dynamical temperature (T_D) at which the characteristic time (t_c) of the system diverges[17].

Below T_D the characteristic time diverges exponentially with the size of the system. This exponentially large time may be computed as follows. We introduce an effective potential $V(q)$ defined as:

$$-NV(q) = \int dP[\sigma] \ln \left(\int dP[\mu] \delta(q - q_{\sigma\mu}) \right), \tag{24}$$

where

$$q_{\sigma\mu} = \frac{\sum_{i=1,N} \sigma_i \mu_i}{N} \tag{25}$$

and for simplicity we use the short notation

$$\int dP[\sigma] \equiv \sum_{\sigma} \exp(-\beta H_U[\sigma]). \tag{26}$$

Using the techniques of [18,19] this potential can be computed by considering the effect of coupling $R = 1 + \epsilon$ replicas in the limit $\epsilon \to 0$. It turns out that this potential has always a minimum at $q = 0$. For $T < T_D$ we finds that this potential acquires a new minimum at $q = q_m$, the two minima being obviously separated by a maximum at $q = q_M$.

If we assume that the dynamics may be approximated by the motion of the system in this potential, we find that

$$t_c \sim \exp(N\delta(T)), \tag{27}$$

where the function $\delta(T)$ is different from zero below T_D and is given by

$$\delta(T) = V(q_M) - V(q_m), \tag{28}$$

i.e. by the free energy barrier that the system has to cross.

A characteristic time which increases exponentially with the size of the system cannot be the correct answer in a system with a short range Hamiltonian. It is well known that in this case metastable states do not exist: false vacua do decay by thermodynamic tunnelling and the characteristic time can be computed by evaluating the free energy of the critical droplet (or instantons in a field theoretical language).

If we use the thecnique of [20] to study the instantons of these theories we find out that the characteristic time is finite also below T_D:

$$t_c \sim \exp(\Delta(T)), \tag{29}$$

where the function $\Delta(T)$ diverges at the critical temperature as

$$\Delta(T) \propto (T - T_c)^{-(D-1)} \tag{30}$$

These results can be qualitatively easily explained. The situation is quite similar to the usual enucleation theory in which we have to estimate the size of the critical droplets. The time for escaping from the false vacuum is given by the exponential of the instanton action (or the exponential of beta times the free energy barrier). An instanton of radius R has two contributions to its action

$$A(R) = -\tau R^D + \Sigma R^{(D-1)} \tag{31}$$

The first is a bulk contribution, which vanishes at the critical temperature ($\tau \propto T - T_c$) and the second is an surface term (the free energy of the interface). The radius for which $A(R)$ is maximum diverges as the inverse of τ and the free energy excess at this point is proportional to $(T - T_c)^{-(D-1)}$.

Near the critical temperature the evolution of the systems is dominated by the rearrangement of spins in large domain, whose radius R diverges at the critical temperature. Larger and larger barriers must be crossed when we go near T_c and the characteristic times diverges at this temperature.

In the Z_2 case discussed in the previous section there are no metastable states already in the mean field approximation [21,22]. In short range models the characteristic time is divergent at the transition, but it divergent as a power law, i.e. as $(T - T_c)^{-\lambda}$, where $\lambda = 2$ in the mean field approximation. The new feature of the non Abelian models is the exponential divergence of the characteristic time at the phase transition. This divergence makes extremely hard to thermalize the systems also at temperatures not so close to T_c.

6. Toward Real Glasses

In spin glasses the Hamiltonian is random as an effect of quenched random disorder. In real glasses the Hamiltonian is not random and the quenched disorder is dynamically generated at low temperature. We can therefore ask how much of the qualitative and quantitative results which have obtained in spin glasses may be transferred to real glasses [23–25].

In order to understand this point one can study models in which the Hamiltonian does not contain quenched disorder and to compare the results with those coming of random Hamiltonian[26–28].

In general our strategy is the following. We want to study the properties of a given Hamiltonian H_G which is not random. We consider a class of Hamiltonians H_R, of which H_G is a particular case. We choose the class H_R in such a way that the statistical properties of H_G and that of a generic Hamiltonian in H_R are as similar as possible. In the best case we can obtain that the two corresponding free energies coincide in the high temperature expansion. In general the behaviour of the system can be controlled better in the high temperature phase.

After having constructed H_R in an appropriate way, we can suppose that the statistical properties of H_G and H_R are the same or, if they are different, we can construct a perturbative expansion which compute this difference. It is clear that this approach may be successful in the high temperature region (more or less by construction) and it may also reproduce the behaviour of the system in the glassy region, included the dynamic and static transitions. However it is cannot certainly reproduce the possible existence of an ordered *crystal* phase.

The proposed strategy for approximating a given Hamiltonian with a random Hamiltonian works very well in the many cases, at least in the framework of the mean field approximation[27–32]. Real glasses have a qualitative behaviour that is quite similar to the one described in the previous section[‖] It is therefore tempting to suppose that also real glasses belongs to the same universality classes of system with a random Hamiltonian. If this conjecture is correct equation (30) should apply also to real glasses.

It well known that glasses show an extremely large increase in the value of the viscosity (which is proportional to the characteristic time) near the glass transition. This increase of fifteen order of magnitude has been measured in many materials. In the case of fragile glasses this increase has been fitted by the phenomenological Vogel-Fulcher law [34]:

$$\eta \propto \exp(C(T-T_c)^{-1}). \tag{32}$$

[‖]Theoretical arguments can be done which point in this direction [33].

The prediction of the replica approach for a three dimensional systems is

$$\eta \propto \exp(C(T-T_c)^{-2}). \tag{33}$$

Unfortunately the data are not sensitive enough to make a clear cut choice among the two proposed formulae, although the data are fitted slightly better by eq. (33) than by the traditional Vogel-Fulcher law. More careful studies of this point are needed.**

It is quite amazing that starting from the study of some peculiar forms of gauge theory on the lattice we finally end up with a partial theoretical understanding of the phenomenological Vogel-Fulcher law, which have been already proposed when Oscar Klein was young.

The possibility of applying these geometrical ideas (which originate from particle physics) to such far away systems as spin glasses and real glasses, shows the substantial unity of theoretical physics, a field to which Oscar Klein gave so many and important contributions.

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** A comparison with other theories[35,36] of the glass transition would make this talk too long.

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