Static chaos and scaling behaviour in the spin-glass phase

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Abstract

We discuss the problem of static chaos in spin glasses. In the case of magnetic field perturbations, we propose a scaling theory for the spin-glass phase. Using the mean-field approach we argue that some pure states are suppressed by the magnetic field and their free energy cost is determined by the finite-temperature fixed point exponents. In this framework, numerical results suggest that mean-field chaos exponents are probably exact in finite dimensions. If we use the droplet approach, numerical results suggest that the zero-temperature fixed point exponent θ is very close to $\frac{d-3}{2}$. In both approaches $d = 3$ is the lower critical dimension in agreement with recent numerical simulations.

1 Introduction

One of the most interesting open problems in spin glasses regards a correct understanding of the nature of the low temperature phase, i.e. the spin-glass phase [1, 2]. Spin glasses are characterized by a strong freezing at a certain critical temperature. Below that temperature a complete description of the nature of the static phase is still missing.

During the last years there have been several developments in the field, the most well known being the mean-field theory [3]. Unfortunately mean-field theory has revealed a complex theoretical structure which is very obscure when applied to non exactly solvable models for which some kind of perturbation theory is needed.

There are other approaches to spin-glasses which are known as phenomenological droplet models, a complete description of them has been given by D. S Fisher and D. Huse [7, 6]. The main idea underlying these models is that the spin-glass behaviour is governed by the zero-temperature fixed point in the renormalization group equations. [4, 5]. Up to now it seems that the Parisi solution to mean-field theory is essentially correct. It has passed the stability analysis [8] and gives also a correct description of the thermodynamics, in agreement with the numerical simulations. It is not clear what is the correct description of the spin-glass phase in short-ranged models. Droplet models are expected to be a good description of the low temperature phase mainly in the case of low dimensions. But droplet models are not suited to describe the physics of high dimensional systems and particularly mean-field theory.

The complexity of the replica approach is found when studying the spectrum of fluctuations around the Parisi solution. The full set of gaussian propagators has revealed a very complex structure [9] and the obtention of the one-loop corrections to the mean-field equations makes progress slow. The main difficulty of this task is the enormous number of sectors within replica space which contribute to the one-loop correction. This explains also why finite-size corrections to the main thermodynamic functions are still unknown in mean-field theory [10]. To all these problems should be added also the fact that, up to now, the major part of the computations have been done only close to T_c within the Parisi approximation.

In this work we will try to introduce a different approach to the problem which can help in understanding the nature of the spin-glass phase. The main idea of the approach is to try to look for one order parameter whose spectrum of fluctuations is easier to take into account. The chaos problem was proposed some time ago and concerns the chaotic nature of the spin glass phase [12, 11]. The term chaotic can be misleading since it can evoke different meanings. In this context, we prefer to use the word static chaos. By this we mean that a small perturbation of the Hamiltonian is enough to reshuffle the Boltzmann weights of the different equilibrium configurations. One constructs a system which is the sum of two Hamiltonians, the initial plus the perturbed one. The full system lives in a larger phase space and allows for a new order parameter. This order parameter is the overlap between the equilibrium configurations of the initial system and the perturbed one. This new order parameter has a longitudinal spectrum of fluctuations without zero modes and hence is stable. The associated correlation functions to this order parameter decay exponentially to zero with a caracteristic correlation length.

The nature of the chaos problem is also interesting concerning numerical techniques like simulated annealing where the the change of the temperature has to be considered as a perturbation to the system. In this case one wants to reach the ground state after a progressive cooling of the system. Let us suppose that the spin glass behaves chaotically against temperature changes. Then, the equilibrium configurations should reorganize completely for any small change of the temperature and a slow cooling would be useless. A small change of temperature would be considered like a new quenching and the system would be always strongly far from equilibrium. Fortunately, as we will discuss later for this particular problem, if there is chaos against temperature changes then it is small and finite-size corrections ensure that a high degree of correlation between the equilibrium configurations at the two temperatures is preserved.

The work is divided as follows. In the following section we will present a quantitative definition for chaoticity and we will introduce different type of chaotic perturbations. We will also present predictions from mean-field theory and phenomenological droplet models. Section 3 is devoted to the study of a particular perturbation, i.e. chaoticity against changes of magnetic field. Starting from the mean-field approach we propose a scaling behaviour within the spin glass phase. We discuss also our predictions in the framework of the droplet approach. Section 4 is devoted to numerical simulation results and finally we present our conclusions and a discussion of the results.

2 A working definition for chaos

The idea underlying the chaoticity of the spin-glass phase relies on the fact that it is a marginal phase [9]. The fact that the spin-glass phase is not fully stable means that a small addition of energy to the system is able to change completely the statistical weights of the equilibrium states with a very small cost of free energy (of order $1/N$ compared to the supplied energy to the system, where N is the size of the system).

Marginality is one of the outstanding results in mean-field theory of spin glasses. It is also a feature of phenomenological droplet models and in general it is related to the fact that in the spin-glass phase spatial (time) correlation functions decay very slowly with distance (time). This decay is not far from a power law in the most general case. The full reorganization of equilibrium states in spin glasses after a small perturbation is a natural feature in mean-field theory. In this case there is an infinite number equilibrium states and all of them contribute to the partition function but with a different weight [14]. This is because they have equal free energies per site except differences of order $1/N$. Any small but finite addition of energy to the system is enough to redistribute these small free energy differences reshuffling the weights of the different pure states.

In droplet models there exists the concept of overlap length (it is sometimes denoted $L_{\Delta T}$ or $L_{\Delta H}$ according to the case if the perturbation is a change of the temperature or the magnetic field). Droplet models suppose that there is only one equilibrium state. When the system is perturbed, the correlation functions reorganize completely in a scale of distances larger than the characteristic overlap length. It is clear anyway that the overlap length in these models has to be always smaller than the correlation length and coincide only in the limit in which the perturbation vanishes.

In what follows $< ... >$ and $\overline{(.)}$ mean thermal and disorder average respectively. Now we want to give an appropiate definition of what is static chaos. For simplicity we will consider Ising spin glasses even though the definition can be generalized to other models. Let us suppose an Ising spin glass system with Hamiltonian $H_1[\sigma]$. Then we apply a perturbation P to the system and the new Hamiltonian for a different copy of spins $\{\tau_i\}$ is given by

$$H_2[\tau] = H_1[\tau] + P[\tau] \tag{1}$$

We consider now a full Hamiltonian which is the initial system $H_1[\sigma]$ plus the perturbed one $H_2[\tau]$, i.e. $H[\sigma,\tau] = H_1[\sigma] + H_2[\tau]$. The phase space has been enlarged and we can consider a new order parameter which, for example, in the case of Ising spin glasses, is given by the overlap $\langle\sigma_i\tau_i\rangle$ between the equilibrium configurations of the system H_1 with the equilibrium configurations of the perturbed one H_2. When there is no perturbation, i.e. $P = 0$, this is the usual order parameter of the spin glass with Hamiltonian $H = H_1 + H_2$.

We now define the chaoticity parameter r by:

$$r(P) = \frac{\overline{\langle\sigma_i\tau_i\rangle_H^2}}{(\overline{\langle\sigma_i^a\sigma_i^b\rangle_{H_1}^2}\;\overline{\langle\tau_i^a\tau_i^b\rangle_{H_2}^2})^{\frac{1}{2}}} \tag{2}$$

where $\langle\sigma_i^a\sigma_i^b\rangle$ denotes the order parameter evaluated taking two copies a, b of the unperturbed system H_1 and similarly for $\langle\tau_i^a\tau_i^b\rangle$ of the perturbed system H_2. The thermal average of the order parameter in the numerator is performed with the full Hamiltonian H. In principle, this order parameter is equal to one if the perturbation is zero. This is trivial because r is the order parameter of the spin glass normalized to itself. Chaoticity in spin glasses reflects the fact that any small but finite perturbation P causes the parameter r to fall abruptly to a value smaller than $r = 1$. Obviously this can only happen in the thermodynamic limit because, for a finite size N, the chaos parameter r will always be a smooth function of the perturbation P. This means that one has to perform the thermodynamic limit before applying the perturbation P. More precisely, the spin-glass phase is chaotic if

$$\lim_{P\to 0}\lim_{N\to\infty} r(P) < 1 \tag{3}$$

It is also possible to define the adimensional quantity

$$a = \frac{\overline{\langle\sigma_i\tau_i\rangle_H^2}}{\overline{\langle\sigma_i^a\sigma_i^b\rangle^2}_{H_1}} \tag{4}$$

where the numerator is obtained by averaging over the Hamiltonian $H[\sigma\tau]$ and the denominator is the order parameter for two copies a and b of the same system $H_1[\sigma]$. The difference between the adimensional quantities a and r is only an appropiate normalization. In fact, the definition of chaos given above in eq.(3) also holds in case of the parameter a. The necessity to distinguish among the parameter a and the parameter r is important for certain types of perturbations. For example, in the case of temperature changes the order parameter $\langle\sigma_i^a\sigma_i^b\rangle$ is very sensitive to the temperature and vanishes at the critical point. Let us suppose the initial Hamiltonian is in the low temperature phase and we change the temperature by putting the system close to the critical point. The chaos parameter a vanishes because the numerator in eq.(4) vanishes close to T_c and the denominator remains finite. On the contrary, the chaos parameter r of eq.(2) has in the denominator a term which also vanishes at T_c and normalizes appropiately the numerator. Because r measures correctly the overlap among equilibrium configurations it is the appropiate parameter to deal with in case of temperature changes. The difference between the chaos parameters a and r is not important in the case of magnetic field changes and other types of perturbations.

Let us now discuss what happens in the case of ordered systems. As an example we take the standard Ising model in a finite number of dimensions. Let us suppose that we are in the low temperature phase, at temperature T below the critical point, and let us take as a perturbation a small change of the temperature. At the temperature T the system has a spontaneous magnetization m. There is only one equilibrium configuration with a fraction m of the spins pointing in a certain direction. When we change a little bit the temperature by a small quantity ΔT, the mean number of spins which point in that direction (i.e. the magnetization) changes linearly with ΔT at least for ΔT small. In this case one sees inmediately that eq.(2) gives the value $r(\Delta T) = 1$ for any small change of the temperature and the system is not chaotic. We represent an equilibrium state by an N dimensional vector $v = (m_1, m_2, ..., m_N)$ where the m_i are the local spin magnetizations. In the Ising model a slight change of the temperature modifies only the length of this vector but not its direction. The chaos mechanism in spin glasses is driven by the fact that as soon as we perturb the system this vector v suffers a sudden rotation because the weight of the different states are changed. In most cases, any small perturbation makes the vector v to become orthogonal to its previous value and $r(P)$ vanishes for a finite perturbation P.

There are many examples of perturbations that one can apply to the system. As was mentioned in the introduction, one can change the temperature or change the magnetic field. These are among the most studied perturbations in the literature. But one can imagine other kinds of perturbation like for instance changing the realization of disorder. In this case, a finite fraction of the J_{ij} couplings is changed (for instance, in case of symmetric distribution of couplings, this change could consist in reversing the sign of the perturbed couplings J_{ij}). One can also imagine to add a small ferromagnetic or antiferromagnetic part to the couplings. In these cases, spin glasses seem to behave chaotically against these perturbations. As an example, we show in figure 1 how the parameter r decreases with the size N, for the case of the SK model for two different perturbations. The first perturbation corresponds to a very large change of temperature ($\Delta T = 0.4$) for an initial temperature $T = 0.5 = T_c/2$ (the perturbed system is at $T = 0.9$ which is very close to T_c but always in the spin-glass phase.) The other perturbation is the application of a small magnetic field $h = 0.2$ to a system initially at zero magnetic field and $T = 0.6$ (the AT line lies at $h \sim 0.4$). The results for N less than 20 have been obtained by calculating exactly the partition function, the remaining ones using Monte Carlo simulations. From figure 1, the system seems much more sensitive against magnetic field perturbations than temperature changes. This is clear also if we observe that under the temperature perturbation, even though it is very strong because it puts the perturbed system close to the paramagnetic phase, the equilibrium configurations at both temperatures still retain a high degree of coherence ($r \sim 0.7$).

In the following, we will focus on the study of a particular perturbation, which has turned out manageable in order to understand its effects in the spin glass phase: the case in which the perturbation consists in applying a small magnetic field to a spin-glass at zero field. This has been the subject of previous research, specially by I. Kondor in the case of mean-field theory [15]. One could also study the case in which the system is at a finite field in the spin-glass phase and the field is slightly changed. This problem is more subtle than the previous one in which the system is initially at zero magnetic field. The main reason is that (at least for short ranged systems) we do not know if the spin-glass phase survives to a magnetic field. If the spin-glass phase survives to the magnetic field then we expect (as predicted in the mean-field approach) that chaoticity will be present in a magnetic field. In the other case (and this is the prediction of droplet models), the system would then be always in the paramagnetic phase and chaos should not be present. Then, according to eq.(2), $r(P) = 1 + O(\Delta h)$ would be continuous for $\Delta h = 0$.

In the case the system is initially at zero magnetic field mean-field approach and droplet models agree in that they both predict that the spin-glass phase is chaotic. More specifically $r(h)$ (we use the intensity of the applied magnetic field h for the pertubation P) is zero for any finite h. But the main mechanism which makes the spin-glass phase chaotic is very different in both pictures. In phenomenological droplet models the spin-glass phase is marginal: the correlation functions decay very slowly with the distance and the correlation length associated to the two point function $C(x) = \overline{\langle\sigma(0)\sigma(x)\rangle^2}$ is infinite. When a magnetic field is applied the spin-glass phase is destroyed and the correlation length becomes finite. It is given by [7]:

$$\xi \sim (q_{EA}\, h^2)^{\frac{1}{2\theta - d}} \tag{5}$$

with q_{EA} the Edwards-Anderson order parameter and θ the thermal exponent which gives the characteristic energy scale L^θ of droplet excitations of typical size L. This means that all excitations of droplets of sizes larger than a certain length ξ will be suppressed by the field. The exponent θ is a zero temperature exponent (it is determined by the zero temperature fixed point of the renormalization group equations) and it is expected to be constant in the low temperature phase. In the critical point the associated thermal exponent θ_c is determined by the finite temperature fixed point of the renormalization group equations and is related to the critical exponents by $\theta_c = \frac{d-2+\eta}{4}$ where η is the anomalous dimension exponent an d is the dimension (even though it has been argued that at low dimensions there appears a new exponent θ_c [17]). In general, we expect that θ_c is smaller than θ above the lower critical dimension and both vanish in the lower critical dimension.

Mean-field theory approach gives a completely different mechanism of chaoticity. After applying a small magnetic field, the spin-glass phase is not destroyed. We suppose that the effect of the magnetic field is the dissapareance of a large number (infinite) of equilibrium states. This mechanism is easy to visualize by taking into account the correct order parameter for spin glasses which is the distribution $P(q)$. Its physical meaning was explained some time ago [18, 19] and it gives the probability density that two pure states α and β have a common overlap $q_{\alpha\beta} = q$. This common overlap corresponds to the scalar product of the local spin magnetization in both states. At zero magnetic field the function $P(q)$ is symmetrically distributed around $q = 0$ and non zero within the interval $(-q_{max}, q_{max})$. In a magnetic field the reversal symmetry $\sigma \to -\sigma$ is broken and $P(q)$ is non zero only for q positive and larger than a minimum value q_{min}. The value of q_{max} is nearly independent of the magnetic field (this approximation, which works extremelly well close to T_c, is called the Parisi-Toulouse hypothesis [16]). In some sense the effect of the magnetic field is to suppress those equilibrium states α which had overlaps $q_{\alpha\beta}$ with the other remaining states β smaller than q_{min}. Within the usual picture of the spin-glass phase in mean-field theory [14] there is an infinity of states with a few number of them dominating the Gibbs measure. This infinity of states lay in the tips of an ultrametric tree and the effect of the magnetic field corresponds to progressively cutting those branches which generate the states which are suppressed. The suppression of the states also conserves the ultrametricity property. The understanding of how pure states α are suppressed by the field according to their statistical weight w_α is still an interesting open problem.

3 Chaos in magnetic field

This section is devoted to the study of chaos in case of an Ising spin glass initially at zero field after turning on a magnetic field. We are interested in the case of a d-dimensional Ising spin glass with random J_{ij} couplings defined by the Hamiltonian

$$H = -\sum_{(i,j)} J_{ij}\sigma_i\sigma_j \; - \; h\sum_i \sigma_i \tag{6}$$

where the couplings J_{ij} are quenched variables distributed according to a probability function $P(J)$ of zero mean and finite variance. The interaction is restricted to nearest neighbours and h is the magnetic field. The Ising spins

σ_i take the two possible values ± 1 and live in a d-dimensional hypercubic lattice. In the limit $d \to \infty$ one expects to converge to mean-field theory, i.e. the SK model. In the SK model all spins interact one to each other and the couplings J_{ij} are normalized by a factor $1/\sqrt{N}$ where N is the number of spins.

3.1 The case of mean-field theory

This question was adressed some time ago by I. Kondor [15]. Let us consider two copies of the same realization of disorder, one at zero magnetic field and the other one at finite magnetic field h. The full Hamiltonian of the problem is given by:

$$H[\sigma\,,\tau] = H_1[\sigma] + H_1[\tau] - h\sum_i \tau_i \tag{7}$$

with

$$H_1 = -\sum_{1\le i<j\le N} J_{ij}\,\sigma_i\,\sigma_j \tag{8}$$

This problem can be directly solved using the standard replica trick for the full Hamiltonian $\sum_{a=1,n} H_a$ where H_a is given by eq.(7) and a is the replica index which runs from 1 to the full number of replicas n (at last one takes the limit $n \to 0$). Now one applies the replica trick

$$\overline{\log Z} = \lim_{n\to 0} \frac{\overline{Z_J^n} - 1}{n}, \tag{9}$$

which yields the expression

$$\overline{Z_J^n} = \int dP_{ab}dQ_{ab}dR_{ab}\exp(-NA[PQR]) \tag{10}$$

with

$$\begin{aligned} A[PQR] \;\; = \;\; & \sum_{a<b}(P_{ab}^2 + Q_{ab}^2 + 2R_{ab}^2) + \sum_a R_{aa}^2 - \log Tr_{\sigma\tau}\,\exp\Big(\beta^2\sum_{a<b} P_{ab}\sigma_a\sigma_b \\ & + \beta^2\sum_{a<b} Q_{ab}\tau_a\tau_b \;+\; \beta^2\sum_{a\neq b} R_{ab}\sigma_a\tau_b \;+\; \beta^2\sum_a R_{aa}\sigma_a\tau_a \\ & + \beta h\sum_a \tau_a) \end{aligned} \tag{11}$$

In this way, one is able to reduce the problem in terms of a lagrangian $A[P\,Q\,R]$ with three order parameters corresponding to the different overlaps among the two copies, i.e $P_{ab} = \langle \sigma_a \sigma_b \rangle$; $P_{aa} = 0$, $Q_{ab} = \langle \tau_a \tau_b \rangle$; $Q_{aa} = 0$ and $R_{ab} = \langle \sigma_a \tau_b \rangle$. For finite h there is an inmediate solution for the equations of motion which is given by the P_{ab} and Q_{ab} Parisi matrices with zero and magnetic field h respectively and $R_{ab} = 0$. The free energy of the whole system is the sum of the free energy of one copy at zero magnetic field plus the free energy of the other copy with field h. This is a solution because it gives the full free energy of two uncoupled systems. The order parameter associated to R is

$$q = \sum_i \sigma_i \tau_i \tag{12}$$

In order to study the stability of this solution one computes the spectrum of fluctuations. The full set of fluctuations is very complex. For instance, within the subspaces generated by the diagonal subblocks P and Q, it corresponds to the spin-glass spectrum derived by C. De Dominicis and I. Kondor [9]. Only the fluctuations around $R = 0$ (the off-diagonal subblock) are those which are physically relevant to the problem because they measure spatial correlations between states corresponding to the two Hamiltonians, the initial and the perturbed one. In mean-field theory there are no distances and we want to obtain the spatial behaviour of the system within the mean-field approximation. This can be done using a Ginzburg-Landau approximation by introducing spatially dependent order parameters in the effective action of eq.(10). Now the order parameters P, Q, R depend on the space variable x and we add to the action A a kinetic term of the type $\sum_{a<b} \frac{\partial^2 R_{ab}(x)}{\partial x^2}$. The spectrum of fluctuations is contained in the the momentum space propagator. This is given by the Fourier transform $G(p)$ of the correlation function

$$C(x) = \overline{\langle \sigma_0 \sigma_x \tau_0 \tau_x \rangle} \tag{13}$$

where $\overline{(\cdot)}$ means averaging over disorder and $\langle (\cdot) \rangle$ is the usual thermal average over the Hamiltonian eq.(7).

The problem of computing the propagator reduces to the diagonalization of a hierarchical matrix of the Parisi type. The full expression has been reported in [15]. Its singular part is given by

$$G(p) = \int_0^{q_{max}} dq \int_{Q_{min}}^{Q_{max}} dQ \frac{p^2 + 1 + \lambda(q)\lambda(Q)}{(p^2 + 1 - \lambda(q)\lambda(Q))^3} \tag{14}$$

with

$$\lambda(q) = \beta(1 - q_{max} + \int_q^{q_{max}} dq\, x(q)) \tag{15}$$

where β is the inverse of the temperature. The same expression applies in the case of $\lambda(Q)$. Here $q(x)$ and $Q(x)$ are the order parameter functions for the spin glass at zero and h field respectively.

The correlation function eq.(13) decays to zero for large distances x with a characteristic length ξ which is given by the minimum eigenvalue of the stability matrix. This eigenvalue is non-zero for finite h which demonstrates the stability of the $R = 0$ solution. The correlation length ξ (which is the inverse square root of the minimum eigenvalue) diverges like $(1-\lambda(Q_{min}))^{-\frac{1}{2}}$. Close to $T_c = 1$ we have $Q_{min} \sim h^{\frac{2}{3}}$. This gives $\xi \sim h^{-\frac{2}{3}}$ which diverges when $h \to 0$.

The stability of the $R = 0$ solution implies that the system is chaotic. This means that $r(P)$ (as given by eq.(2)) always vanishes for finite h like $\frac{1}{N}$, where N is the size of the system. The result that ξ diverges when h goes to zero is rather natural because in that case the perturbation vanishes and the two copies are identical. Then the correlation length ξ is the spin-glass correlation length which is infinite because there is marginal stability. In some sense, there is a first order phase transition at $h = 0$ where the probability distribution associated to the order parameter q defined in eq.(12) changes from a delta function peaked in $q = 0$ to the usual order parameter distribution for the spin glass [20]. Also the critical point is chaotic but in this case the correlation lenth diverges like $\xi \sim h^{-\frac{1}{2}}$. We remind the reader that in the paramagnetic phase there could not be chaos of the type defined in eq.(3). This is because the correlation length ξ would never diverge but only converge smoothly to its corresponding finite value at zero magnetic field.

Now we turn to the behaviour of the propagator $G(p)$ of eq.(14) in the limit $p \to 0$. Using the known expressions [21] for $q(x)$ and $Q(x)$ close to T_c in eq.(14) we obtain a divergent expression for $G(0)$. Its most divergent part is

$$G(0) \sim \int_{Q_{min}}^{Q_{max}} \frac{dQ}{(1-\lambda(Q))^{\frac{5}{2}}} \tag{16}$$

which gives $G(0) \sim p^{-4} \sim \xi^4 \sim h^{-\frac{8}{3}}$. This is not new and this result can also be obtained from the study of the intravalley gaussian propagators as derived in [22]. We can define a certain kind of non-linear susceptibility by:

$$\chi_{nl} = \sum_i C(i) = G(0) \sim h^{-\frac{8}{3}} \tag{17}$$

This susceptibility can also be written $\chi_{nl} = N\,\overline{\langle q^2 \rangle}$ with q given in eq.(12). Using the fact that $R = 0$ is a stable solution altogether with eq.(17), the following scaling behaviour holds

$$a \equiv r \sim f(Nh^{\frac{8}{3}}) \tag{18}$$

This result will be derived in the following section using scaling arguments and will be also generalized to short-range models.

3.2 Scaling theory of chaos with magnetic field

Next we want to give a precise physical meaning to the correlation length ξ. As commented in the previous section, the spin glass phase is marginal with an infinity of equilibrium states, none of them having a characteristic correlation length. Under a small magnetic field, a lot of states are suppressed and the correlation length ξ is finite. We interpret ξ as the new typical correlation length of the states which have been suppressed. This is the natural continuation of what comes out in the critical point. In this case there is only one marginal state. After applying a magnetic field, the correlation length becomes finite and the system goes into the paramagnetic phase. In the spin-glass phase the suppressed states acquire finite correlation length and their free energy increases respectively to the remaining ones. We are still within the spin-glass phase because the remaining states dominate the partition function and they still have infinite correlation length. In the spin-glass phase all equilibrium states are non equivalent. Some of them have a much higher statistical weight. This means that only those states $\{\alpha\}$ which give overlaps $\{q_{\alpha\beta} \leq q_{min}\, \forall\beta\}$ are simply erased by the magnetic field. When all states are suppressed we reach the AT line [13]. This can only happen in case when an infinity of equilibrium states coexist in the low temperature phase.

If we want to be more precise we have to generalize these ideas to the case of short-range models. Two basic assumptions are enough to this aim. The first one concerns the physical interpretation on the effect of the magnetic field on the equilibrium states. The second one uses information in the critical point to understand what happens in the spin-glass phase. More precisely, we suppose that the cost in free energy of the dissapearing states in the spin-glass phase, scales in the same way as in the critical point. We argue that the low temperature spin-glass phase is determined by the finite temperature fixed point of the renormalization group equations. This is the contrary assertion of droplet models in which the spin-glass beahviour is governed by the zero-temperature fixed point. Our assumptions give exact results in mean-field theory. The existence of some critical properties in the low temperature phase has been also seen in a different context. For example, it has been proved that the exponent which characterizes the decay of the tail of the $P(q)$ around $q = q_{max}$ freezes below the critical point in mean-field theory [10]. In short-range Ising spin glasses there are also numerical results which suggest that the freezing of some critical exponents really takes place in the low temperature phase [23, 24].

At the critical point we know that the singular part of the free energy (per site) is given by

$$f_{sing} \sim h^2 q \sim q^{\frac{d}{[q]}} \tag{19}$$

Here q is the usual order parameter defined in eq.(12), d is the dimension and $[q]$ is the dimension of the operator Q_{ab} in units of the inverse of the correlation length. the value $[q]$ is connected to the critical exponents β, ν and η via the relation $[q] = \frac{\beta}{\nu} = \frac{d-2+\eta}{2}$.

Now we generalize this expression to the case in which replica symmetry is broken, i.e. in the spin-glass phase. First of all, we need a general expression for the singular part of the free energy which is invariant under the permutation group of the different replicas. The most easy expression of this type is

$$f_{sing} \sim \sum_{a<b} Q_{ab}^{\frac{d}{[q]}} \tag{20}$$

where the exponent $[q]$ is given by the critical exponents.

One can easily derive the correct behaviour of the singular part of the free energy for an order parameter $q(x)$ of the type shown in figure 2. To obtain the correct singular part of the free energy corresponding to the states which are suppressed by the field we have to take the difference of eq.(20) with $h \neq 0$ and $h = 0$

$$\begin{aligned} f_{sing} &= \sum_{a<b} Q_{ab}^{\frac{d}{[q]}}(h) - \sum_{a<b} Q_{ab}^{\frac{d}{[q]}}(0) = \\ & \int_0^1 (q_h^{\frac{d}{[q]}}(x) - q_0^{\frac{d}{[q]}}(x))\, dx = x_{min}\, q_{min}^{\frac{d}{[q]}} = q_{min}^{\frac{d}{[q]}+1} \end{aligned} \tag{21}$$

The main ingredient that we have used in this derivation is the fact that the order parameter $q(x)$ in short-range models is characterized by a continuous part plus a plateau. Under the application of a magnetic field a new plateau appears with $q(x) = q_{min}$ and $x_{min} \sim q_{min}$. This last result is connected with the fact that the order parameter distribution $P(q)$ at zero magnetic field is finite for $q = 0$. Because $P(q) = \frac{dx(q)}{dq}$ (where $x(q)$ is the invers of the $q(x)$) this means that $q(x) \sim x$ for x close to zero. In the critical point the previous derivation applies with $x_{min} = 1$ and $q \sim h^{\frac{2}{\delta}}$

where $\delta = \frac{d+2-\eta}{d-2+\eta}$. For droplet models the same derivation is valid but now $x_{min} = 1$ and $P(0)$ vanishes in the thermodynamic limit like $L^{-\theta}$ with θ the zero-temperature fixed point exponent. The singular part of the free energy scales like ξ^{-d}, ξ being given by eq.(5).

Now we apply eq.(21) to mean-field theory. Mean-field critical exponents together with $\eta = 0$ give $[q] = 2$. Because $q_{min} \sim h^{\frac{2}{3}}$ we obtain $f_{sing} \sim h^{\frac{8}{3}}$ and the global singular free energy scales like $N\, h^{\frac{8}{3}}$. Because the parameters a and r are adimensional we reproduce the scaling behaviour of eq.(18).

3.3 Estimate of the AT line and the lower critical dimension

The first result which comes out from the previous subsection is that $d = 4$ plays a role as a special critical dimension. This deserves some explanation. The upper critical dimension in Ising spin glasses is 6. Above 6 dimension the critical exponents coincide with the mean-field ones. These exponents are associated with the order parameter $Q(x)$ corresponding to the overlap $\sigma^a(x)\sigma^b(x)$ between two copies a, b with the same Hamiltonian. The correlation length associated to the two point function $\langle Q(0)Q(x)\rangle$ diverges at the critical point and remains infinite in the low temperature phase. The chaos correlation length is associated to the two point function $\langle R(0)R(x)\rangle$ and corresponds to a different order parameter $R(x) = \sigma(x)\tau(x)$ which couples two systems with different Hamiltonians. We argue that the exponent of the chaos correlation length ξ associated to $R(x)$ lies in a different universality class of that to which the order parameter $Q(x)$ belongs.

We can find the appropiate upper critical dimension associated to the criticality of chaos. From eq.(18) and using $\xi \sim h^{-\frac{2}{3}}$ we obtain an argument of the scaling function for a of the form L/ξ in four dimensions. This means that $d_u = 4$ has the role of an upper critical dimension.

In the most general case we can introduce the exponent λ defined by

$q_{min} \sim h^{\frac{2}{\lambda}}$. We expect the following scaling to be satisfied

$$a \equiv r \sim f(N\, h^{\frac{2(d+[q])}{\lambda[q]}}) \tag{22}$$

The value $[q]$ depends on the critical exponents and this scaling contains only one non critical parameter (λ) and thus is easily measurable in a simulation.

The exponent λ is theoretically unknown and there is no numerical prediction on its value. From the value of λ one obtains the AT line using the condition $q_{min} \sim q_{max}$. Because $q_{max} \sim \tau^{\beta}$ with $\tau = \frac{T_c - T}{T_c}$ the AT line is given by the equation $h \sim \tau^{\frac{\beta\lambda}{2}}$. In mean-field theory $\beta = 1$, $\lambda = 3$ gives $h \sim \tau^{\frac{3}{2}}$. In $d = 4$ depending on the value of λ and β, a different expression is found. This and the special case $d = 3$ are left as a discussion in the next section.

One can also estimate what is the lower critical dimension d_l. In fact, we expect that the scaling eq.(22) for $T_c = 0$ should be of the form $a \equiv f(N\, h^2)$. This gives $d+[q] = \lambda[q]$, i.e. $[q](\lambda-1) = d$. The exponent λ should diverge as d approaches d_l because q is discontinuous at $T_c = 0$ when a magnetic field is applied. So $[q] = 0$, i.e, $d_l - 2 + \eta = 0$ which is the usual relation determining the lower critical dimension [25] (in principle this relation should at least be satisfied for Hamiltonians with a countinuous distribution of couplings). Furthermore, in case $d = d_l$ one expects $\xi \sim h^{-\frac{2}{d_l}}$. Because $\xi \sim h^{-\frac{2}{3}}$ for $d = d_u = 4$ this means that $d_l = 3$ at zero order of approximation or mean-field level. We call it mean-field level or zero order because in this case we suppose the exponent for the correlation length ξ does not vary between $d_u = 4$ and $d = 3$. The fact that 4 and 3 are very close assures that this is a good approximation which is probably exact.

We should now recall that all these predictions have to be apropiately modified for droplet models. For these models there is no transition in a magnetic field. The condition $d_l - 2 + \eta = 0$ also applies and the exponent θ is well approximated by the result $\theta = \frac{d-3}{2}$. This is in agreement with the numerical results of the following section. Recent numerical simulations for case $d = 2$ show that the zero temperature exponent $\theta \sim -0.46$ [26] is surprisingly close to the chaos prediction -0.5. This suggests that in the framework of droplet models also $d_l = 3$ and the previous expression for θ are probably exact.

4 Numerical results

In this section we present Monte Carlo numerical simulations in order to test these ideas. We should note that the chaos parameters a and r defined in this work and all the scaling laws based on them are computable using standard numerical simulations. The standard technique is to consider two parallel Monte Carlo simulations, one for the system H_1 and the other one for the perturbed system $H_2 = P[H_1]$. The first copy is at zero magnetic field while the second one has a magnetic field h. Both copies evolve in time and, once they have thermalized, one computes the corresponding order parameters. Since one is interested in scalings within the spin-glass phase, the main difficulty is that samples have to be equilibrated in the low temperature phase where metastability is very strong. All the results in this section are for small lattices and we have paid attention that they are fully equilibrated. The general schedule of the simulation is as follows. An initial cooling is performed until the first copy at zero magnetic field thermalizes at the working temperature and the second copy thermalizes with an applied magnetic field equal to a maximum value h_{max}. Then, the first copy evolves without any perturbation and the field of the second copy is progressively decreased step by step down to zero. In general, for each different value of the magnetic field of the second copy, a long enough thermalization is done after which statistics is collected. Then, the order parameters a and r of eqs.(4) and eq.(2) can be computed.

The Hamiltonian under study is given by eq.(6). In all simulations we have used the heat-bath algorithm and spins are updated sequentally. In case of short-range models we impose periodic boundary conditions. The distribution of the coupling J is discrete (the J_{ij} can take the values ± 1 with equal probability). If there is a finite temperature phase transition we expect universality to apply (anyway see [27]) and the results for discrete couplings should be equivalent to the case in which the distribution is continuous. Our main goal is now to test scaling laws of the type eq.(22). Scaling fits work well if we use the parameter r or the parameter a (in all cases they differ very slightly, approximately by 5 per cent). Then, we will present results only for the parameter a.

We now show the results in case of mean-field theory. The results for different magnetic fields ranging from 0.2 up to 1.0 at $T = 0.6$ are shown in figure 3 for several sizes. We show the parameter a versus $h\,N^{\frac{3}{8}}$ (we have chosen this argument instead of $N\,h^{\frac{8}{3}}$ in order to compare directly these mean-field results with those corresponding to short-range models.) There is an agreement with the prediction of eq.(18). At the critical point $T = 1$ the appropiate scaling argument is $N\,h^3$ and in order to compare with the scaling law eq.(18) of figure 3 we show results for $T_c = 1$ in figure 4. If in figure 3 we plot the chaos parameter a in function of $N\,h^{\frac{8}{3}}$ (instead of $h\,N^{\frac{3}{8}}$) we discover that the scaling functions of figures 3 and 4 are clearly different suggesting that the criticality of chaos in the critical point and in the low temperature phase are in a different universality class.

Next we present results for the case $d = 4$. Figure 5 shows the parameter a as a function of $L\,h^{\frac{2}{3}}$. This is the mean-field scaling which is in full agreement with data. Simulations were performed at $T = 1.5$ ($T_c \simeq 2.05$ [28]) which is $\simeq 0.7T_c$. Metastability effects are very strong and thermalization is more difficult (in the sense that one needs more thermalization steps) than in the SK model case. Error bars are not shown because they are very small (of order of the size of the symbols). The agreement with the theoretical prediction is good.

Then we can derive results for the AT line. Using eq.(22) we get $\lambda = 4.2$ which gives $q_{min} \sim h^{0.48}$. In the critical point taking $\eta \sim -0.25$ and using the hyperscaling relation $\delta = \frac{d+2-\eta}{d-2+\eta}$ where $q \sim h^{\frac{2}{\delta}}$ one gets the result $\delta \sim 3.6$. In the critical point, q scales with the magnetic field with a larger exponent $q \sim$

$h^{0.55}$ and one has the impression that this is a general feature at any dimension (in the mean-field case q scales linearly with the field in the critical point while the minimum overlap scales like $h^{\frac{2}{3}}$ in the spin-glass phase.) Because the critical exponent $\beta \sim 0.6$ and $\eta = -0.25 \pm 0.1$ the corresponding AT line should scale like $h \sim \tau^{1.3 \pm 0.1}$ which is close to the mean-field theory result (even though there is no reason that it should coincide). Unfortunately we have no means to test if this prediction is correct, mainly because the question of the existence of the AT line is still unsolved [29, 30, 31]. In the framework of droplet theory we can derive the correct value of the zero-temperature exponent θ. It gives $\theta = 0.5$ (in the critical point the finite-temperature exponent θ is $\theta_c \sim 0.43$). Figure 5 also gives valuable information in the case there is no AT line. The chaos correlation length should correspond to the correlation length of the spin-glass in the paramagnetic phase as given in eq.(5). In the case of $d = 4$ we obtain $\xi \simeq 5h^{-\frac{2}{3}}$ if we estimate ξ as the distance over which the chaos paarmeter a decreases by an order of magnitude. In order to search numerically for the existence or not of phase transition in magnetic field at $T = 1.5$ one should study lattice sizes $L > \xi$ where ξ is given by the previous expression.

We analyze now the data for $d = 3$. Simulations were performed for the $\pm J$ nearest-neighbour Ising spin glass. Recent numerical simulations suggest that there is only a singularity at $T = 0$ [32]. But, due to the so large correlation length, we expect that the system will have some kind of pseudocritical behavior for small lattices. In fact, standard numerical simulations for small sizes show that $T_c \sim 1.2$ with $\eta \sim -0.1\,, \beta \sim 0.5$ [33, 34]. This means that, even if there is no true phase transition in the thermodynamic limit, simulations for small lattices should be sensitive to the pseudocritical behaviour and finite-size scaling for the chaos parameter a should give information regarding this pseudocritical point. What comes out is very interesting and has been plotted in figure 6. The mean-field result $\xi \sim h^{-\frac{2}{3}}$ fits very well the data. This is in agreement with the fact that $d_l = 3$ which is the value for the lower critical dimension if mean-field theory is exact. If $d_l = 3$ then any finite T belongs to the paramagnetic phase which is characterized by the true finite correlation length ξ_T (in order to distinguish it from the chaos correlation length ξ). The chaos correlation length ξ would behave like $h^{-\frac{2}{3}}$ in the regime $\xi << \xi_T$. In the regime $\xi_T \sim \xi$ the value of the chaos correlation length ξ should progressively match the value of ξ_T and remaining finite. We can derive in this regime of sizes the location of the pseudocritical AT line. From eq.(22) we derive $q_{min} \sim h^{0.26}$ which gives $h \sim \tau^{1.9}$ for the critical exponent $\beta \sim 0.5$. This result is not far from experimental determinations of the AT line in bulk CuMn spin glasses [35] (where a scaling of the form $h \sim \tau^{1.8}$ with an exponent slightly larger than the mean-field result 3/2 is compatible with experiments.)

5 Conclusion

It seems that the study of static chaos in spin glasses can give interesting predictions of the nature of the spin glass phase. The information obtainable from the subject is great because of the different ways one can perturb the system. In this work we have focused in a magnetic field perturbation. In this case it is possible to establish a physical picture in which states are supressed by the action of the magnetic field. By 'suppression of the states' we mean that these states increase their free energy and do not contribute any more to the partition function. Similarly to what happens at the critical point where there appears a finite correlation length after applying a magnetic field, these suppressed states acquire a finite correlation length (the chaos correlation length). To prove this result we should know what is the real mechanism of the modification of the free energies of the different equilibrium states. This means to understand how states are suppressed by the magnetic field depending on their statistical weights. This is an interesting analytical problem in mean-field theory which is possibly not out of reach.

Using this ideas we have been able to derive a scaling behaviour for the chaos parameters a and r which depends on the critical exponents at T_c.

We argue that some critical exponents survive in the spin-glass phase which means that the low temperature phase is governed by the finite-temperature fixed point. Curiously this is the opposite assertion of droplet models in which the spin-glass behaviour is governed by the zero-temperature fixed point. The complete understanding of the correct description of the low temperature phase in spin-glasses is one of the major still open problems. Mean-field approach yields the value 2/3 for the chaos correlation length exponent. We expect this exponent to be exact down to $d = 4$ which is the upper critical dimension if hyperscaling applies for the singular part of the free energy of the supressed states. Our numerical results are in very good agreement with this prediction. Surprisingly this 'mean-field' behaviour seems exact down to $d = 3$ which should correspond to the lower critical dimension. In the framework of droplet picture our numerical results suggest that the relation $\theta = \frac{d-3}{2}$ is probably exact. Both approaches predict that 3 is the lower critical dimension even though we expect some kind of pseudocritical behaviour in the regime in which lattice sizes are smaller than the true correlation length. This pseudocritical behaviour is expected also with finite magnetic field giving a pseudocritical AT line. Our results predict a transition line in agreement with some experimental results. For $d > d_l = 3$ the physics is determined by the existence or not of a spin-glass phase with magnetic field. A definite answer on the existence or not of the AT line in finite dimensions is a prioritary task in order to clarify the controversy on the real nature of the spin-glass phase (see [36] for some recent numerical results.)

Now we say few words in case of temperature changes. In this case, as shown in figure 1, chaos is much weaker. This is a interesting result which finds also a natural explanation in the context of mean-field theory. This result has already been shown in [15] doing the same kind of analysis as has been performed in section 3.1. Namely, when the initial system at temperature T and the perturbed one at temperature T' both lie within the spin-glass phase then $\lambda(q)$ in eq.(15) is always equal to one which corresponds to an infinite correlation length. This means that if chaos exists then it is marginal and the chaos parameter r for any finite perturbation ΔT goes to zero like $N^{-\alpha}$ with an exponent $\alpha < 1$. Obviously this result does not exclude the possibility that chaos is absent and $\alpha = 0$. Why chaos is much weaker in case of temperature changes than for magnetic field perturbations can be intuitively understood if one imagines that by lowering the temperature new equilibrium configurations emerge from previous ones and that the system suffers a continuous bifurcations into new states. In this case, a degree of coherence has to be preserved between the new and the old states and this is in agreement with mean-field calculations on the chaos problem.

It would also be very interesting to understand the chaotic nature of the low temperature phase in other spin-glass models, random-field problems and the vortex-glass phase in superconductors. The chaos approach could reveal as a good starting point to obtain (like happens in Ising spin glasses) the lower critical dimension for several models in which there is still much controversy.

¿From the study of chaoticity in spin glasses we also expect to give some hint regarding some real dynamical experiments in spin glasses [37, 38]. Cycling temperature experiments show that by lowering the temperature some degree of correlation is preserved between the probed states[39, 40]. Even though experimental spin glasses never reach equilibrium we think that a correct answer to the statics is relevant to a qualitative understanding of the effect of perturbations in the out-off equilibrium relaxations [41, 42]. There are also in course [43] some cycling magnetic field experiments which we hope will be in agreement with the main conclusions of this work.

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FIGURE CAPTION

Fig. 1 Chaos parameter r in the SK model for two different perturbations. In one case the system is at $T = 0.6$,$h = 0.$ and we apply a field $h = 0.2$, T staying constant. In the other case the system is at $T = 0.5$ and we increase the temperature by $\Delta T = 0.4$. Error bars in the second case are smaller than the size of the symbols. More details in the text.

Fig. 2 Mean-field theory order parameter function $q(x)$ in the spin-glass phase with magnetic field. It is characterized by a minimum overlap q_{min}, a maximum overlap q_{max} and the corresponding breakpoints x_{min},x_{max}.

Fig. 3 Chaos with magnetic field in the SK model at $T = 0.6$. Field values range from $h = 0.2$ up to $h = 1.0$ for the smaller sizes and up to $h = 0.4$ for the largest ones.The number of samples range from 200 for $N = 32$ down to 40 for $N = 1632$. Typical error bars are of order 5 per cent in all cases.

Fig. 4 Chaos with magnetic field in the SK model at $T = T_c = 1$. Field values range from $h = 0.1$ up to $h \sim 0.8$. The number of samples range from 200 for $N = 32$ down to 50 for $N = 736$.

Fig. 5 Chaos with magnetic field in the $4d$ $\pm J$ Ising spin glass at $T = 1.5$. Magnetic field values range from $h = 0.1$ up to $h = 1$. The number of samples is approximately 100 for all lattice sizes. Typical error bars in this case are of the size of the symbols.

Fig. 6 Chaos with magnetic field in the $3d$ $\pm J$ Ising spin glass at $T = 1.5$. Magnetic field values range from $h = 0.1$ up to $h = 1$. The number of samples is approximately 200 for all lattice sizes except for $L = 7$ in which there are 100 samples. Typical error bars are shown in case $L = 6$.