RSB in Finite D Systems

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You get the crunchy part...

(Realistic) Spin Glasses (and real glasses): many issues we want to understand.

Work with: Giorgio Parisi, Federico Ricci-Tersenghi, Juan J. Ruiz-Lorenzo and Francesco Zuliani.

A short summary.

1. $P(q)$, interfaces, window overlaps.
2. Coupling replicas: Migdal-Kadanoff Approximation (MKA) versus Monte Carlo simulations of the $3D$ Edwards Anderson model.
3. Analysis of the dynamical behavior. About the intricacies of data analysis.

Lyon, May-June 1999

Mean Field solution of spin glasses is known.

$\longrightarrow$ finite D? similar solution?

Since MF (RSB) solution is very atypical:

i.e. many states,
$P(q)$ non-trivial,
Ultrametricity,
de Almeida Thouless line,
dynamics, ...
how many of these features survive in finite D?

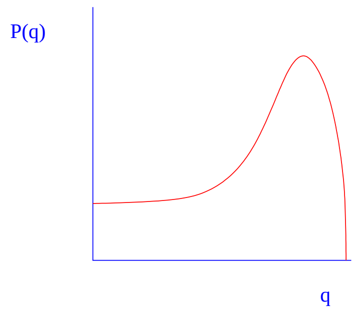
Usual numerical simulations: basically

$P(q)$ looks like RSB

But: many delicate points and doubts.

Here we discuss some of them.

A non-trivial $P(q)$ is surely a crucial signature of a spin-glass transition.



But it is not enough.

$P(q)$ could look non-trivial in trivial systems because of the presence of interfaces.

But RSB theory recognizes and distinguishes among these two situations. Let us discuss an example.

Consider a $3d$ ferromagnet, $T < T_c$, $m \neq 0$ for $V \to \infty$.

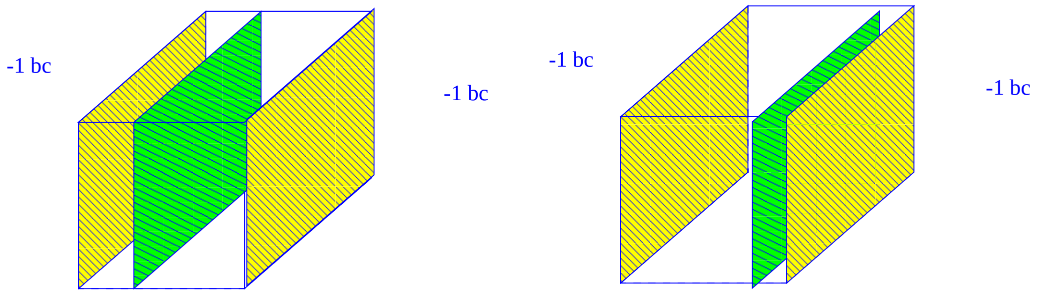
We will consider a situation such to create an interface, and T small such that the interface is not rough.

We consider a finite volume system with N spins.

y, z: periodic boundary conditions

x: antiperiodic boundary conditions

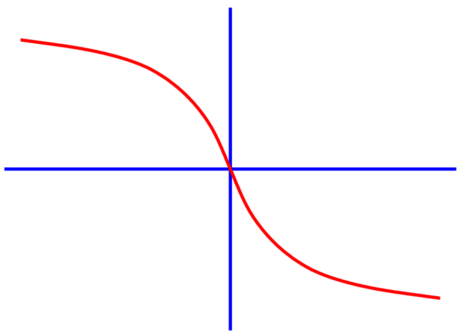
Low T: interface, domain wall.



$$m(i_x) \equiv \frac{1}{N_y N_z} \sum_{i_y=1,N_y,i_z=1,N_z} \sigma(i_x, i_y, i_z)$$

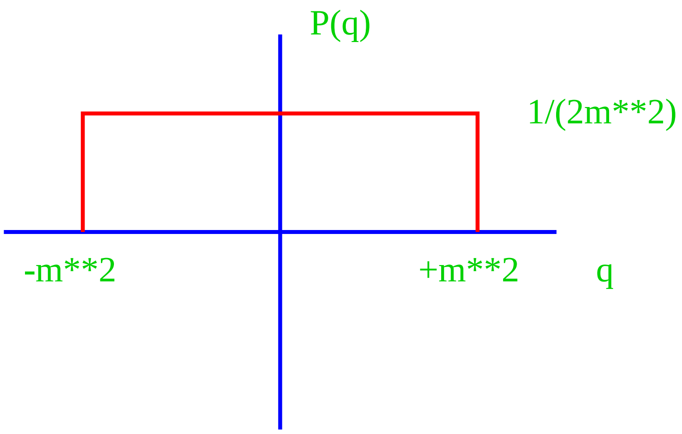
Large N: classify configurations from the magnetization profile: $m(i_x) \equiv f(i_x - I)$ I: position of the interface. f: characterizes the profile.

$f(i_x \to 0) \to +m$ and $f(i_x \to N_x) \to -m$ or the other way around.



This classification can be done for generic configurations (measure 1 for $N \to \infty$).

In this situation, for $N \to \infty$, $P(q) = \frac{1}{2m^2}$ for $-m^2 \leq q \leq m^2$



So, $P(q)$ is non-trivial, while the system is not a spin glass...

But the situation is different from a RSB situation and, as we will see, RSB easily distinguishes it.

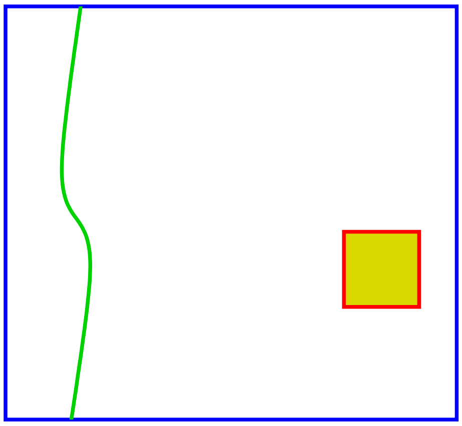
1. No fluctuations among $P_J(q)$ (but this is maybe not so crucial).

2. Compute $P(q)$ in a finite part of the lattice (window, block,...).

$$V = L^3,\ 1 \ll B \ll L$$

Region of size B^3: $q_B \equiv \frac{1}{B^3} \sum_{i \in B} q_i$.

$$P_B(q) = \lim_{L \to \infty} P_B^L(q)$$



Here

$$\lim_{B \to \infty} P_B(q) = \frac{1}{2} \left(\delta(q - m^2) + \delta(q + m^2) \right)$$

since for a local observable the probability of hitting an interface is zero.

In the Replica Symmetry Breaking Ansatz one finds that $\lim_{B \to \infty} P_B(q) = P(q)$

Numerical simulations show a clear, non-trivial $P(q)$. $P(q \simeq 0) \neq 0$, and the maximum at q_{MAX} becomes sharper with increasing L.

Maybe this is an effect faked by interfaces? From the work by Newman and Stein:

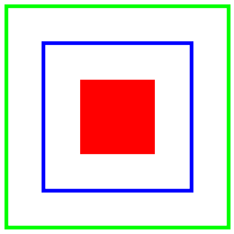
Essentially all the simulations of which we are aware compute the overlap distribution in the entire box. [...]

We suspect that the overlaps computed over the entire box are observing domain wall effects arising solely from the imposed boundary conditions rather than revealing spin glass ordering [...]

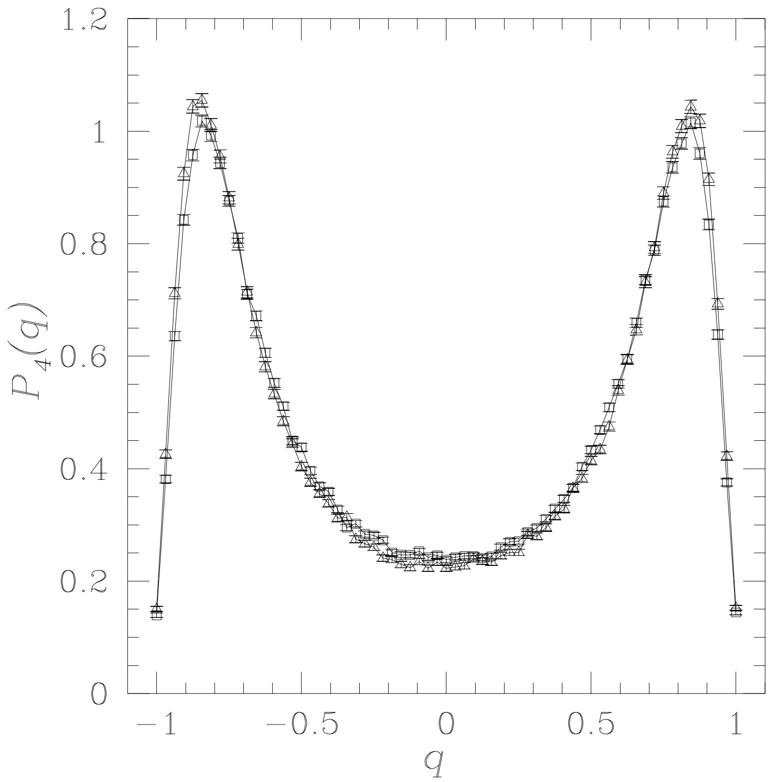
In other words if overlap computations were measured in *small* windows far from any boundary, one should find only a pair of δ–functions [...]

This is a very interesting issue, worth to check.

Block size $B = 4$. Triangles $L = 8$. Squares $L = 12$.
$T \simeq 0.7\ T_c$, $3D$.



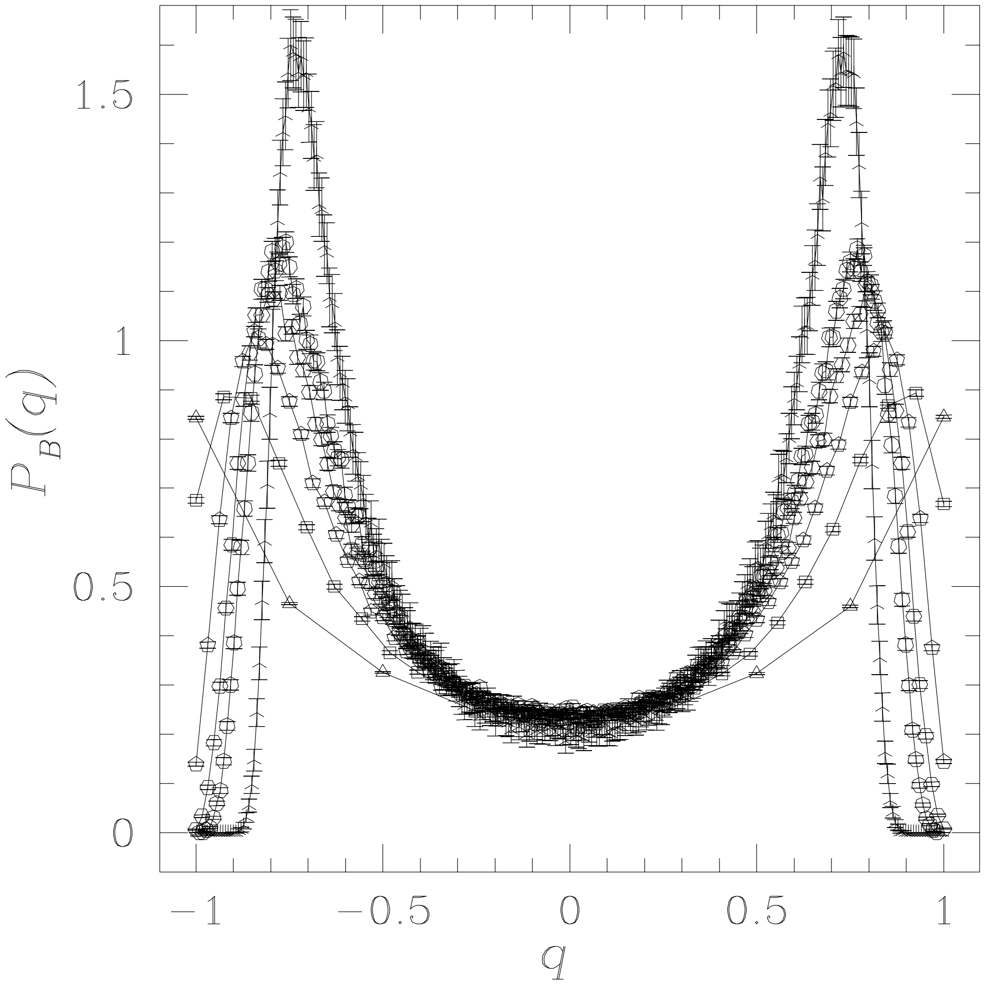
Parallel Tempering $\Longrightarrow$ equilibrium ok.



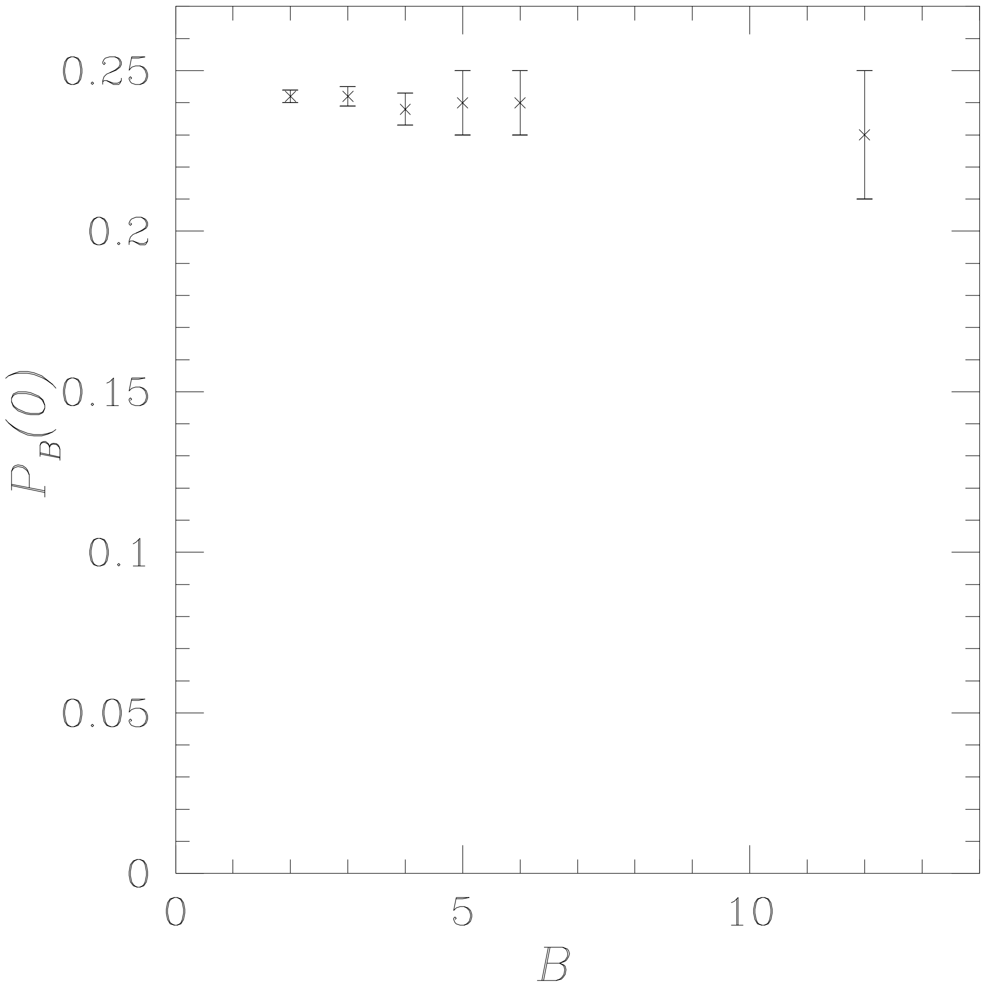
1. Small finite size effects: better than full $P(q)$
2. non-trivial
3. no substantial dependence on L
4. no approach to δ-function

$T \simeq 0.7\ T_c$, $3D$. Fixed lattice size $L = 12$. From $B = 2$ (lower curve, triangles). To $B = 12$ (sharper peaks, arrows.

The major changes in $P_B(q)$ are for increasing B. They are related to the usual (strong) L-dependence of the full $P(q)$.

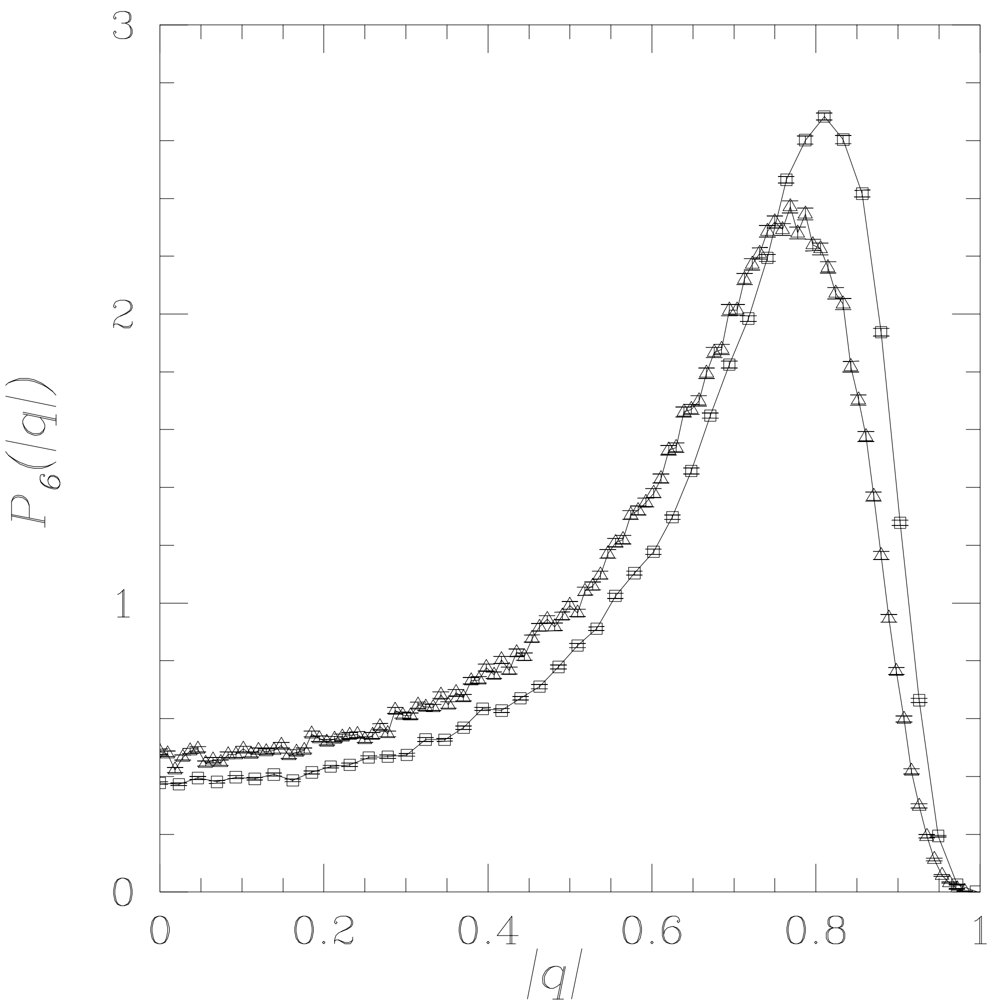


$P_B(q \simeq 0)$ does not become smaller for smaller B values. There is only very weak dependence on B (and in the opposite direction).



$B = 6$, squares $L = 6$, triangles $L = 12$.

$B = 6$ in $L = 12$ is less δ-like than $B = 6$ in $L = 6$: this is the opposite effect than in an interface scenario.



These data are from cond-mat/9804017, published in J. Phys. A.

Coupling Replicas

Couple two copies of the system. You can use

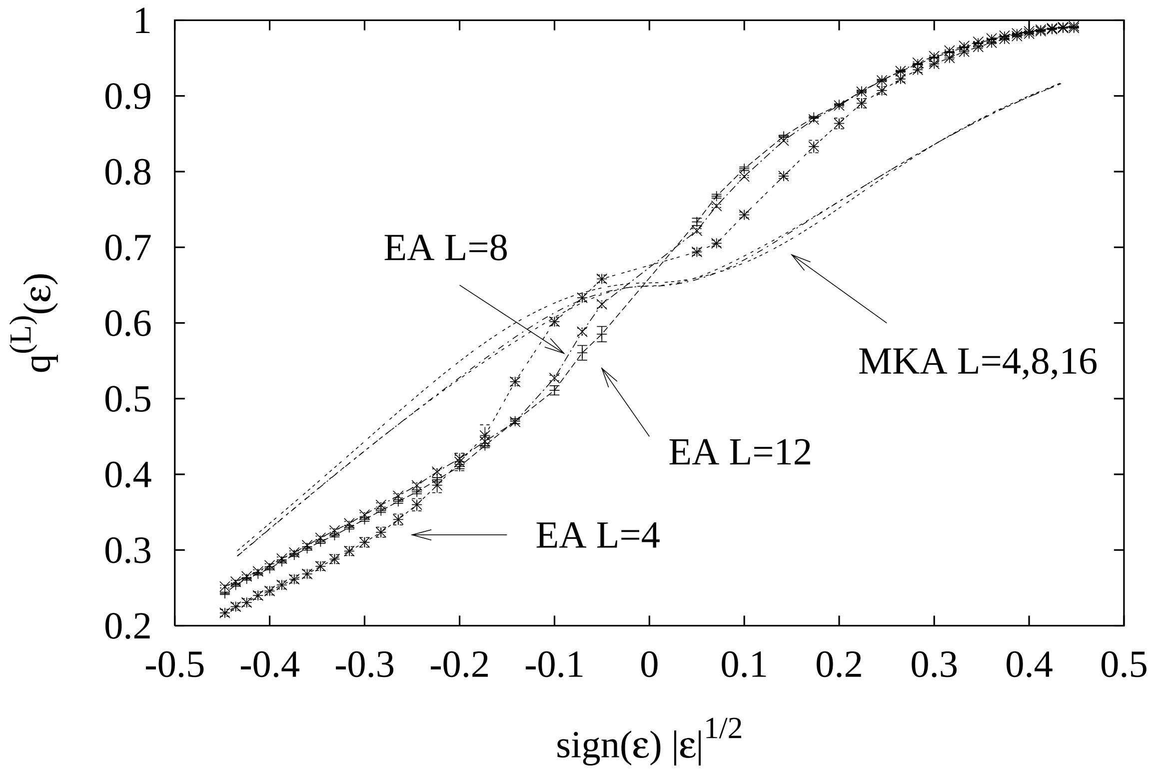
$$H_q[\sigma,\tau] = H_0[\sigma] + H_0[\tau] + \frac{\epsilon}{V}\sum_i \sigma_i\tau_i$$

or for example

$$H_{q_E}[\sigma,\tau] = H_0[\sigma] + H_0[\tau] + \frac{\tilde{\epsilon}}{V}\sum_{i,\mu} \sigma_i\sigma_{i+\mu}\tau_i\tau_{i+\mu}$$

Compute $\lim_{\epsilon\to 0^+} q(\epsilon)$ and $\lim_{\epsilon\to 0^-} q(\epsilon)$ or do it for $\tilde{\epsilon}$ (avoid possible troubles due to interfaces).

$$\text{RSB:}\quad \lim_{\epsilon\to 0^+} q(\epsilon) = q_+ + A_+\epsilon^\lambda$$
$$\lim_{\epsilon\to 0^-} q(\epsilon) = q_- + A_-|\epsilon|^\lambda$$

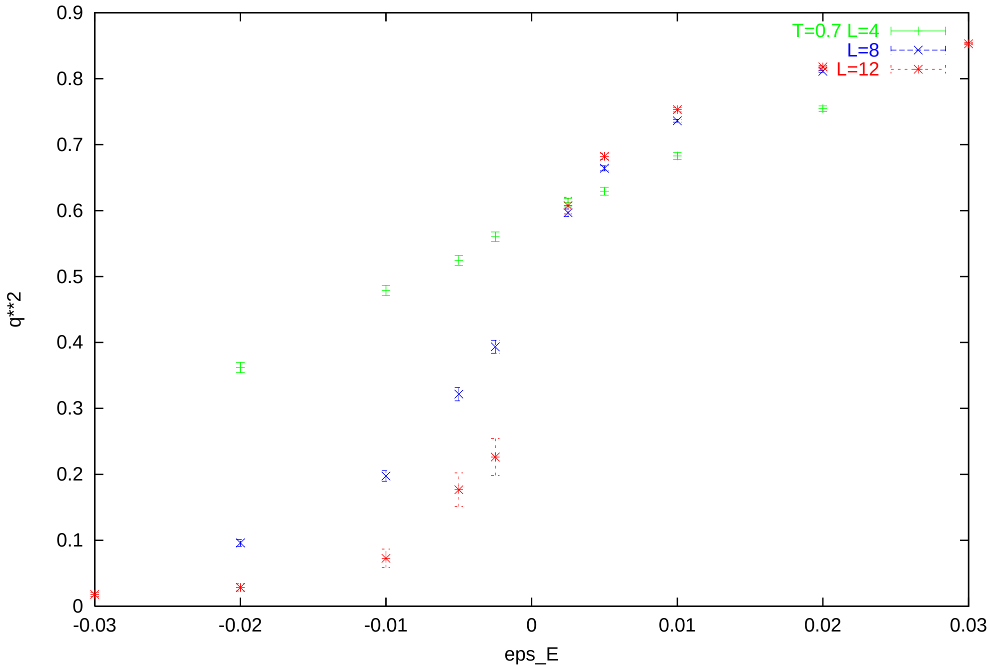


It is interesting to look at q^2 when coupling two replicas with an energy overlap term ϵq^E, with

$$\epsilon < 0$$

RSB: we are forcing the system to a value of the energy overlap $q^E = q^E_{\mathrm{min}}$, that also forces $q^2 = q^2_{\mathrm{min}} = 0$ in zero magnetic field.

In a droplet picture a small ϵq^E with $\epsilon < 0$ would only imply a small change in q^2.



On the need of a careful analysis of numerical data, and on the danger of scaling plots.

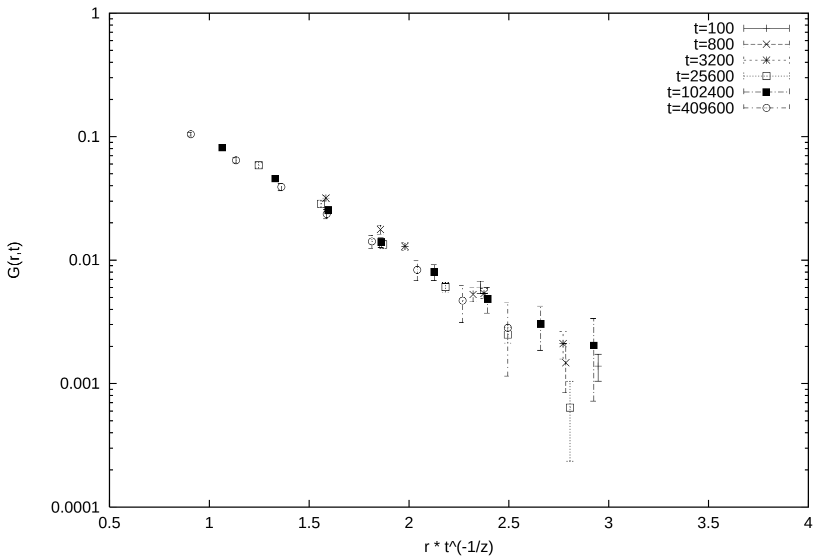
See among other work by Rieger and coworkers, Marinari, Parisi, Ritort and Ruiz-Lorenzo and Komori, Yoshino and Takayama, containing lot of nice new evidences about many aspects of the issue (distribution of free energy barriers, analysis of the relaxation process,...)

$G(r,t) = \langle q_0(t) q_r(t) \rangle$, at equal times. A simple droplet growth implies $G(r,t) \simeq \overline{q} + A e^{-\frac{r}{\xi(t)}}$, $\xi(t) \simeq t^{\frac{1}{z}}$, $z \simeq 10$.

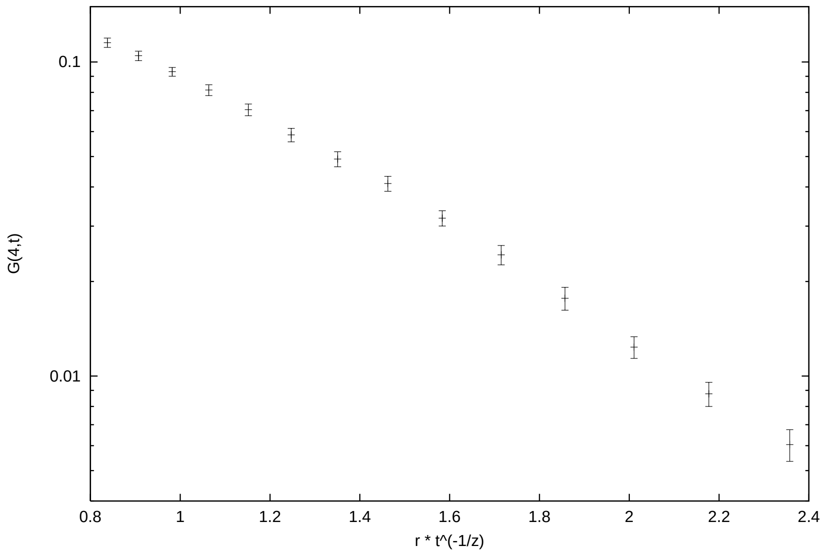
RSB implies a more complex behavior. $q \simeq 0$ sector: $G(r) \simeq r^{-\mu}$, $\lim_{r\to\infty} G(r) = 0$.

We had verified the Ansatz $G(r,t) = \frac{g}{r} e^{-\left(\frac{r}{\xi(t)}\right)^{\frac{3}{2}}}$.

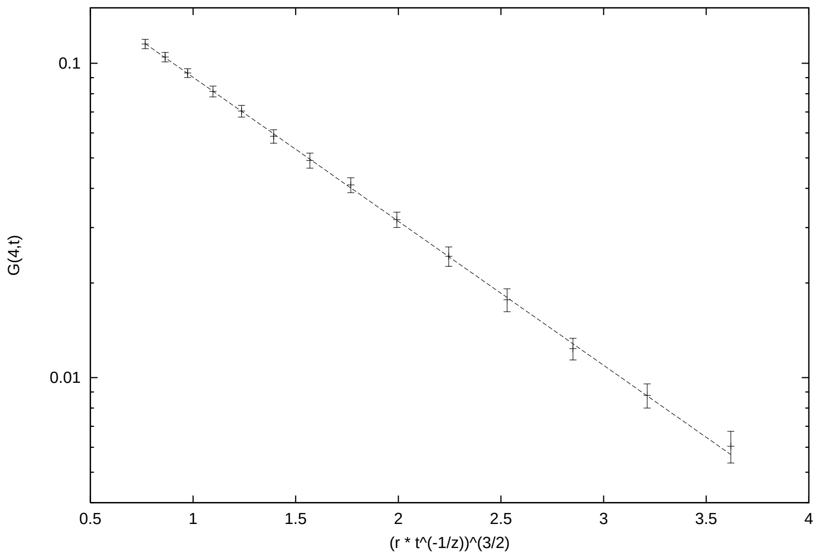
But scaling plot seems linear (T. Konori et al.):



Let us look more carefully. This can still look linear:



but it is not:



All the detailed analysis completely confirms our scaling Ansatz, not compatible with simple domain growth (and other evidences point toward the same direction).