Finite-dimensional spin glasses in the aging regime: replica symmetry breaking and ultrametricity

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One of the main features of spin glasses is their very slow dynamics: a real sample never reaches equilibrium on the experimental (and maybe even geological!) time scales.

As a consequence, the result of any measure depends on the time the sample spent in the frozen phase: the waiting time.

Typical experiment (or dynamical simulation):

1. quench a system in the frozen phase at time $t = 0$
2. wait a time t_w (with or without an external field)
3. start the measurements (the result will depend on t_w)

At the initial time in dynamical simulations, we quench from $T = \infty$ (random configuration) to $T < T_c$.

Equilibrium simulations ($L < \xi(t)$)
to measure the "statics"

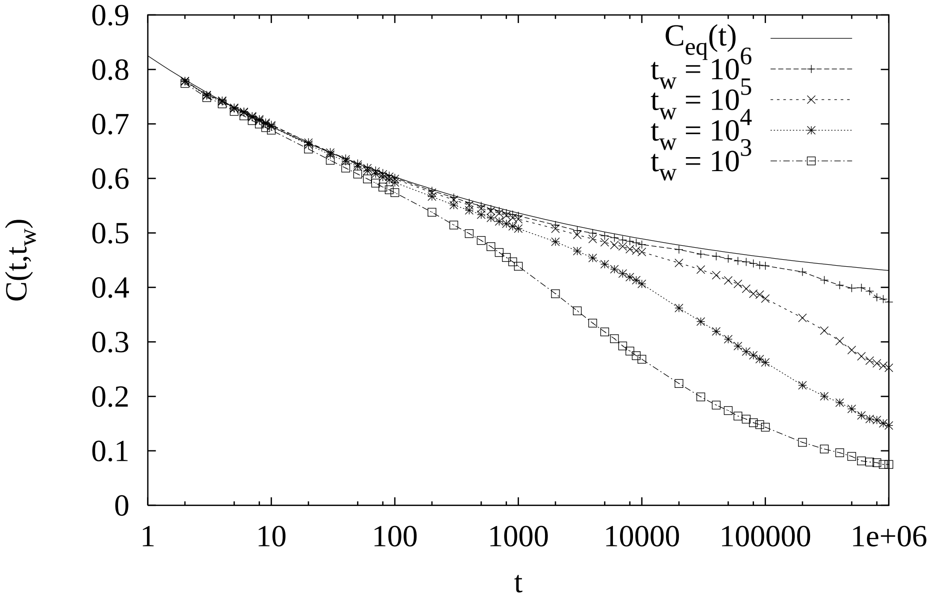
vs.

Off-equilibrium simulations ($L > \xi(t)$)
to measure the "dynamics"

Advantages of a dynamical study:

- necessary to study a system which never reaches equilibrium (like realistic spin glasses);
- numerically: bigger volumes, faster simulations, no thermalization times;
- critical exponents via dynamical scaling (once we know T_c);
- non self-averaging equilibrium quantities can be measured from off-equilibrium self-averaging ones.

$$C(t, t_w) \equiv \frac{1}{N} \sum_{i=1}^{N} s_i(t_w) s_i(t_w + t)$$



$t < t_w$: "quasi-equilibrium" regime,
$C(t, t_w) \simeq C_{\mathrm{eq}}(t)$;

$t > t_w$: aging regime. If simple aging holds
$C(t, t_w) \propto \mathcal{C}(t/t_w)$.

$$q_{\mathrm{EA}} \equiv \lim_{t \to \infty} \lim_{t_w \to \infty} C(t, t_w)$$

Fluctuation Dissipation Theorem (FDT)

We define

$$C(t,t') \equiv \langle A(t)A(t')\rangle$$
$$R(t,t') \equiv \left.\frac{\delta\langle A(t)\rangle}{\delta\epsilon(t')}\right|_{\epsilon=0}$$

with $H' = H + \epsilon(t)A(t)$

Assuming time translation invariance (TTI)

$$C(t,t') = \mathcal{C}(t-t')$$
$$R(t,t') = \mathcal{R}(t-t')$$

we have

$$\boxed{R(t,t') = \beta\ \theta(t-t')\ \frac{\delta C(t,t')}{\delta t'}}$$

Off-equilibrium Fluctuation Dissipation Relation (OFDR)

In the aging regime the FDT can be modified introducing a multiplicative violation factor X, that for large times only depends on the correlation.

$$\boxed{R(t,t') = \beta\ X[C(t,t')]\ \theta(t-t')\ \frac{\delta C(t,t')}{\delta t'}}$$

In the simulations, we perturb the system switching on a random magnetic field at time t_w: $h(t) = h_0\ \theta(t-t_w)$.

$$C(t,t') = \frac{1}{N}\sum_i \langle s_i(t) s_i(t') \rangle$$

$$M_S[h](t) = \frac{1}{N}\sum_i \frac{\langle h_i s_i(t) \rangle}{h_0} =$$

$$= \int_{-\infty}^{t} \mathrm{d}t' R(t,t') h(t')$$

In the linear response regime (very low fields) we define

$$\chi(t,t_w) = \lim_{h_0 \to 0} \frac{M_S[h](t)}{h_0}$$

and with a little algebra we have

$$\chi(t,t_w) = \beta \int_{C(t,t_w)}^{1} X(u)\ \mathrm{d}u$$

Defining

$$S(y) = \int_y^1 X(u)\ \mathrm{d}u$$

we can simplify the previous formula to

$$T\ \chi(t,t_w) = S(C(t,t_w))$$

FDT: $X = 1 \Rightarrow T\ \chi(t,t_w) = 1 - C(t,t_w)$

Link with the Statics

In the large times limit:

$t \to \infty$ and $t_w \to \infty$

such that $C(t, t_w) = q$

$$\boxed{X[C(t, t_w)] \longrightarrow x(q)}$$

where

$$x(q) = \int_0^q \mathrm{d}q' P(q')$$

Equivalently

$$S(y) \longrightarrow s(y) = \int_y^1 x(q)\ \mathrm{d}q$$

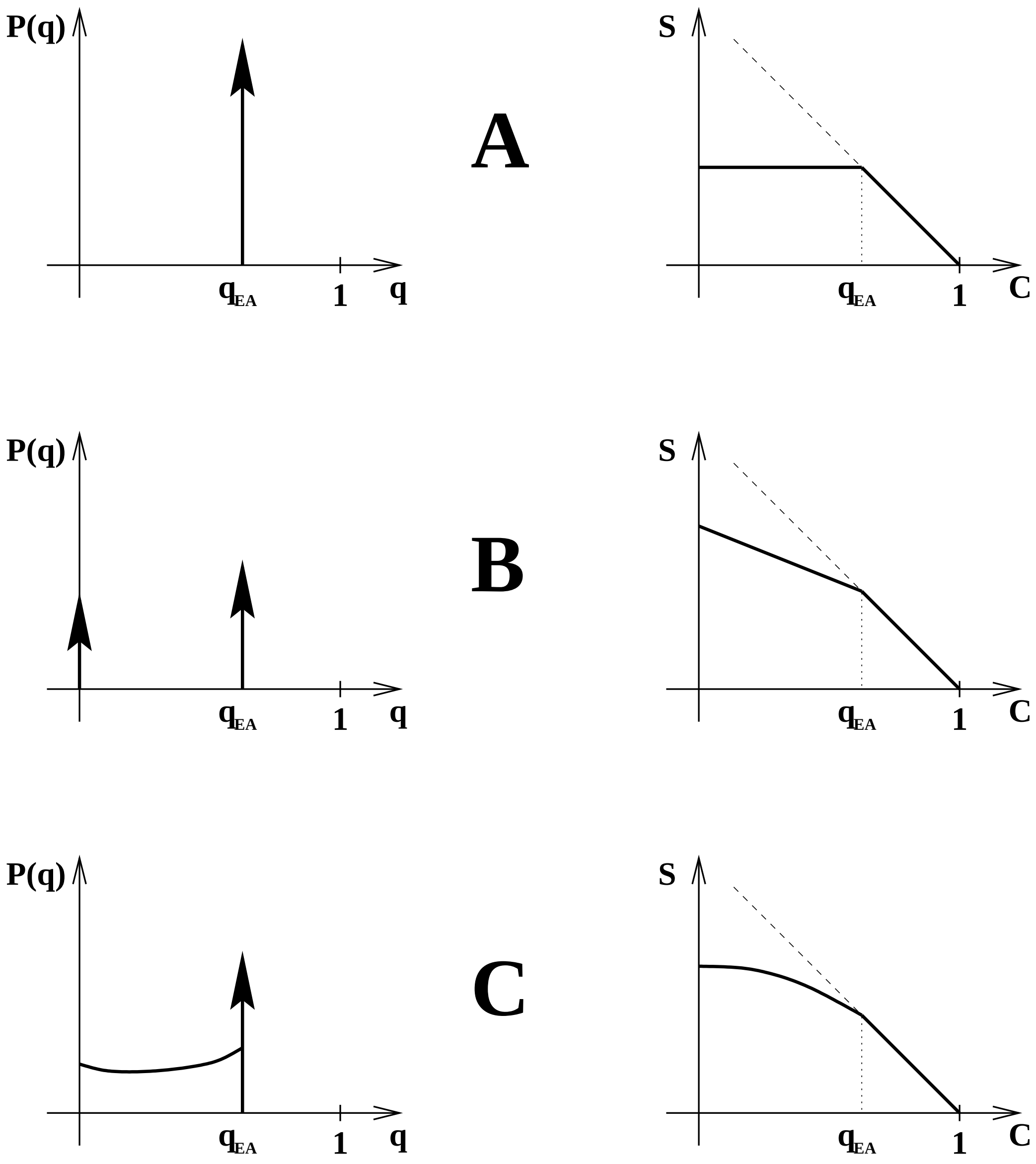
This link has been proved, under very general assumptions,

by Franz, Mèzard, Parisi and Peliti

(Phys. Rev. Lett. **81** (1998) 1758).

Classification

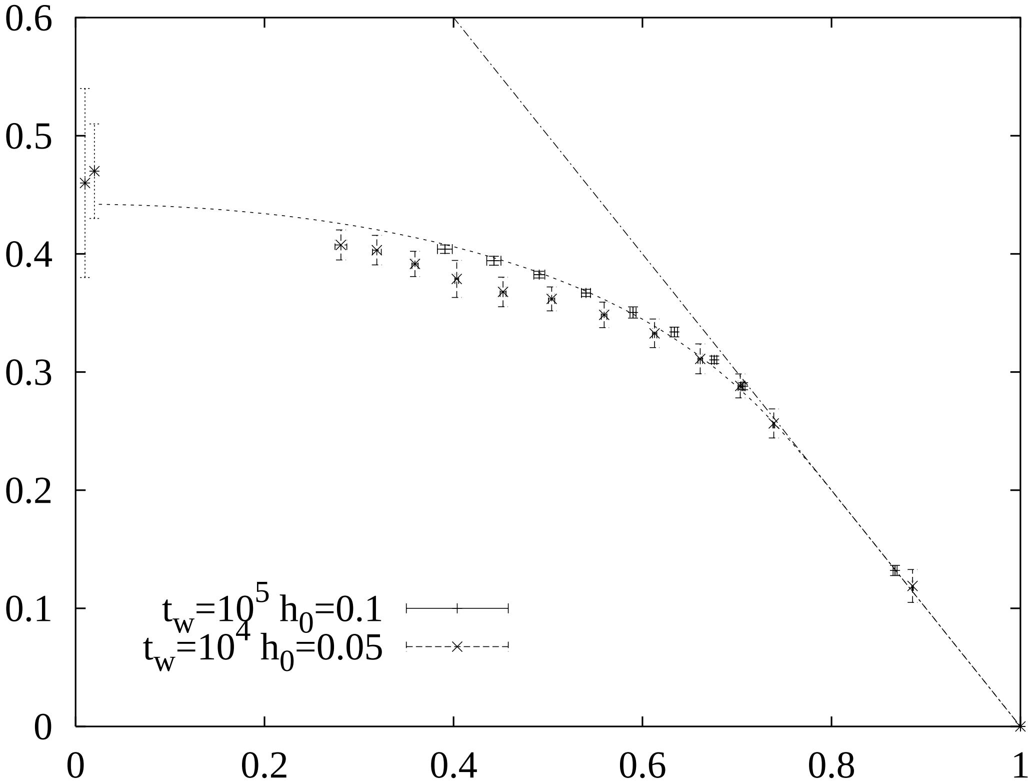
Three main kinds of dynamical response $S(C)$, that correspond two different shapes of the static $P(q)$.



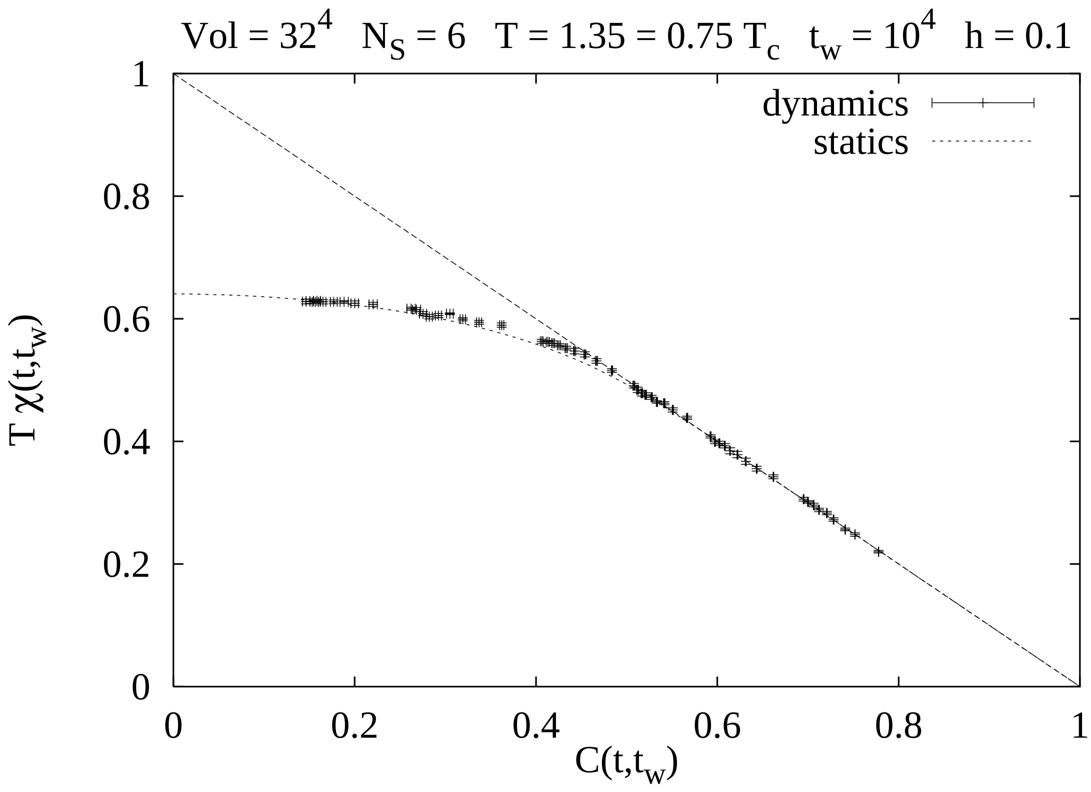
Numerical Results

Edwards-Anderson (EA) model

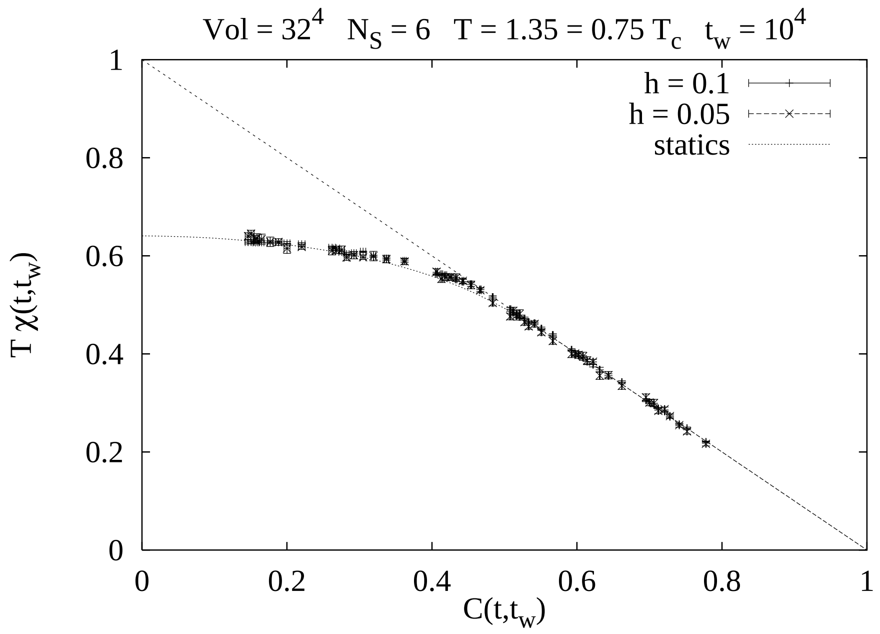
D=3 Volume=64^3 $T = 0.7 \simeq \frac{3}{4}T_c$



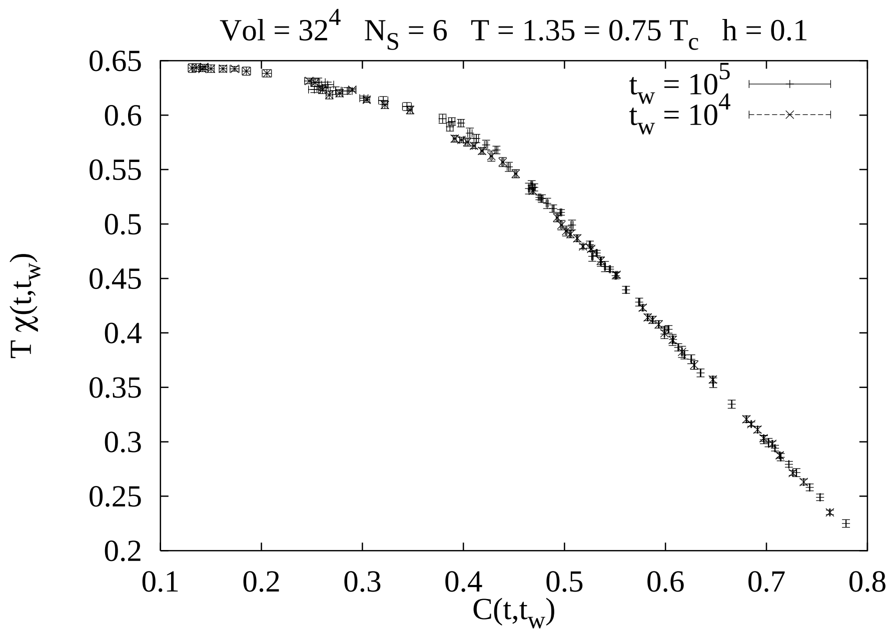
EA D=4


Vol = 32^4 N_S = 6 T = 1.35 = 0.75 T_c t_w = 10^4 h = 0.1
dynamics
statics
T $\chi(t,t_w)$
$C(t,t_w)$
0
0.2
0.4
0.6
0.8
1

Check for the linear response to hold.



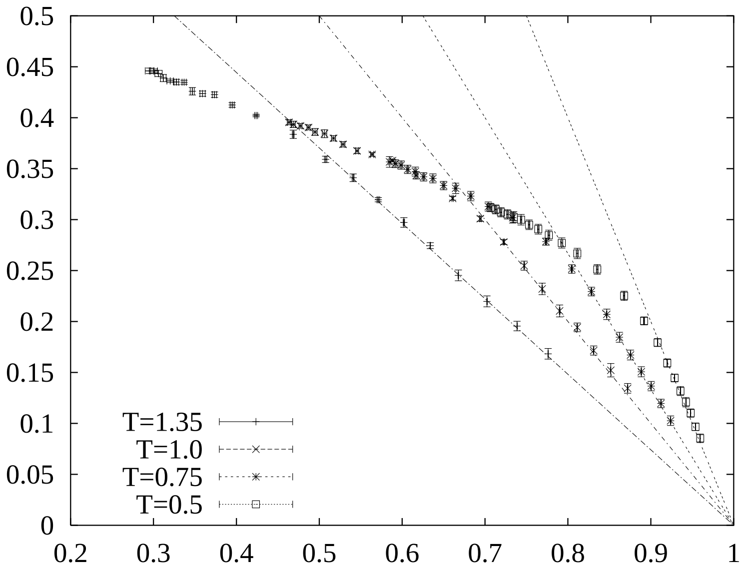
Check for the large times regime.

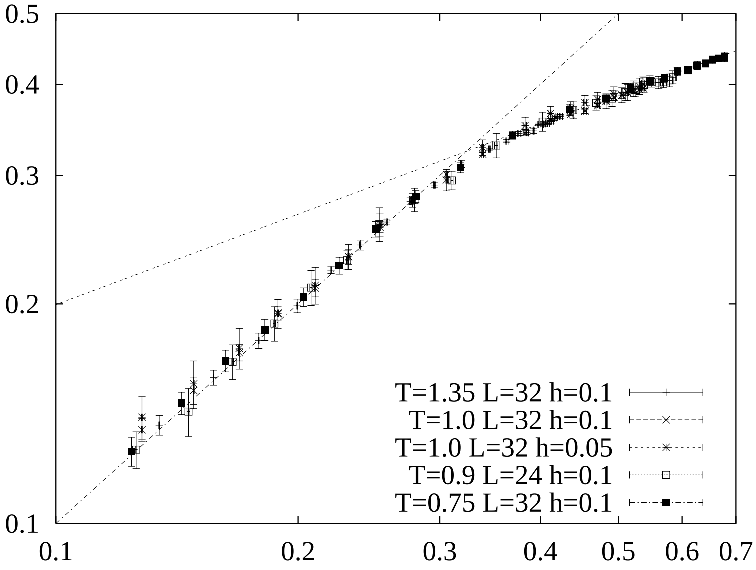


Parisi-Toulouse (PAT) hypothesis

$$q(\beta, x) = \tilde{q}(\beta x)$$

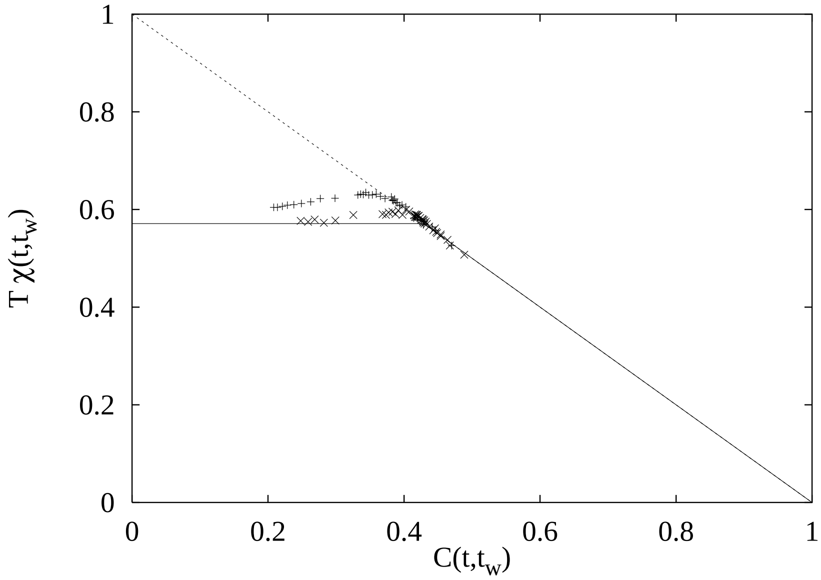
It is one more feature of the SK model which seems to hold in the EA model.



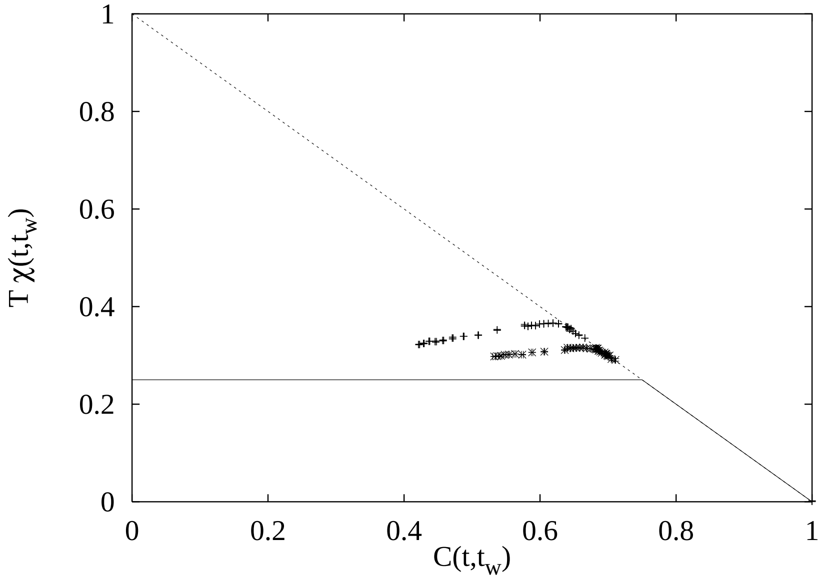


As a main check we have applied the same method to models where there is non RSB.

Diluted ferro: 400^3, $T = 0.9T_c$, $t_w = 10^3, 10^4$



RFIM: 440^3, $T = 0.9T_c$, $t_w = 10^2, 10^3$



Three main remarks:

1. The method based on the OFDR does work.

   - We have verified numerically all the analytical assumptions.
   - It gives information on the off-equilibrium dynamics (efective temperature, memory effects, ...) and on the right equilibrium behavior, no matter of how complex the off-equilibrium dynamics is.
   - Has many advantages with respect to equilibrium simulations (large volumes, small number of samples, no thermalization times, ...)

2. The classification based on it has a physical meaning.

3. The same kind of measure can be done also experimentally on real samples.

Ultrametricity

Given three spin configurations the ultrametric inequalities read

$$q_{12} \geq \min\{q_{13}, q_{23}\} \quad , \quad d_{12} \leq \max\{d_{13}, d_{23}\}$$

Dynamically the ultrametricity constrains the correlation functions

$$C(t_1, t_3) = \min\{C(t_1, t_2), C(t_2, t_3)\}$$

Very hard to check numerically!

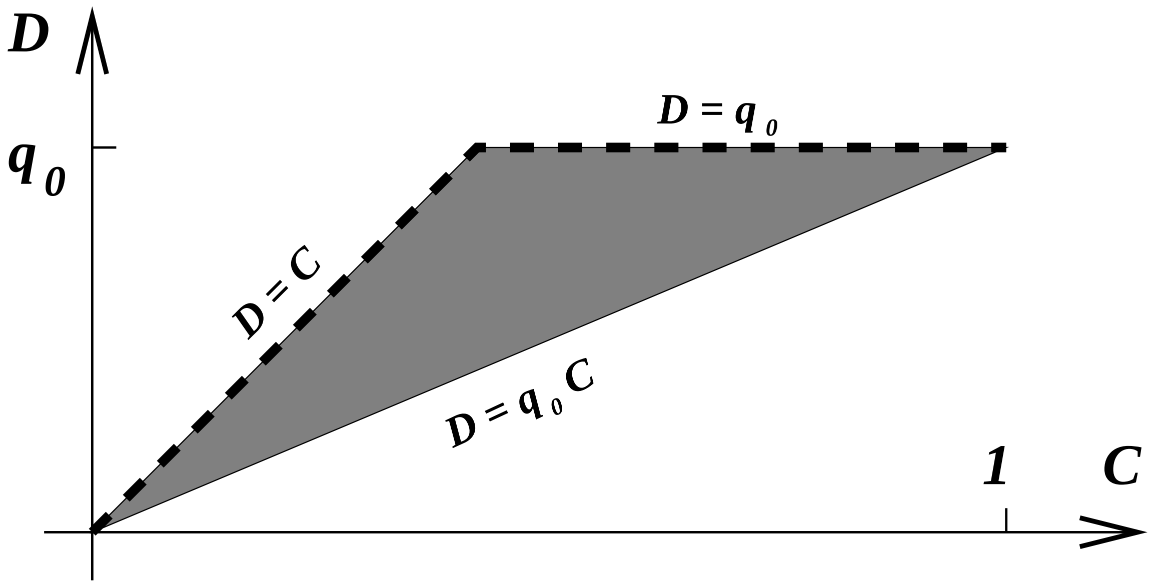
We consider 2 real replicas and define the auto- and cross-correlation function as

$$C(t, t') = \frac{1}{N} \sum_i S_i^1(t) S_i^1(t')$$

$$D(t, t') = \frac{1}{N} \sum_i S_i^1(t) S_i^2(t')$$

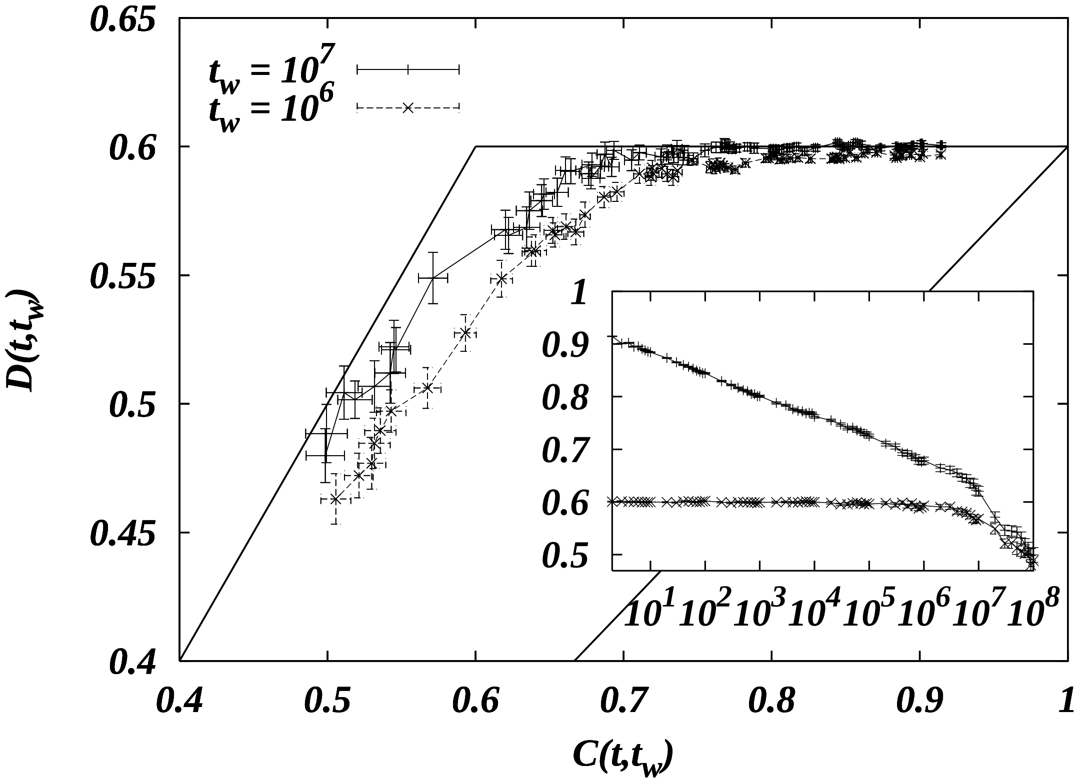
If we constrain the 2 replicas to have, at every time, a fixed overlap $q_0 \in [0, q_{\rm EA}]$, then ultrametricity implies

$$D(t,t') = \min\{C(t,t'), q_0\}$$



We evolve simultaneously 2 replicas constrained to have an overlap q_0 and we check if the data stay on the ultrametric bound.

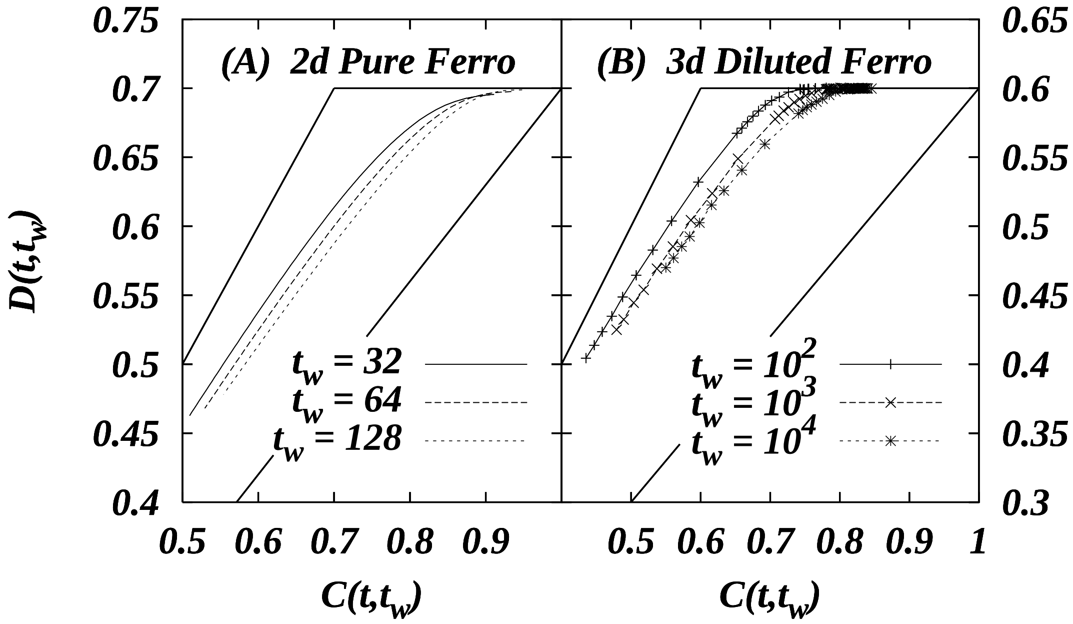
EA $3D$: $T = 0.7 \simeq \frac{3}{4} T_c$, $t_w = 10^6, 10^7$



In a pure or diluted ferromagnet ultrametricity is not expected to hold.

Ferro $2D$: $T = 1.5$, $t_w = 32, 64, 128$

Diluted Ferro $3D$: $T = 1.67$, $t_w = 10^2, 10^3, 10^4$



Zooming in the region near the ultrametric bound and comparing the EA model (where we expect ultrametricity) and the diluted ferromagnetic (which is not ultrametric), we can see the different behavior.