Physics of glassy systems

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A short summary.

1. Glassiness and metastability.
2. The three susceptibilities
3. Glassy models: spin glasses, tilings and glasses
4. The probability distribution of the overlap
5. General principles: stochastic stability and separability
6. Ageing, fluctuation dissipation relations and real experiments

Pisa, July 1999

What is glassiness? $\Longrightarrow$ Ubiquitous metastability.

Usual metastability: you have a first order phase transition in a parameter h (e.g the magnetic field for a ferromagnet).

Crossing a first order phase transition you go into a metastable state.

$$\tau(h) \propto \exp(A/h^\alpha) \qquad \alpha = d - 2$$

We can define a linear response:

$$\chi_{LR} = \lim_{h \to 0^+} \sum_i \langle \sigma(i)\sigma(0) \rangle_c$$

Let us start with the state $m > 0$ at $h = 0$ and

add a magnetic field h at time 0.

$$\chi_{LR} = \lim_{t \to \infty} \frac{\partial}{\partial h} m(t, h)$$

$$\chi_{eq} = \frac{\partial}{\partial h} \lim_{t \to \infty} m(t, h)$$

$$\chi_{LR} \neq \chi_{eq} = \infty$$

THESIS:

In glassy systems there is an open region in parameter space where

$$\chi_{LR} \neq \chi_{eq}$$

$$\Delta m(t) = \chi_{LR} \Delta h \quad \text{for} \quad 1 << t << \tau(\Delta h)$$

$$\Delta m(t) = \chi_{eq} \Delta h \quad \text{for} \quad \tau(\Delta h) << t$$

Here χ_{eq} must be finite (at least almost everywhere).

We define χ_{irr}

$$\chi_{eq} = \chi_{LR} + \chi_{irr}$$

Glassy systems can be divided into two categories

- System with quenched disorder, e.g. spin glasses
- Translational invariant systems, e.g. tilings and glasses.

SPIN GLASSES

- $U(i,k)$ is a Z_2 gauge field on the lattice at infinite temperature:
- σ_i is a matter field.
- $H_U(\sigma) = -\sum_{i,k} U(i,k)\sigma(i)\sigma(k) - \sum_i h\sigma(i)$
- $F(\beta) = \beta^{-1}\overline{\ln(Z_U(\beta))}$
  $Z_U(\beta) = \sum_{\{\sigma\}} \exp(-\beta H_U(\sigma))$

Comments:

- The overbar is the average over the U: we are in the quenched approximation. The unquenched case in not physically interesting.
- h is the magnetic field, which breaks gauge invariance.
- At $h = 0$ "to find the minimum of $H_U(\sigma)$" is equal " to find the Landau gauge", σ being the gauge fixing.
- Gribov ambiguity $\Longleftrightarrow$ many minima of $H_U(\sigma)$.

Glassy region (at least in mean field theory):

(De Almeida Touless line).

TRANSLATIONAL INVARIANT MODELS

WANG TILINGS

$\sigma(i) \in Z_M \qquad H(\sigma) = \sum_i \sum_{\mu=1,d} E_\mu(\sigma(i), \sigma(i+\mu))$

The model depend on $E_\mu(\sigma, \tau) = 0, 1$

$\#$ Models $= 2^{DM^2}$ (some related by symmetries).

Ising model Potts models.... well known. (know as Wang tilings).

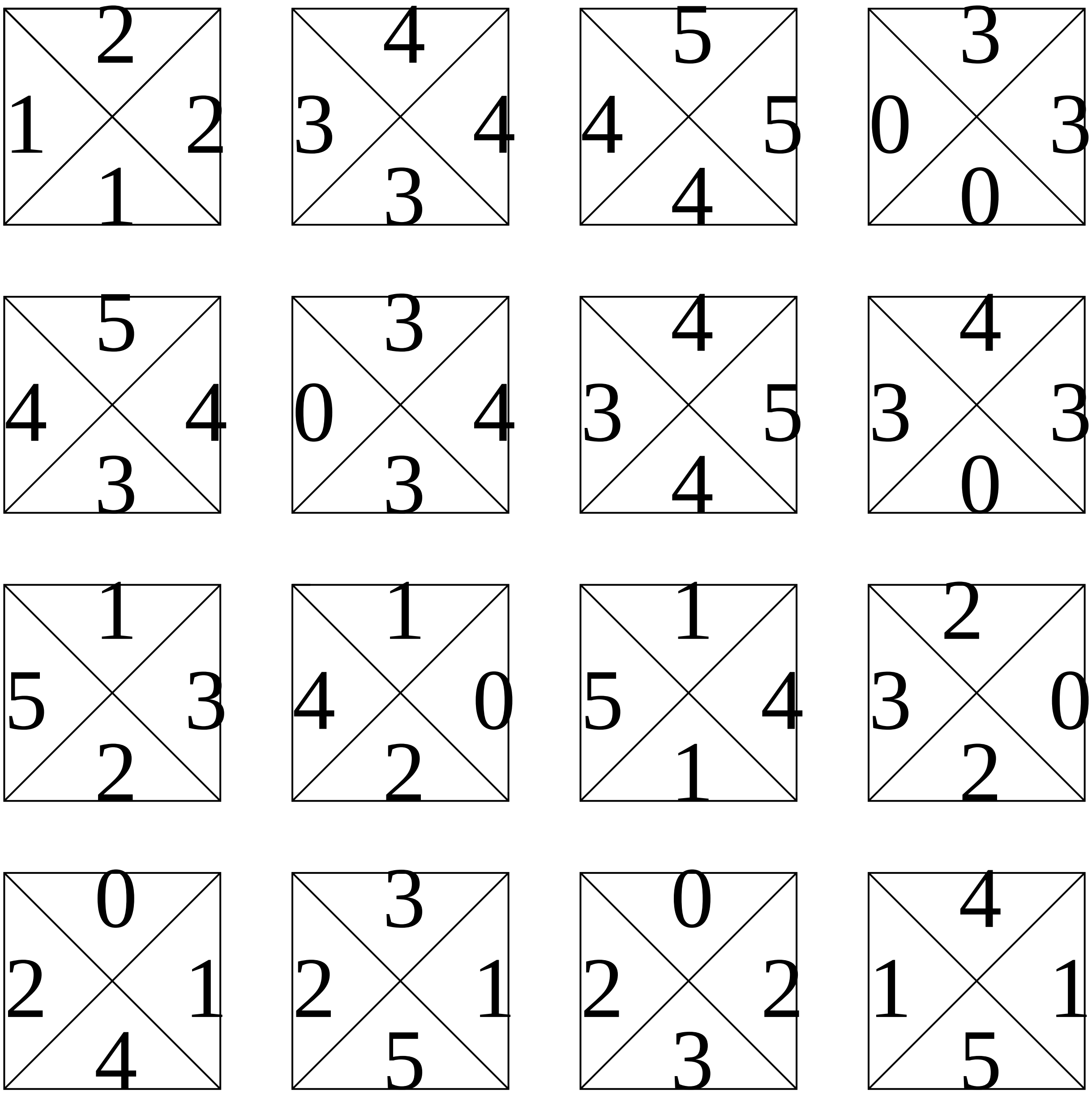
For large M weird things may happen.

For $M \geq 13$ $D = 2$ $\exists$ E

such that the minima are never periodic.

For $M \geq 56$ $D = 2$ $\exists$ E

such that the the problem of deciding if a configuration of spins on a finite volume can be imbedded in an infinite volume ground state is not decidable.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4 3 4 3
5 4 5 4
3 0 3 0
5 4 4 3
3 0 4 3
4 3 5 4
4 3 3 0
1 5 3 2
1 4 0 2
1 5 4 1
2 3 0 2
0 2 1 4
3 2 1 5
0 2 2 3
4 1 1 5

GLASSES

$$a = 1, M \;\; N(a) = Nc(a) \;\; \sum_a c(a) = 1$$

$$\text{e.g.} \;\; M = 2 \;\; c(1) = .8 \;\; c(2) = .2$$

$$H = \sum_{a,b=1,M} \sum_{i=1,N(a)} \sum_{k=1,N(b)} V_{a,b}(x_a(i) - x_b(k))$$

$M(M+1)/2$ functions $V_{a,b}(x)$ describe the model

INTERPRETATION

Large finite system the phase space can be decomposed into many finite volume states (or valleys or lumps)

Breaking of the ergodicity.

The Gibbs measure is the sum of smaller unrelated pieces.

- Connected correlation functions in each state go to zero al large distances (cum grano salis).
- The time to go from one state to an other state is exponentially large with the volume.
- Chaos: when we change parameters we have state crossing: stable states becomes metastable.
- No Gibbs rule: states coexist in an open region of parameter space.

Decomposition of the Gibbs measure for a finite system into states:

$$\langle\cdot\rangle = w_\alpha \langle\cdot\rangle_\alpha \ \sum_\alpha w_\alpha = 1 = \ldots$$

STATE EQUIVALENCE: all states have similar properties:

$$\chi_\alpha \equiv N^{-1}\langle M^2\rangle_\alpha^c = \chi_{LR}$$

$$N^{-1}\langle M\rangle_\alpha = m + N^{-1/2}\delta_\alpha$$

However $\chi_{irr} = \sum_{\alpha,\gamma} w_\alpha w_\gamma (\delta_\alpha - \delta_\gamma)^2$

How to be more quantitative?

We introduce a generalized overlap. $A(i)$ is a local operator

$$q^A_{\alpha,\gamma} = N^{-1}\sum_i \langle A(i)\rangle_\alpha \langle A(i)\rangle_\gamma$$

Examples

$$A(i) = \sigma(i) \longrightarrow q^A = q \text{ (usual overlap)}$$

$$A(i) = H(i) \longrightarrow q^A = q^E. \text{ (energy overlap)}$$

Description of a given system: all the w_α and all the $q^A_{\alpha,\gamma}$.

Problem: this description depends on N and on the quenched disorder if present.

INTRODUCTION OF AN ENSEMBLE.

- Average over quenched disorder at fixed N
- Average over N in a window if no quenched disorder is present.
- Average over small added disorder for a given system

Introduce $\mathcal{P}(\{w\},\{q\})$

J denotes an element of the ensemble $P_J(q) =$

$$Z^{-2} \sum_{\{\sigma\},\{\tau\}} \exp(-\beta(H(\sigma)+H(\tau))\delta(q(\sigma,\tau)-q) =$$

$$\sum_{\alpha,\gamma} w_\alpha w_\gamma \delta(q - q_{\alpha,\gamma})$$

where $q(\sigma,\tau) = N^{-1}\sum_i \sigma(i)\tau(i)$

If we define :

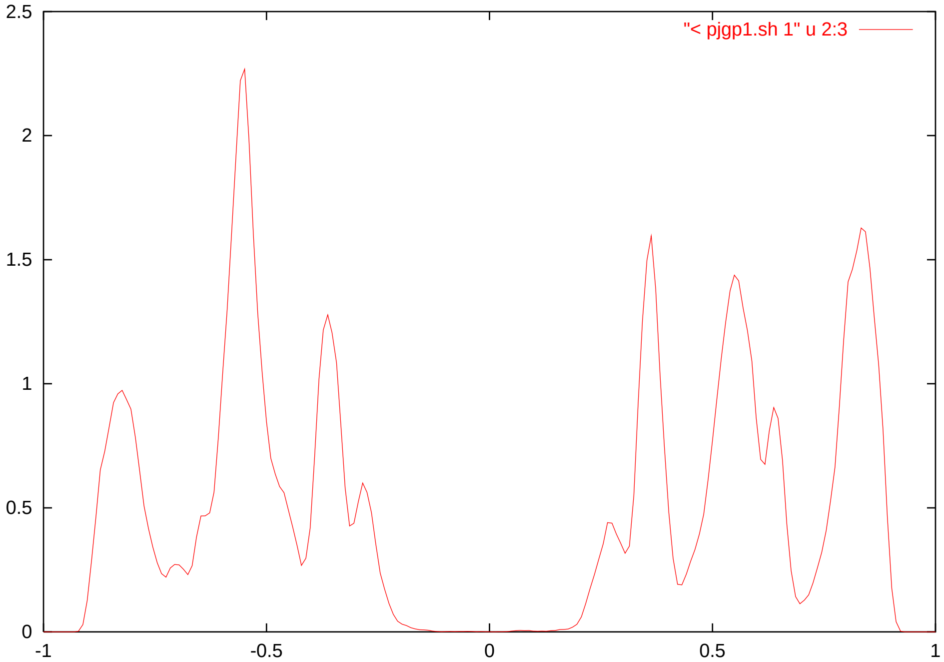
$$P(q) = \overline{P_J(q)} \qquad P(q_1,q_2) = \overline{P_J(q_1)P_J(q_{12})}$$

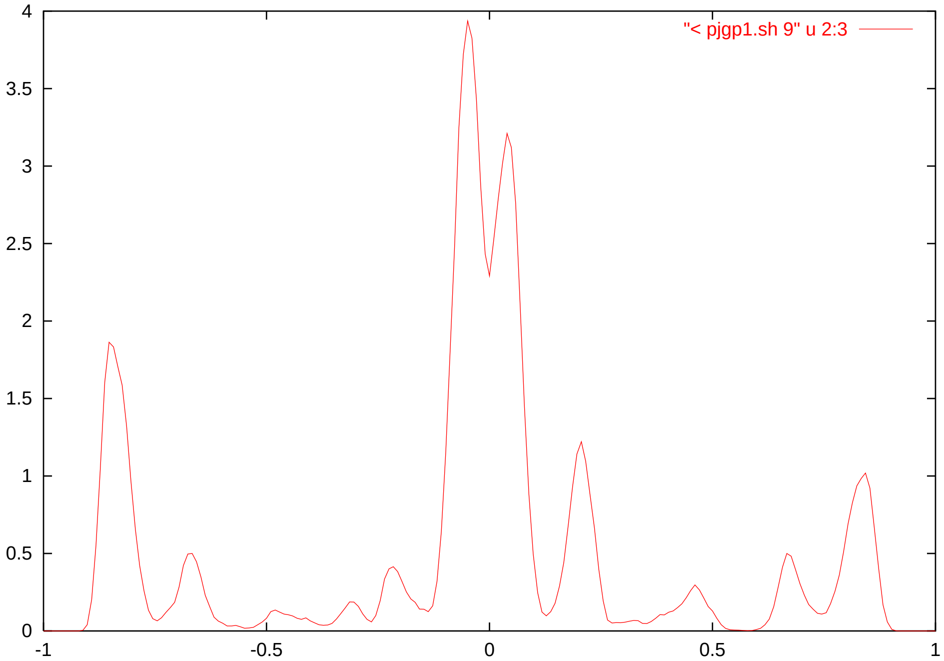
$\mathcal{P}(\{w\},\{q\}$ is equivalent to an infinite set of functions

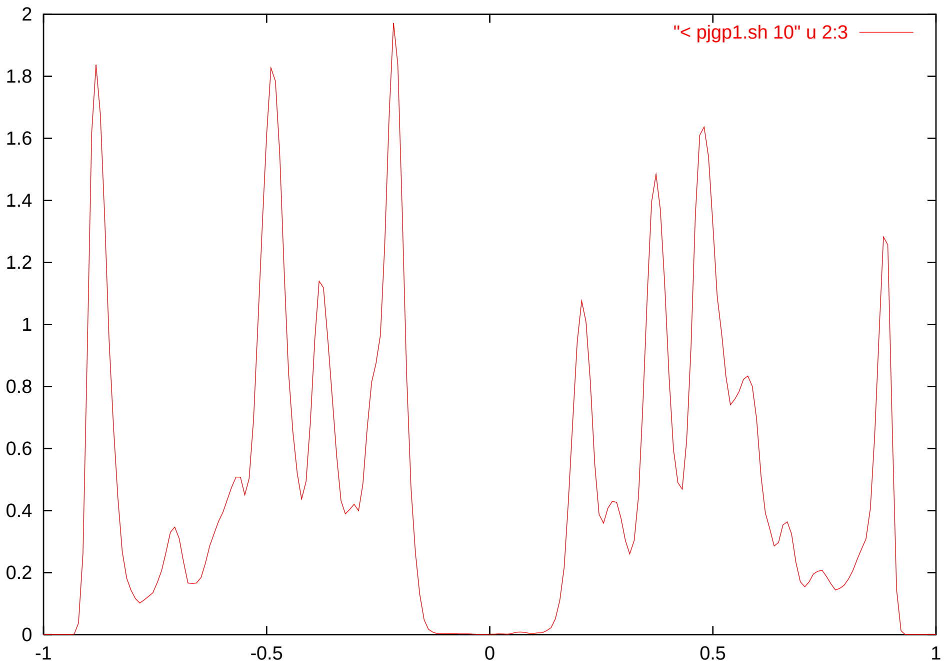
$$(P(q), P(q_1,q_2), P(q_1,q_2,q_3)$$

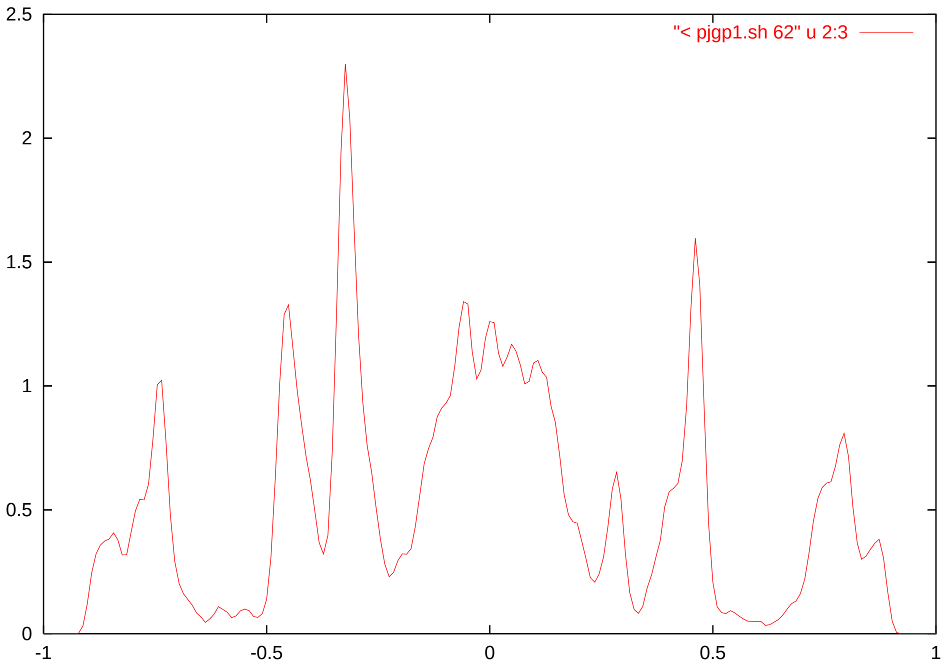
plus similar functions constructed with the overlaps q^A.

A few sample for $D = 3 \quad L = 16$

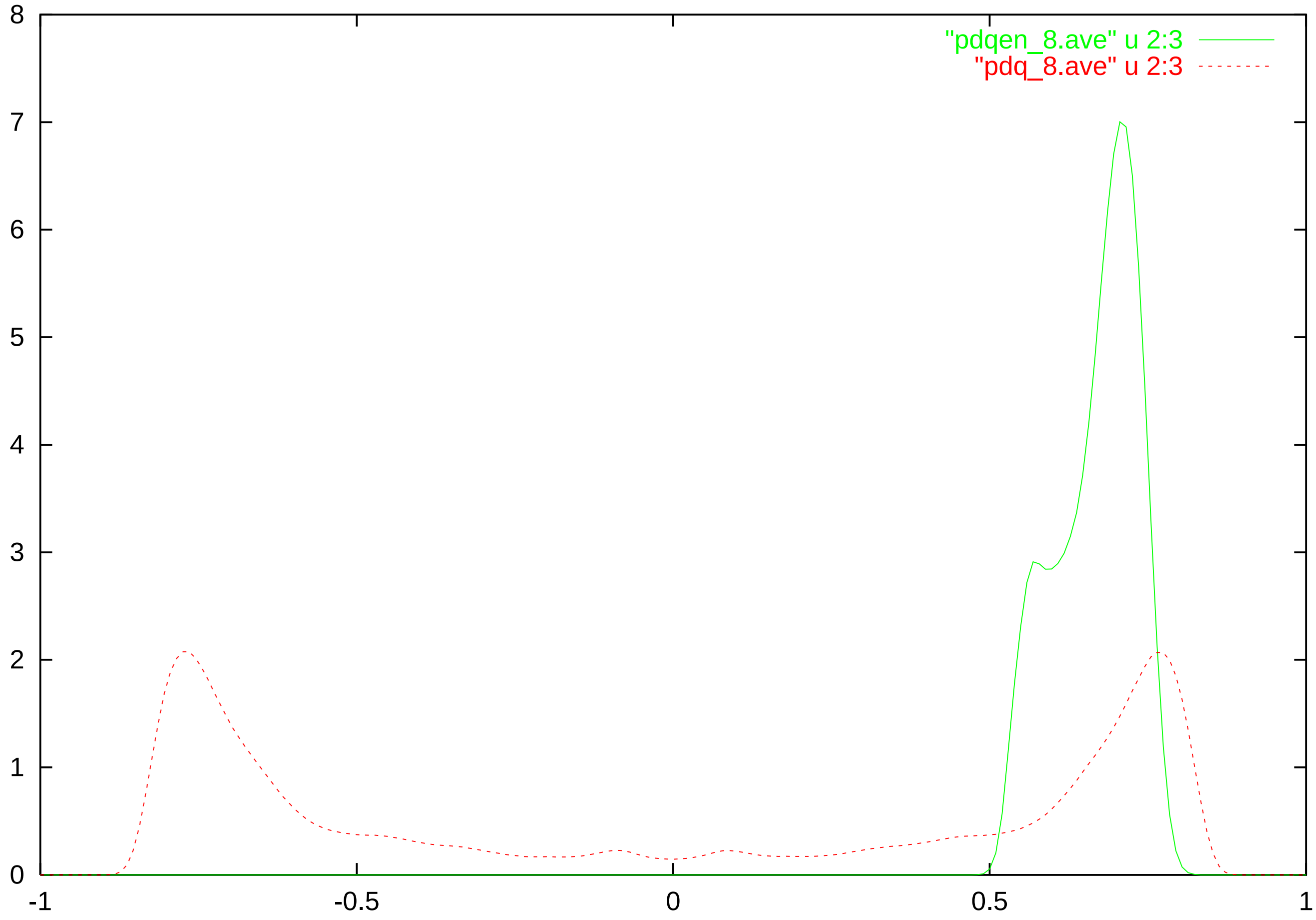




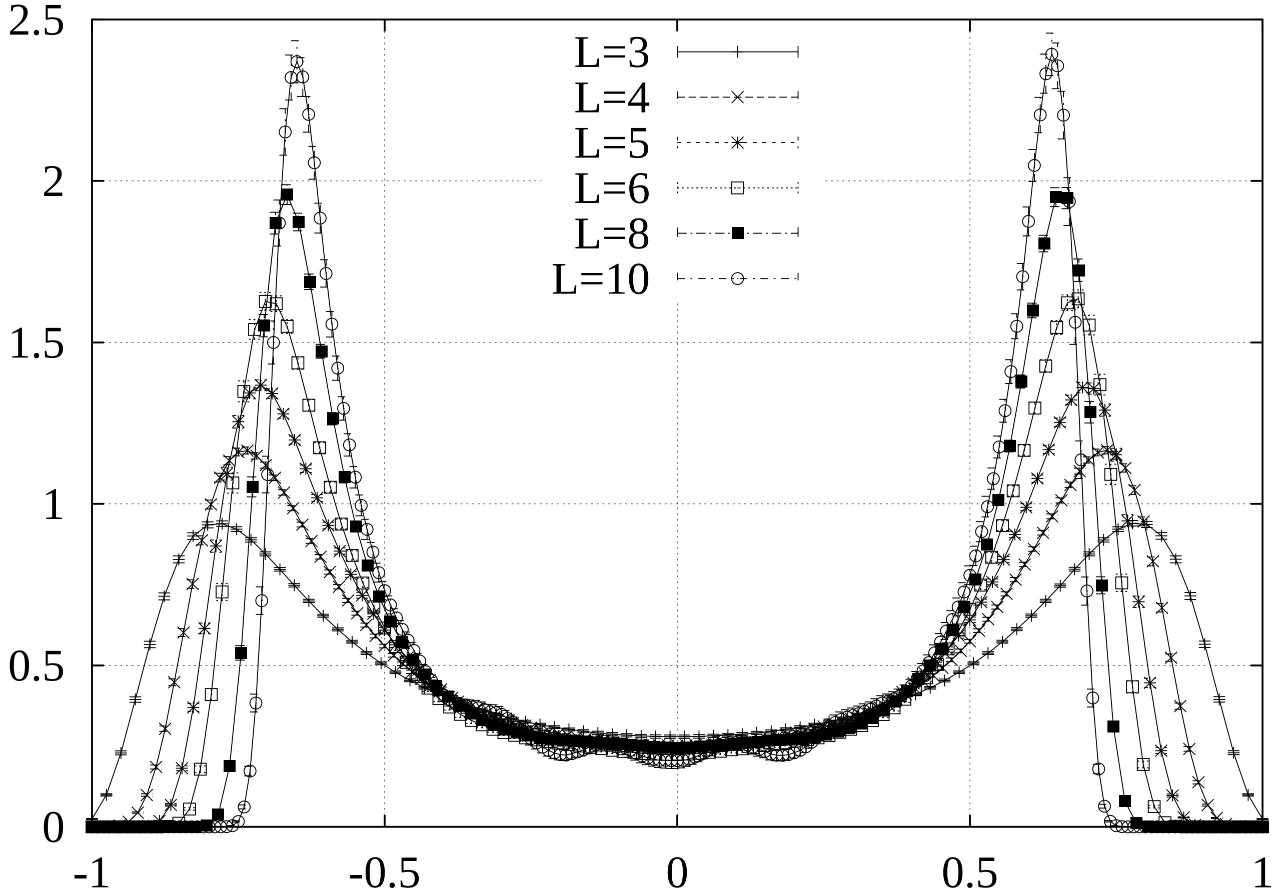

"< pjgp1.sh 10" u 2:3
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-1
-0.5
0
0.5
1


"< pjgp1.sh 62" u 2:3
2.5
2
1.5
1
0.5
0
-1
-0.5
0
0.5
1

After average over many sample (D=3)



After average over many sample (D=4)



ORDER INTO THE MESS:

STOCHASTIC STABILITY (very reasonable)

Roughly speaking stochastic stability implies that the system is generic.

$$H(\epsilon)(\sigma) = H(\sigma) + \epsilon H_R(\sigma)$$

where $H_R(\sigma)$ random Hamiltonian,

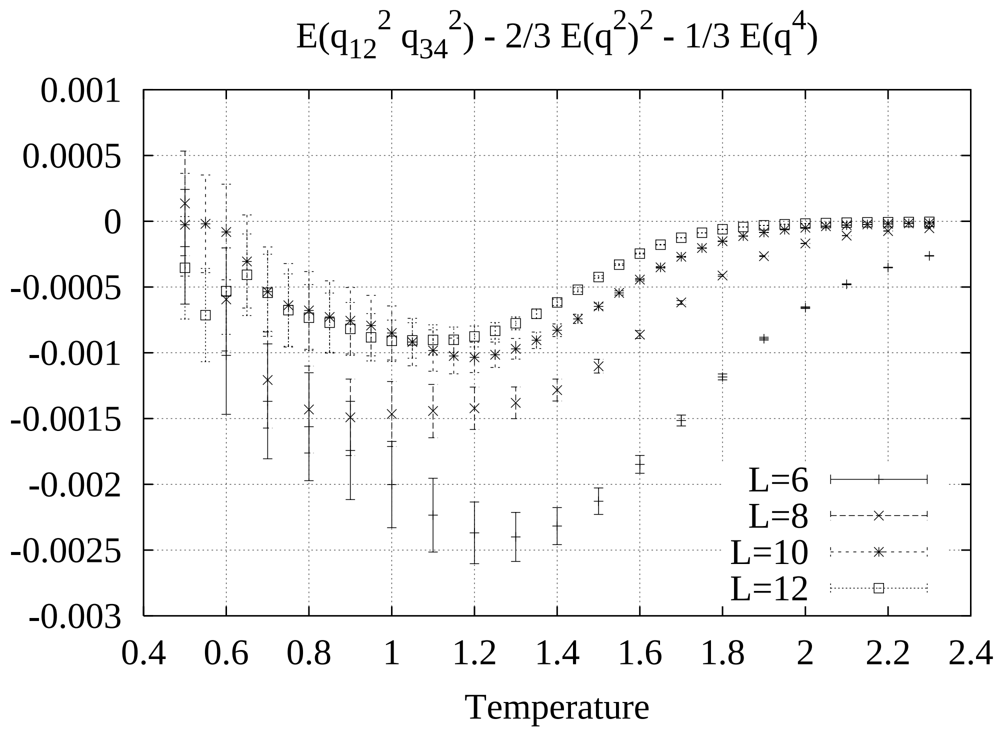
everything is smooth in ϵ.

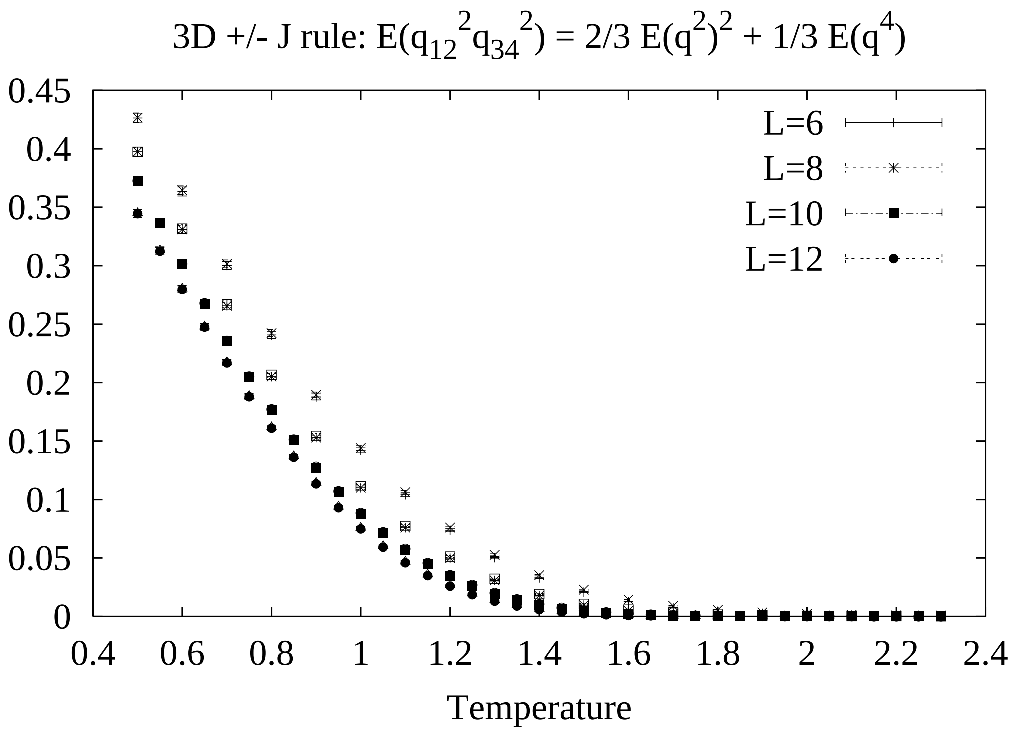
Stochastic stability implies strong constraints on $\mathcal{P}(\{w\},\{q\}$

e.g. Guerra relations:

$$P(q_1,q_2) = \frac{1}{3}P(q_1)\delta(q_1-q_2) + \frac{2}{3}P(q_1)P(q_2)$$

$$\overline{\langle q_{12}^2 q_{34}^2 \rangle} = \frac{1}{3}\overline{\langle q_{12}^4 \rangle} + \frac{2}{3}\overline{\langle q_{12}^2 \rangle}^2$$


E(q12^2 q34^2) - 2/3 E(q^2)^2 - 1/3 E(q^4)
0.001
0.0005
0
-0.0005
-0.001
-0.0015
-0.002
-0.0025
-0.003
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
Temperature
L=6
L=8
L=10
L=12


3D +/- J rule: E(q12^2 q34^2) = 2/3 E(q^2)^2 + 1/3 E(q^4)
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
Temperature
L=6
L=8
L=10
L=12

SEPARABILITY (one significant overlap)

All overlaps (depending on the operator A) are given functions of the spin overlap.

$$\forall A\ \exists f^A(q) \text{ such that } q^A_{\alpha,\gamma} = f^A(q_{\alpha,\gamma)}).$$

In general case an infinite number of quantities (i.e. $q_{\alpha,\gamma)}$ $\forall A$) characterizes the mutual relations among α and γ.

Here this infinite number is reduced to one.

Separability seems to imply ultrametricity

$$q_{\alpha,\gamma)} > \min(q_{\alpha,\beta)}, q_{\beta,\gamma}) \ \forall \beta.$$

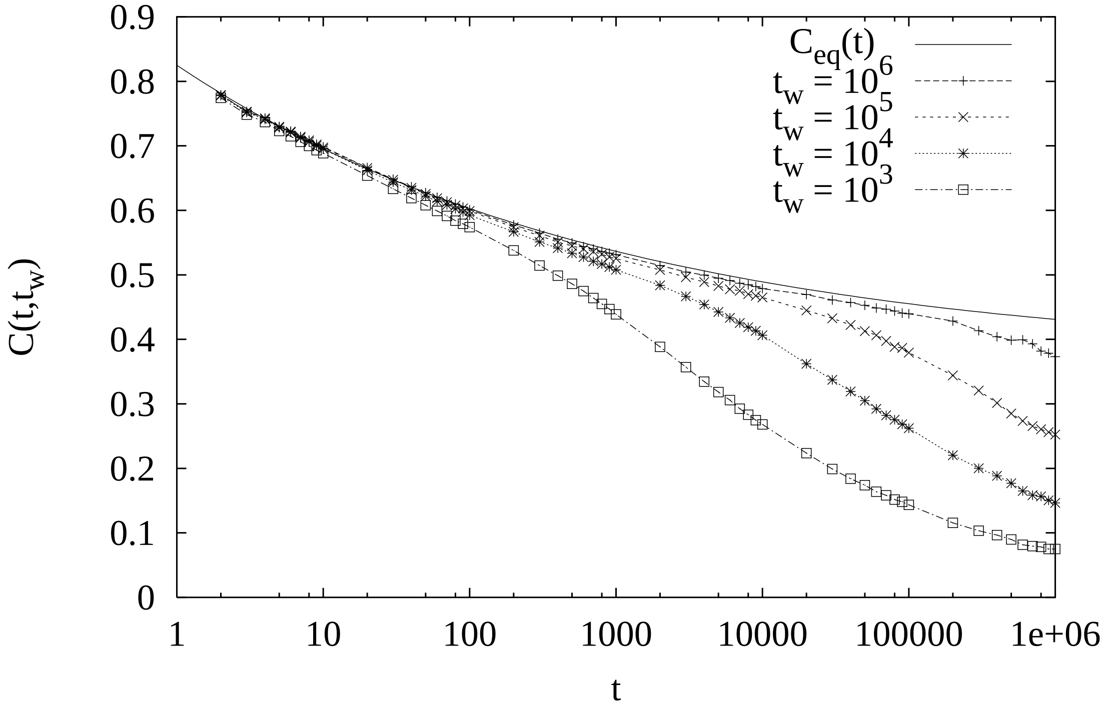
Separability may be less compulsory of stochastic stability.

Dynamics

What happens if we cool the system starting a random (high temperature) configuration at time zero?

- Ageing
- Two time correlations and responses
- Scaling R versus C
- Fluctuation dissipation relations

$$C(t,t_w) \equiv \frac{1}{N}\sum_{i=1}^{N}\langle\sigma_i(t_w)\sigma_i(t_w+t)\rangle = q(t_q, t_w+t)$$



$t < t_w$: "quasi-equilibrium" regime, $C(t,t_w) \simeq C_{\rm eq}(t)$;

$t > t_w$: aging regime. If simple aging holds
$C(t,t_w) \propto \mathcal{C}(t/t_w)$.

$$q_{\rm \ EA} \equiv \lim_{t\to\infty}\lim_{t_w\to\infty} C(t,t_w)$$

$$H = H_0 + \theta(t - t_w) \sum_i h_i \sigma_i$$

$$R(t, t_w) \equiv \frac{1}{N} \sum_{i=1}^{N} \langle \frac{\partial \sigma_i(t_w + t)}{\partial h_i} \rangle$$

$\frac{dR}{dC} = 1 \Longleftrightarrow$ Fluttuation Dissipation Theorem (FTD)

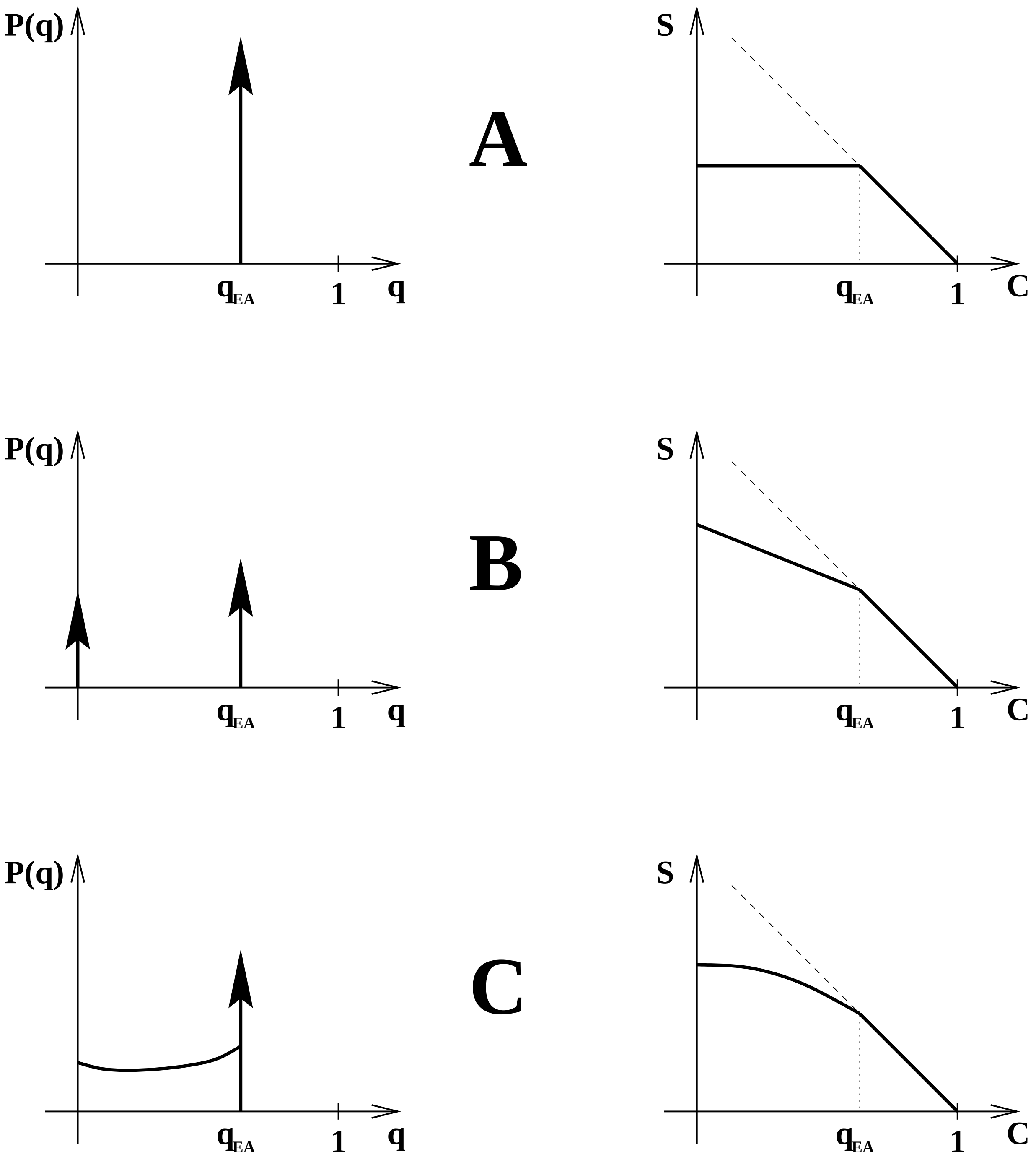
valid for $t << t_w$.

Classification

R versus $C \Longleftrightarrow P(q)$

$R(C) = \int_0^C dq P(q)$

Three main kinds of dynamical response $S(C)$, that correspond to different shapes of the static $P(q)$.

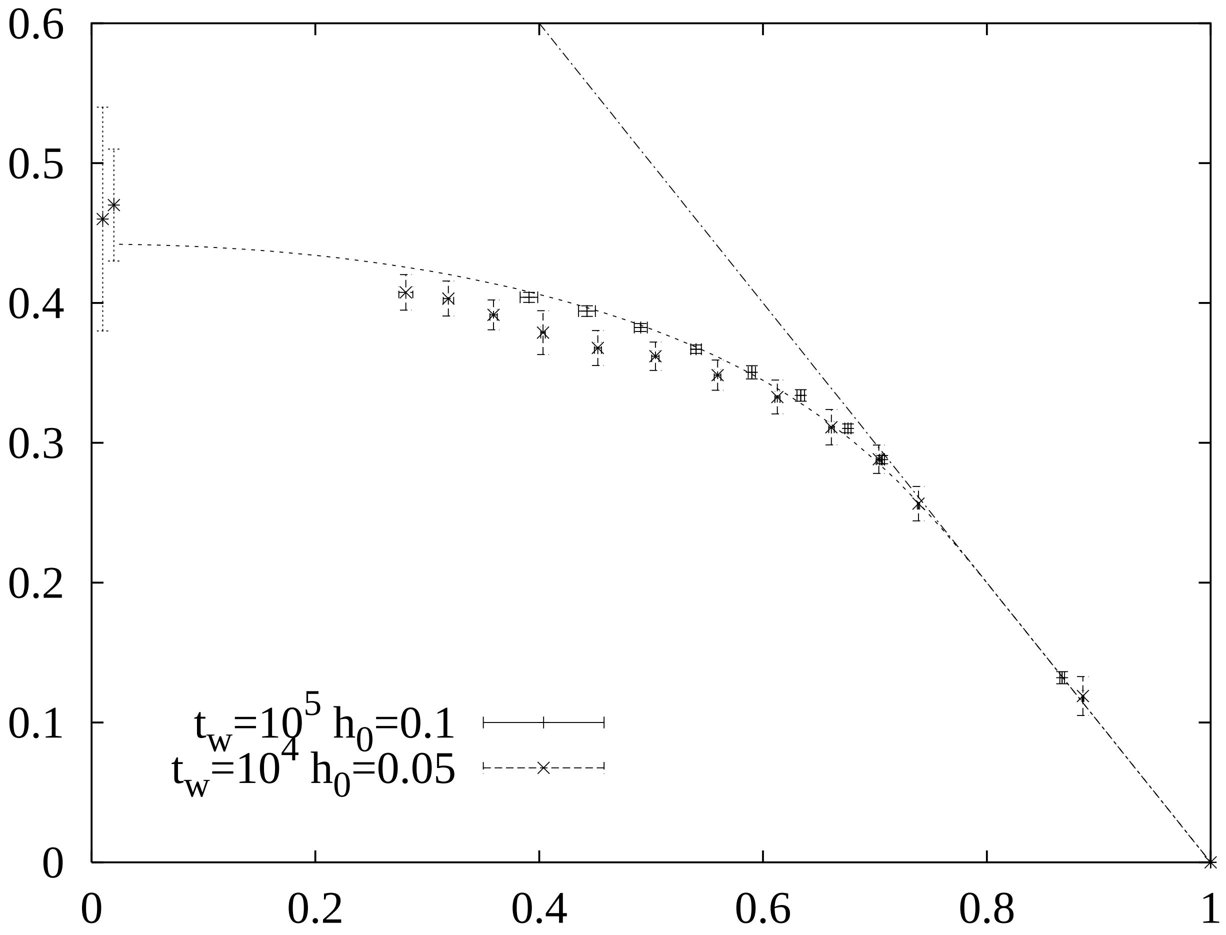


P(q)
q
EA
1
q
A
S
q
EA
1
C
P(q)
q
EA
1
q
B
S
q
EA
1
C
P(q)
q
EA
1
q
C
S
q
EA
1
C

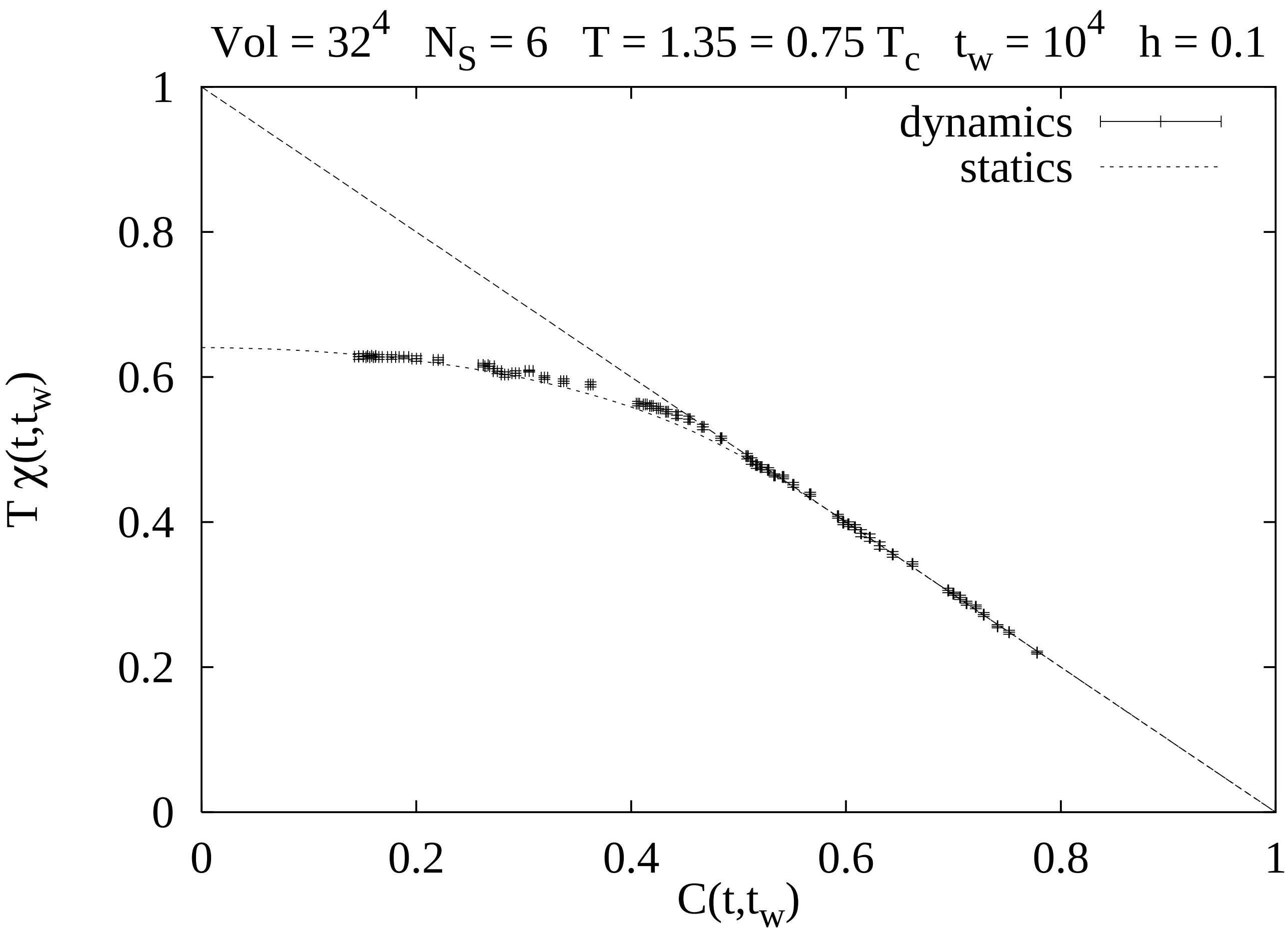
Numerical Results

Edwards-Anderson (EA) model

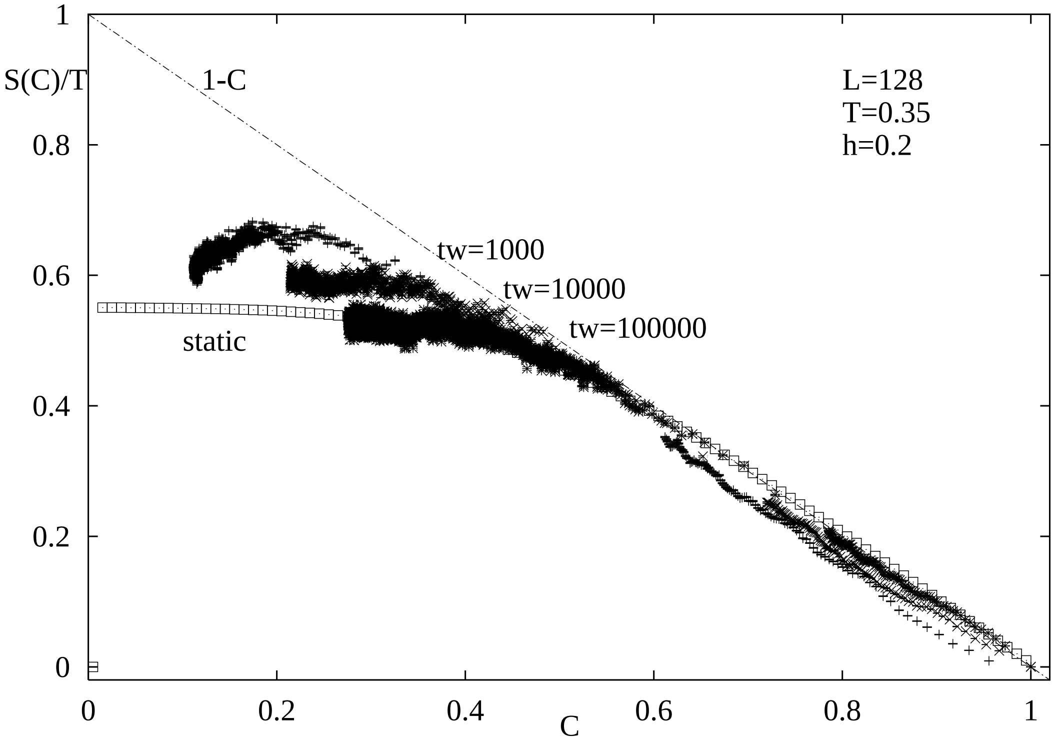
D=3 Volume=64^3 $T = 0.7 \simeq \frac{3}{4}T_c$



EA D=4


Vol = 32^4 N_S = 6 T = 1.35 = 0.75 T_c t_w = 10^4 h = 0.1
dynamics
statics
T χ(t,t_w)
C(t,t_w)
0
0.2
0.4
0.6
0.8
1

Tilings M=16


1
S(C)/T
1-C
L=128
T=0.35
h=0.2
0.8
0.6
tw=1000
tw=10000
tw=100000
static
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
C

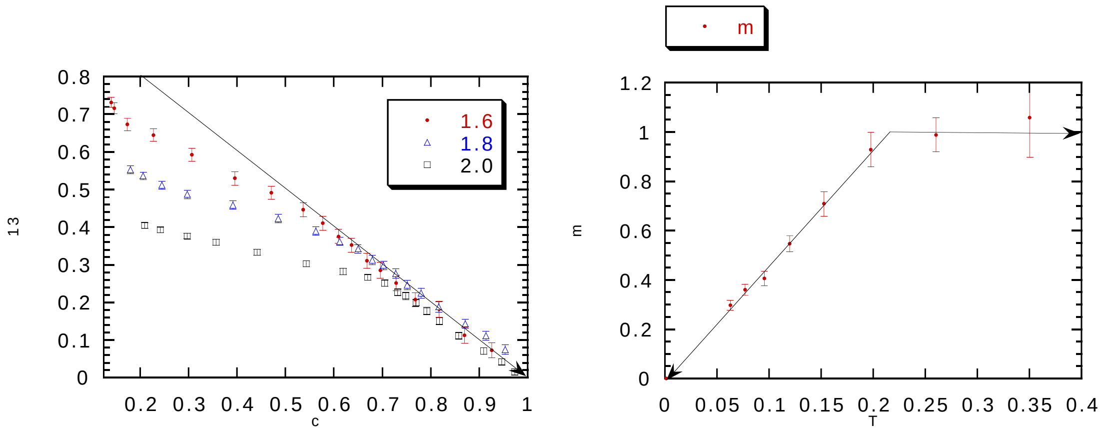
Glasses Analytic computations support

$$X(C) = m \quad \text{for} \quad C < C^*$$

$$X(C) = 1 \quad \text{for} \quad C < C^*$$

$$m \approx T/T_c \text{ (i.e } m = 1 \text{ at } T = T_c,\ m = 0 \text{ at } T = 0)$$

$$m\beta \approx constant = \text{effective temperature of the valleys}$$



Direct measurements in experiments for glasses and spin glasses in the next future on this screen.

- cond-mat/9906076

  Title: Replica Symmetry Breaking in Short Range Spin Glasses: A Review of the Theoretical Foundations and of the Numerical Evidence

  Authors: E. Marinari, G. Parisi, F. Ricci-Tersenghi, J. Ruiz-Lorenzo, F. Zuliani

- cond-mat/9903370

  Title: The response of glassy systems to random perturbations: A bridge between equilibrium and off-equilibrium

  Authors: S. Franz, M. Mezard, G. Parisi, L. Peliti

- cond-mat/980100

  Title: Testing replica predictions in experimentd

  Authors: Giorgio Parisi

- cond-mat/9903129

  Title: Thermodynamics of binary mixture glasses

  Authors: Barbara Coluzzi, Marc Mezard, Giorgio Parisi, Paolo Verrocchio

- Title: Thermodynamics of a tiling model (In preparation)

  Authors: Luca Leuzzi, Giorgio Parisi