

**Chaotic, memory and cooling rate  
effects in spin glasses:**

**Is the Edwards-Anderson model a good  
spin glass?**

**Marco Picco**

*LPTHE, Université Paris VI and VII*

**Felix Ritort**

*University of Barcelona*

**F. R.-T.**

*ICTP, Trieste*

## Motivations for this work

Comparison with [experiments](#), in particular with

- recent [memory](#) and [chaos](#) experiments by Saclay and Uppsala groups;
- [AC](#) measurements experiments;
- [temperature cycling](#) and temperature [shifting](#) experiments.

From the [theoretical](#) point of view, chaos effects in spin glasses are not well understood, especially chaos with respect to temperature and dynamical chaos.

## The Edwards-Anderson model

$$H[\sigma] = - \sum_{\langle i,j \rangle} J_{ij} \sigma_i \sigma_j - h \sum_i \sigma_i$$

$\langle i, j \rangle$  represent nearest neighbours pairs on a cubic lattice

$\sigma_i = \pm 1$  are Ising spins

the  $J_{ij}$  couplings are extracted from a Gaussian distribution with zero mean and unitary variance

## Static Chaos

$$\beta H[\sigma, \tau] = \beta_1 H_1[\sigma] + \beta_2 H_2[\tau]$$

The difference between the two terms is a small perturbation:  $\beta_1 \neq \beta_2$  or  $h_1 \neq h_2$  or  $\{J^1\} \neq \{J^2\}$ .

How similar are two typical configurations  $\sigma$  and  $\tau$  ?

They are similar up to the *chaos length*

$$L(\Delta x) \simeq (\Delta x)^{-1/\zeta}$$

where  $\Delta x$  can be  $\Delta T$ ,  $\Delta h$  or  $\Delta J$ .

**Droplet Model**       $\zeta = d_s/2 - \theta$

It predicts same chaotic effects with respect to  $\Delta J$  and  $\Delta T$ .

**Mean Field**       $\zeta = 1, \frac{3}{2}, 2$

Stronger chaos with respect to  $\Delta h$  and  $\Delta J$  than with respect to  $\Delta T$ .

Maybe no temperature chaos in the SK model !

## Dynamic chaos

Only temperature chaos has been studied.

No numerical evidences for chaos have been found.

Up to now the **Edwards-Anderson (EA)** model has been considered the **best candidate** for describing real spin glasses.

It is the **simplest** model (cubic lattice, nearest neighbours random interactions, Ising spins).

It correctly **reproduces many experimental facts**:

- aging in zero-field-cooled magnetization  $M_{\text{ZFC}}(t, t_w)$ ;
- saturated magnetization decay  $M(t) \simeq t^{-\lambda(T)}$  with  $\lambda(T) \propto T$ ;
- correlation length growth  $\xi(t, T) \simeq t^{1/z(T)}$  with  $z(T) \propto 1/T$ ;
- AC susceptibilities  $\chi'$  and  $\chi''$ .

Nevertheless in the last years many very interesting experiments have been performed, which is now time to reproduce in the EA model.

Moreover very few studies have been focused on the EA model in the presence of an AC external field.

## Large scale simulations (on parallel computers too)

- sizes up to  $100^3$ ;
- times up to  $10^8$  MCS.

### Observables

Autocorrelation function

$$C(t_w, t_w + t) = \frac{1}{N} \sum_i \sigma_i(t_w) \sigma_i(t_w + t)$$

$$C(t_w, t_w + t) = C_{st}(t) + C_{ag}(t_w, t_w + t)$$

Zero-field-cooled magnetization

$$\chi(t_w, t_w + t) = \frac{1}{Nh} \sum_i \sigma_i(t_w + t)$$

$$\chi(t_w, t_w + t) = \chi_{st}(t) + \chi_{ag}(t_w, t_w + t)$$

Fluctuation dissipation theorem

$$\chi_{st}(t) = \frac{1 - C_{st}(t)}{T}$$

Fluctuation dissipation ratio

$$X[C] = -T \left. \frac{\partial \chi(s, t)}{\partial C(s, t)} \right|_{C(s, t) = C}$$

## AC measurements

$$h(t) = h_0 \cos(2\pi\omega t) \quad \omega = \frac{1}{P}$$

$$\chi(t) = \chi_0 \cos(2\pi\omega t + \phi)$$

$$\chi' = 2 \int_0^P \chi(t) \cos(2\pi\omega t) dt$$

$$\chi'' = 2 \int_0^P \chi(t) \sin(2\pi\omega t) dt$$

In all our simulations we closely follow **experimental procedures**: coolings rates, quenchings, waiting times, etc...

The only (big) difference is a factor  $\sim 10^{10}$  in the absolute time scales.

When not differently specified, at the beginning of any simulation the system has been quenched from a random configuration ( $T = \infty$ ) to a temperature in the frozen phase.

## $C(t_w, t_w + t)$ scaling

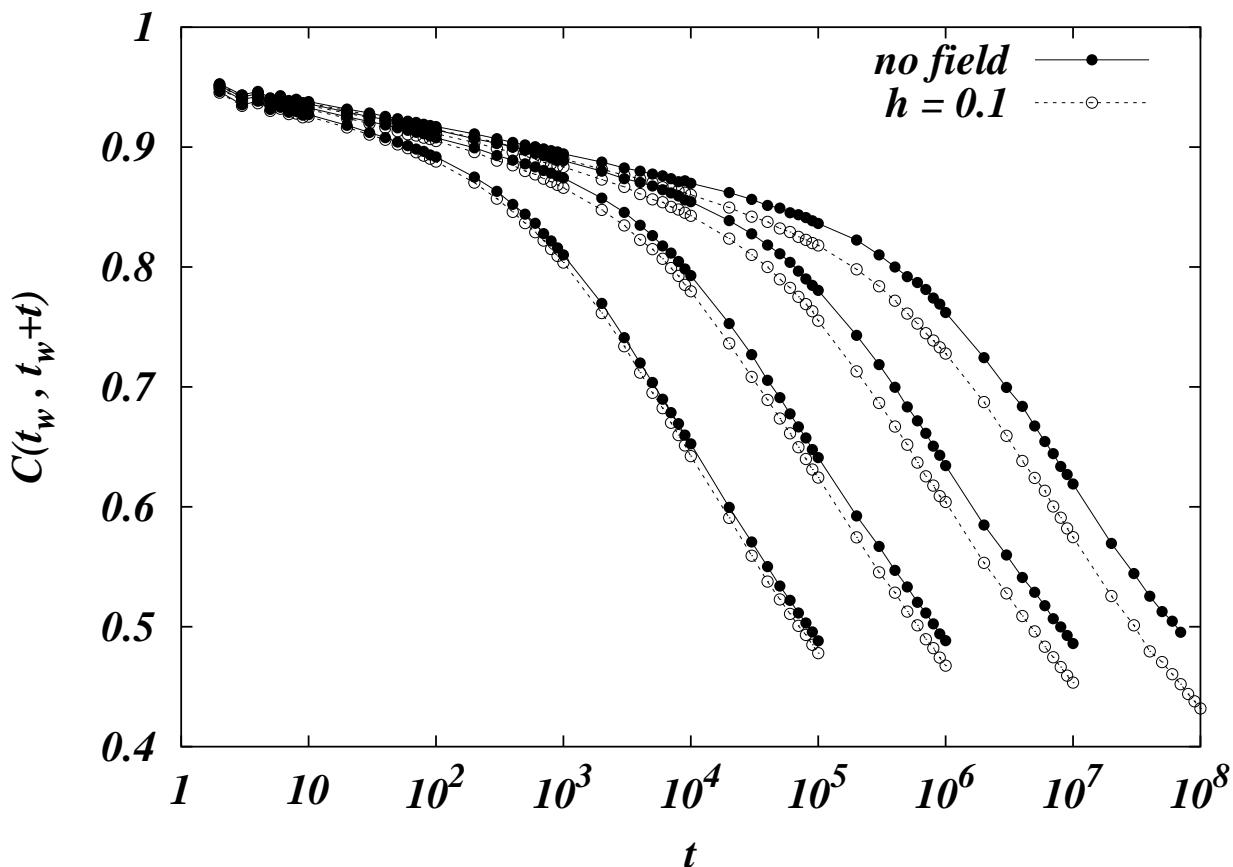
The scaling of the correlation function in the EA model is interesting by itself and, moreover, it can be compared with the scaling of experimental  $M_{\text{TRM}}$ .

$$C(t_w, t_w + t) = C_{st}(t) + C_{ag}(t_w, t_w + t)$$

$$\text{with} \quad C_{st}(t) = q_{\text{EA}} + At^{-\alpha}$$

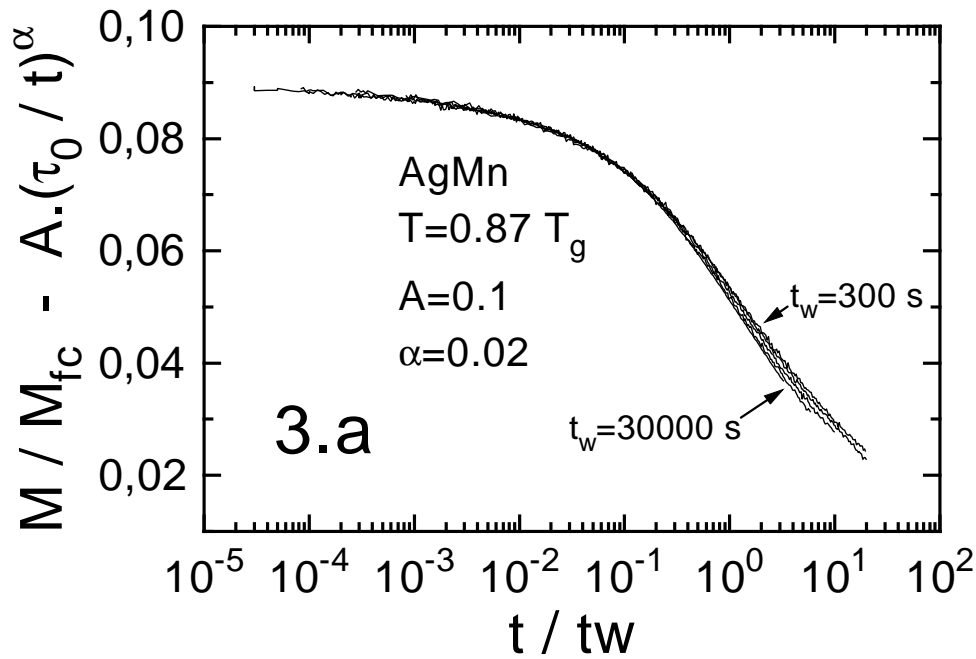
$$\text{Full (or simple) aging:} \quad C_{ag} = \tilde{C}(t/t_w)$$

Autocorrelation functions in a  $64^3$  system,  
 $T = 0.5$  and  $t_w = 10^3, \dots, 10^6$ .

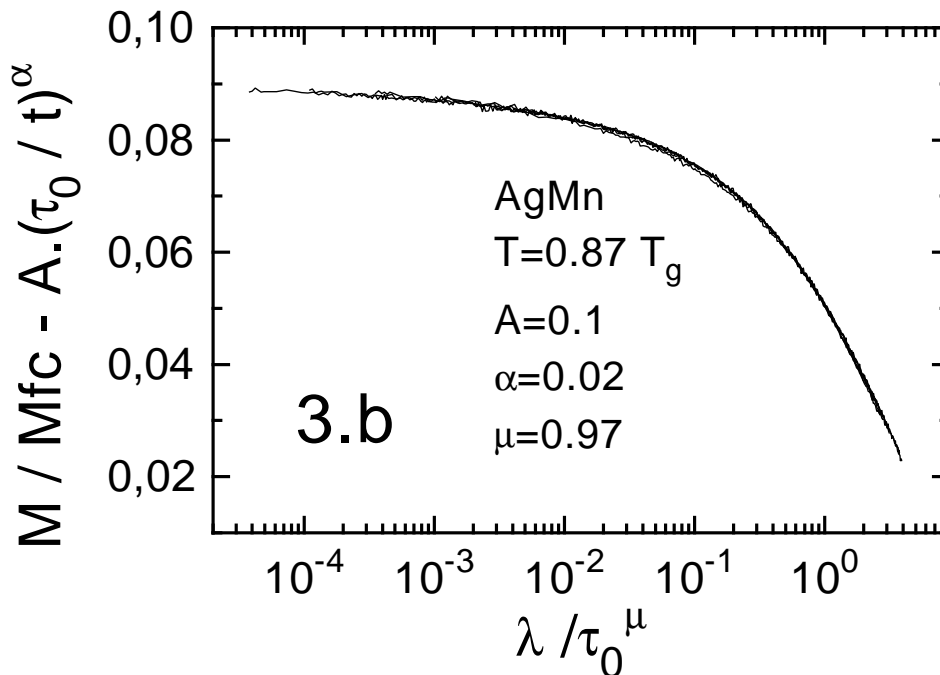


**Note** that experiments are always performed in the presence of an external magnetic field.

Experimental data for  $M_{\text{TRM}}$  do not seem to collapse completely using  $t/t_w$  as the scaling variable.

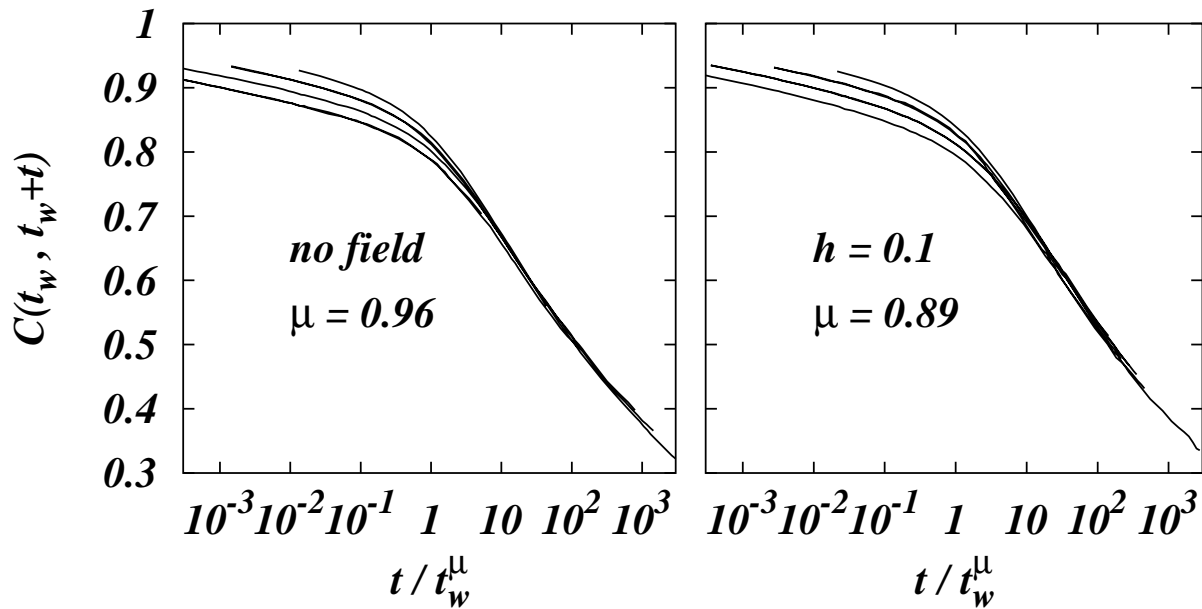


The best scaling seems to be achieved with the scaling variable  $t/t_w^\mu$  with  $\mu = 0.97$ .





A naive scaling in the aging regime would predict  $\mu < 1$  also in the EA model.



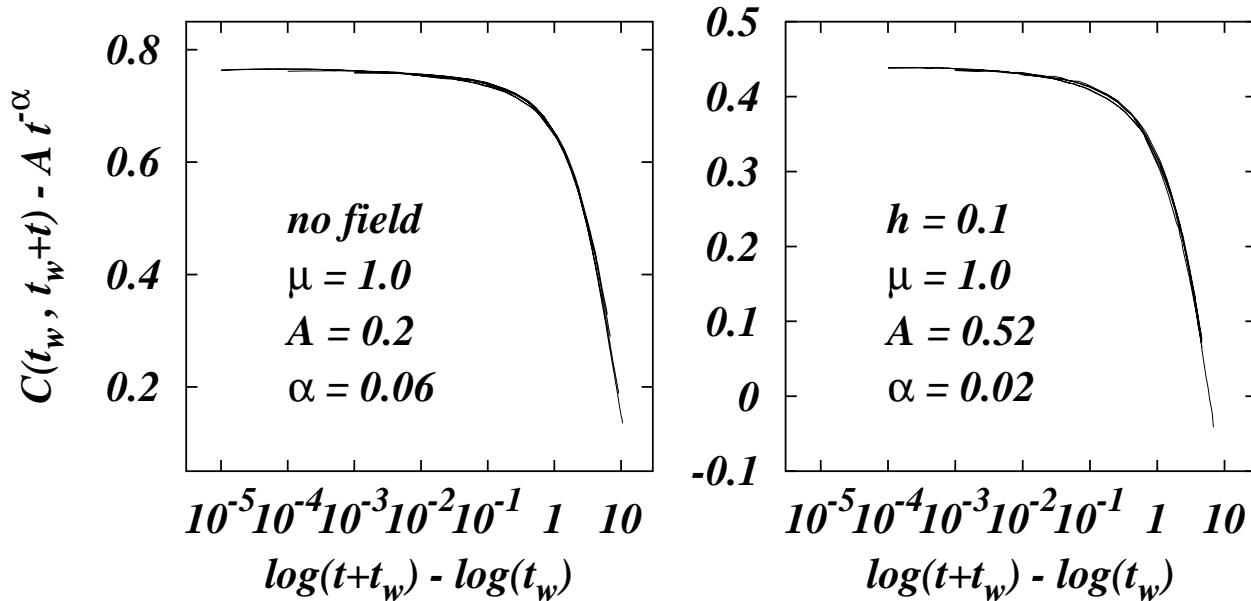
The best value for  $\mu$  seems to decrease in the presence of an external field.

In order to do the same analysis performed on experimental  $M_{\text{TRM}}$  data we use the scaling variable

$$\frac{(t_w + t)^{1-\mu} - t_w^{1-\mu}}{1 - \mu}$$

$$\Downarrow \mu \rightarrow 1$$

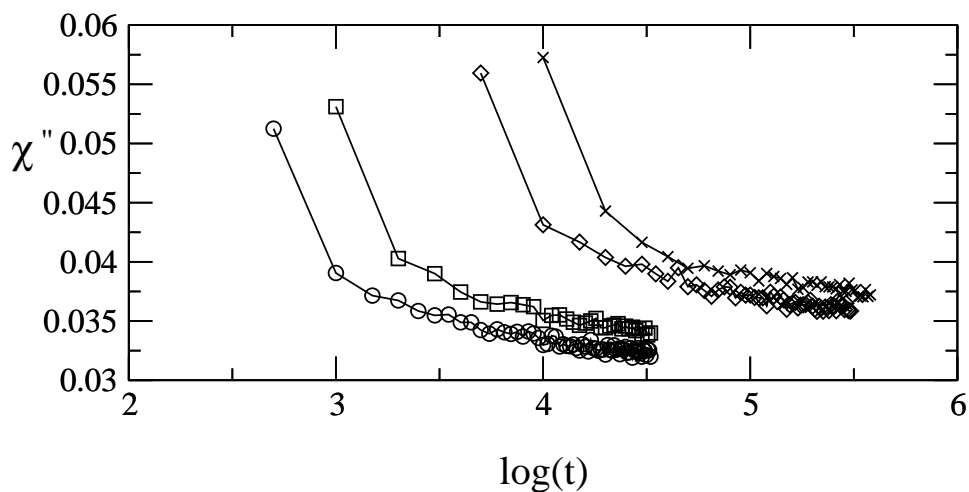
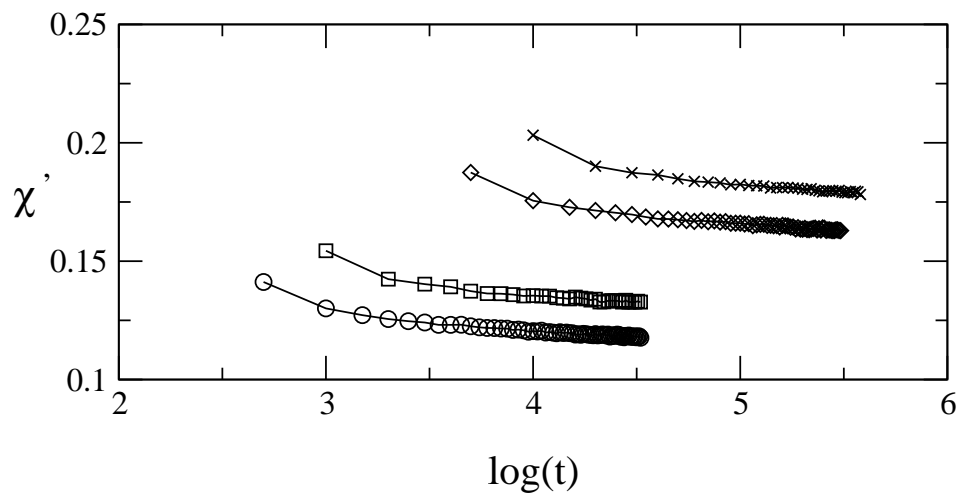
$$\ln(t_w + t) - \ln(t_w)$$



**Note** that data collapse (maybe also the experimental one) *apparently* can be improved if the scaling variable  $\ln(t_w + t) - \ln(t_w)$  is used instead of  $t/t_w$ .

## Relaxation of AC susceptibilities

We quench a  $100^3$  system to  $T = 0.6$  and we immediately start measuring the  $\chi'$  and  $\chi''$  susceptibilities applying an AC field of intensity  $h_0 = 0.1$  and frequency  $\omega = 1/P = 0.02, 0.01, 0.002, 0.001$ .

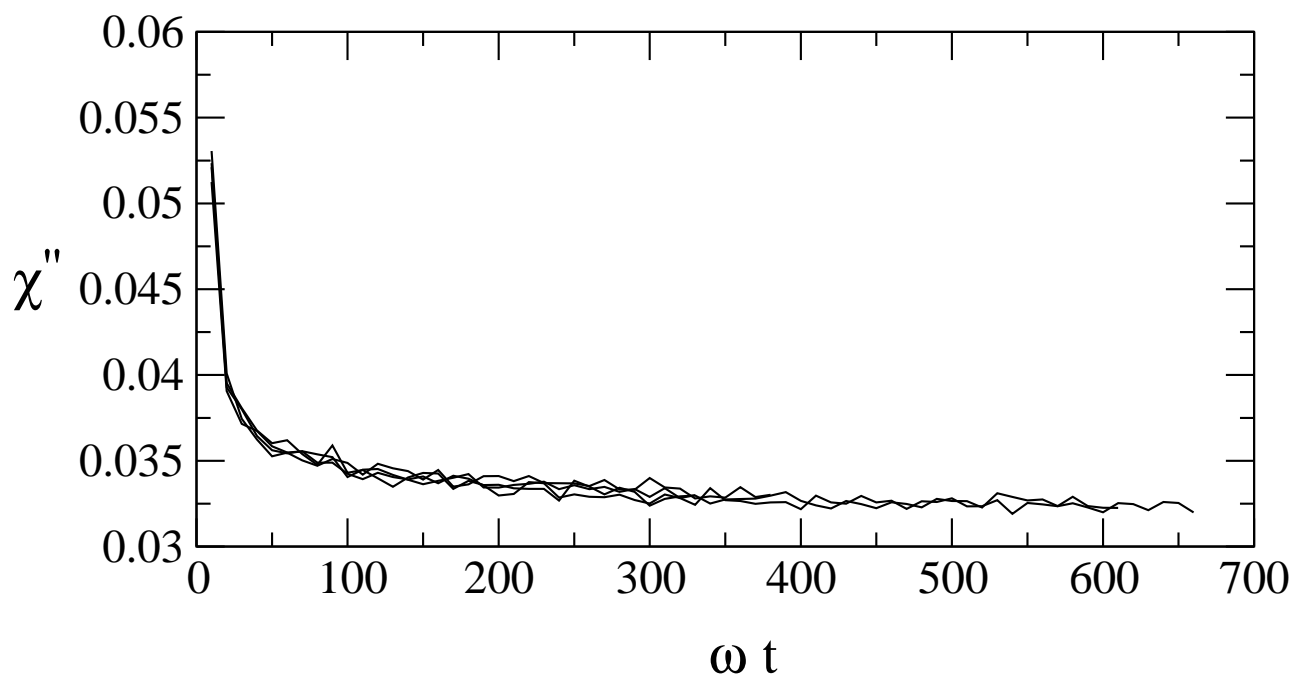
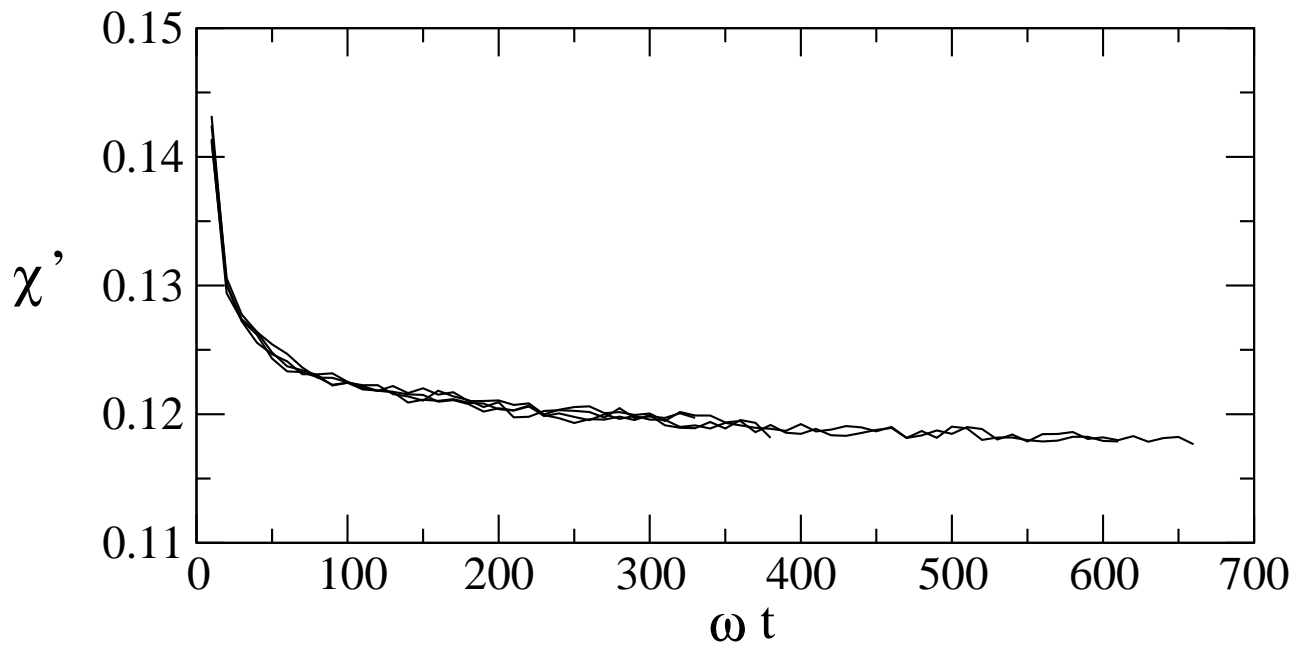


Data can be well fitted by the law

$$\chi(\omega, t) = \chi(\omega, \infty) + A t^{-\alpha}$$

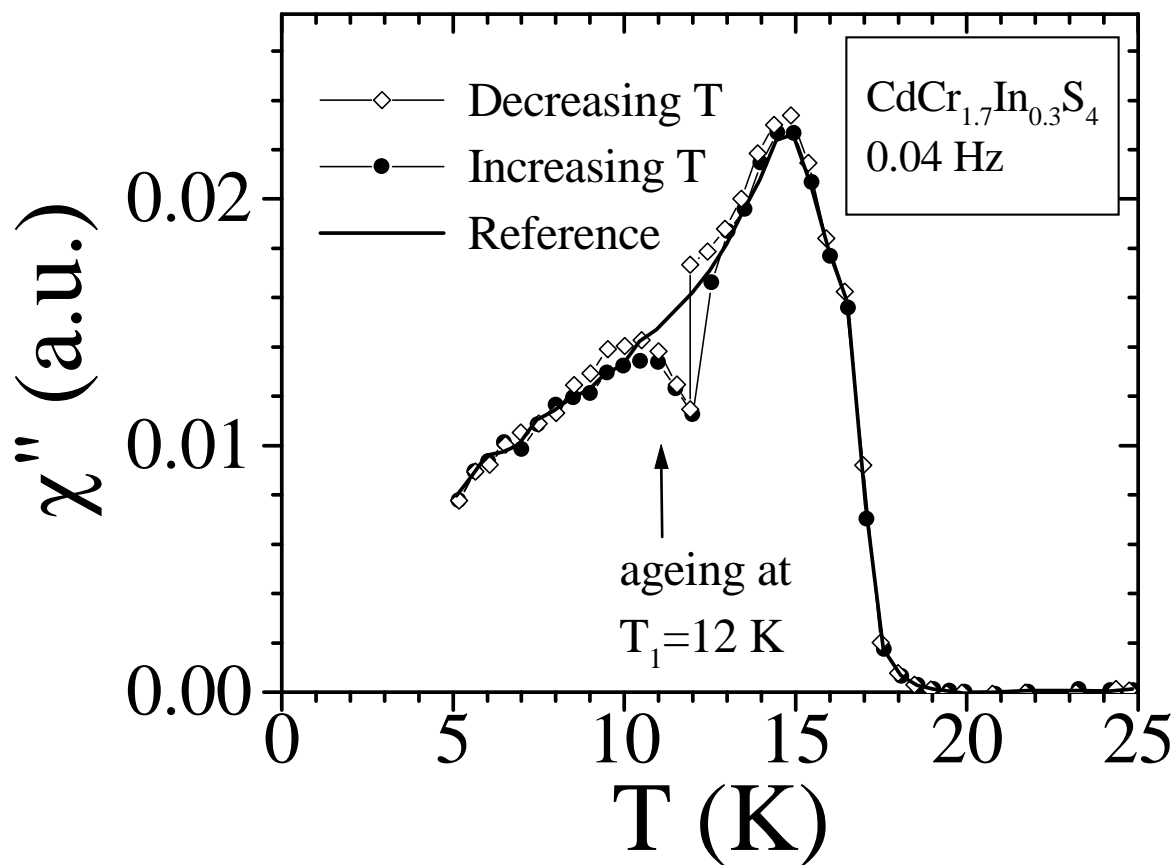
## $\omega t$ scaling

In the AC measurements, on time-scales we have access to, we find the same  $\omega t$  scaling experimentalists find.



## "Cooling and stop" experiment

In the last years many experiments of this kind have been performed by Saclay and Uppsala groups. We have tried to reproduce the first and simplest one.



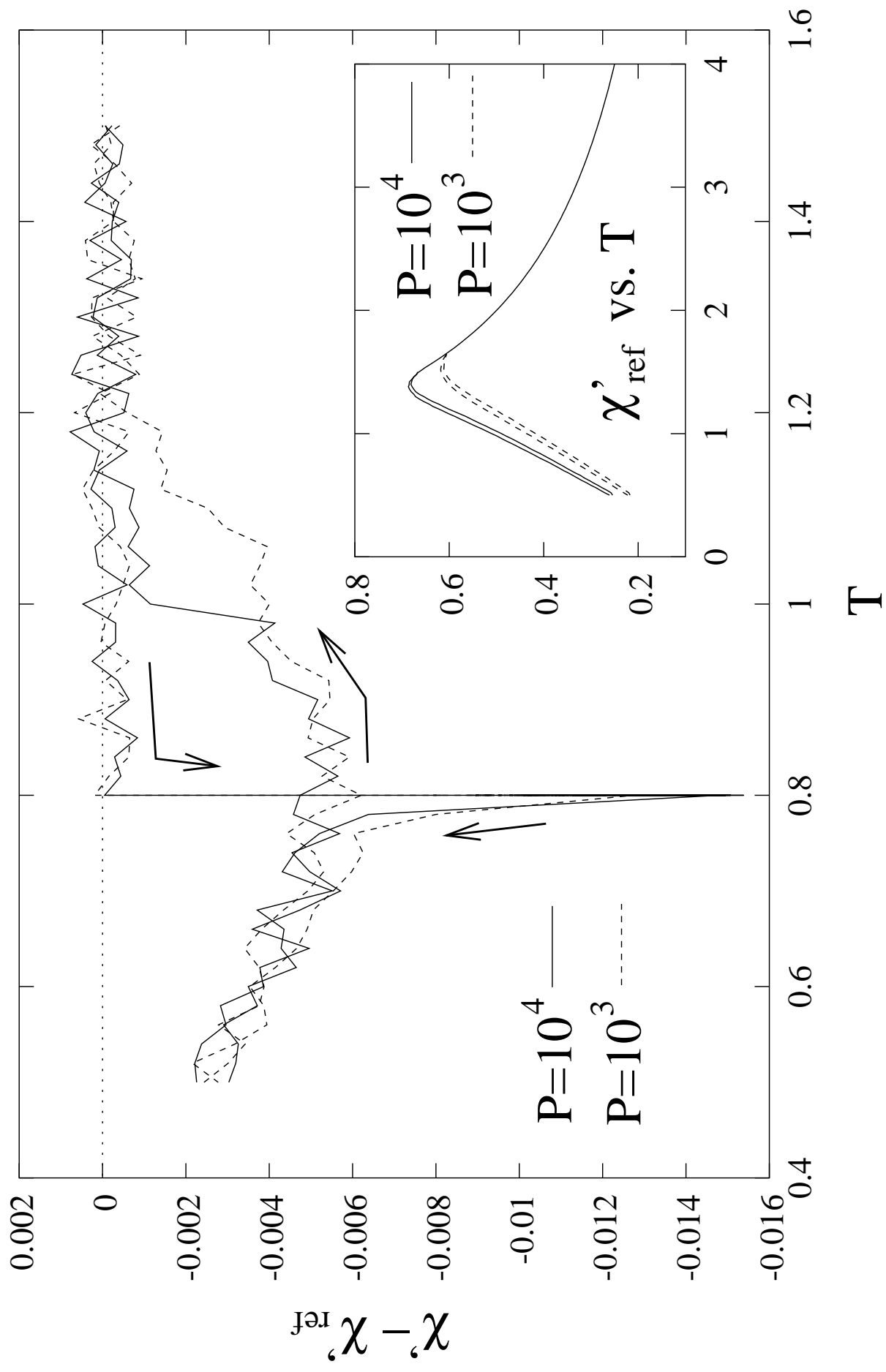
Experimentalists usually presents results for the  $\chi''$  susceptibility because its relaxation is larger (relatively to the asymptotic value).

In our experiment we work without any external fields in order to improve the signal. We measure correlation functions (in 100 samples of  $64^3$  systems) and we obtain the in-phase susceptibility as

$$\chi' = \frac{1 - C}{T}$$

Some notes on the **procedure**:

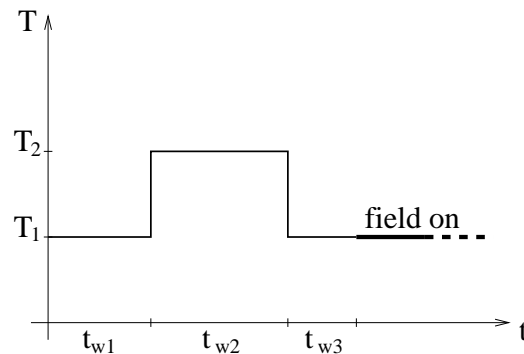
- **Cooling** from high temperature down to  $T = 0.5$  and then heating back again.
- The time of the simulation is divided in **intervals** of duration  $P$ .
- For every interval, we measure the **correlation** between the first and the last times belonging to the same interval, i.e.  $C(t + P, t)$ .
- **Reference curves** have been obtained with constant cooling and heating rates and no stops. At the end of every interval we change the temperature by  $\Delta T = \pm 0.02$ .
- The **stop** at  $T = 0.8$  is 100 intervals long, i.e.  $100 \cdot P$  MCS.



## Experiments with temperature changes

How much is a relaxation process at temperature  $T_1$  affected by a relaxation at temperature  $T_2$ ?

$T$ -cycle



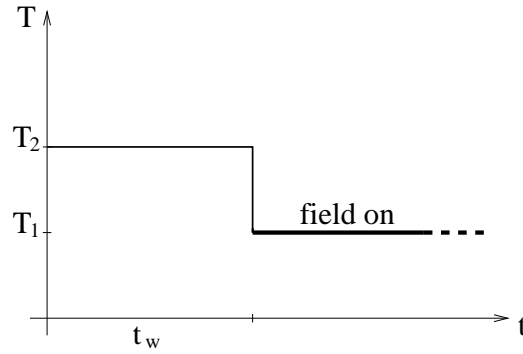
In real spin glasses for  $\Delta T = T_2 - T_1 > 0$  the *effective waiting time* decreases (rejuvenation).

In the EA model, the effective waiting time (i.e. the correlation length) **always grows** with time, without any reinitialization when the temperature is changed.

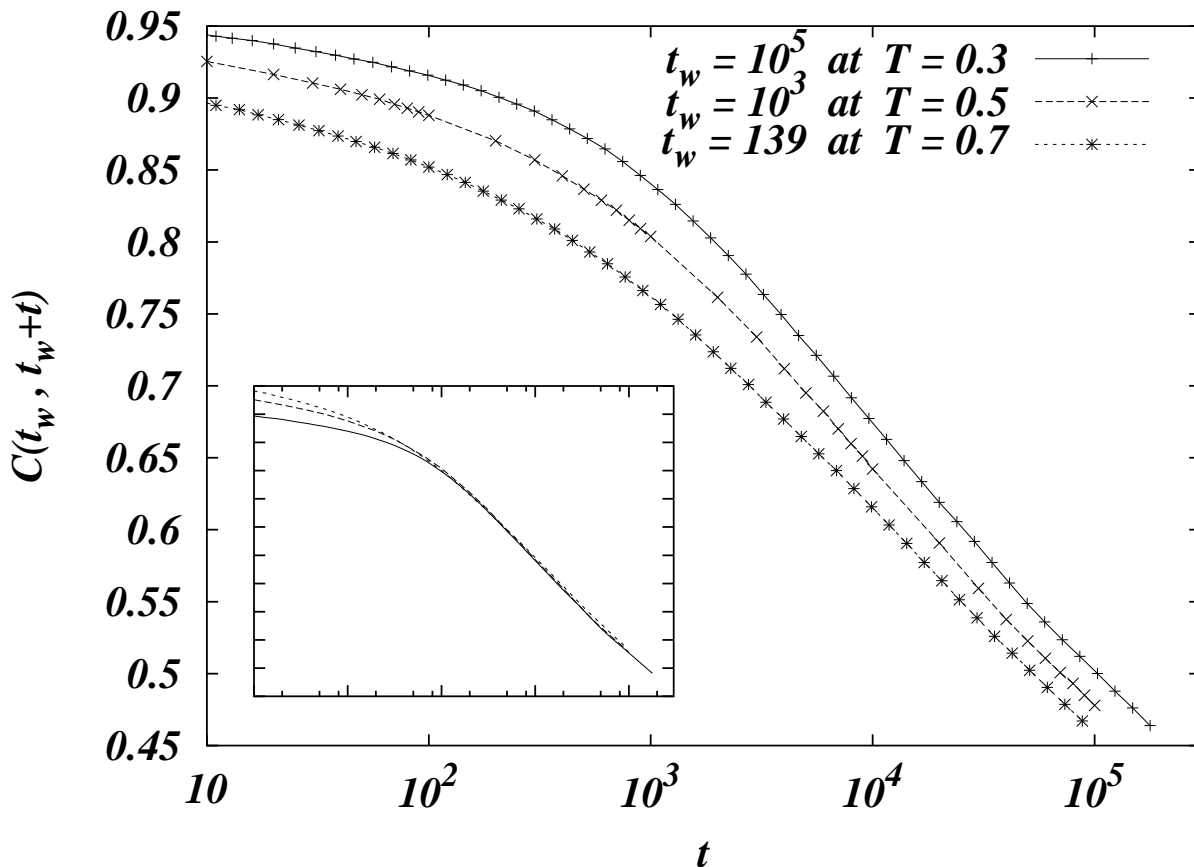
Strong **cooling rate effects**.



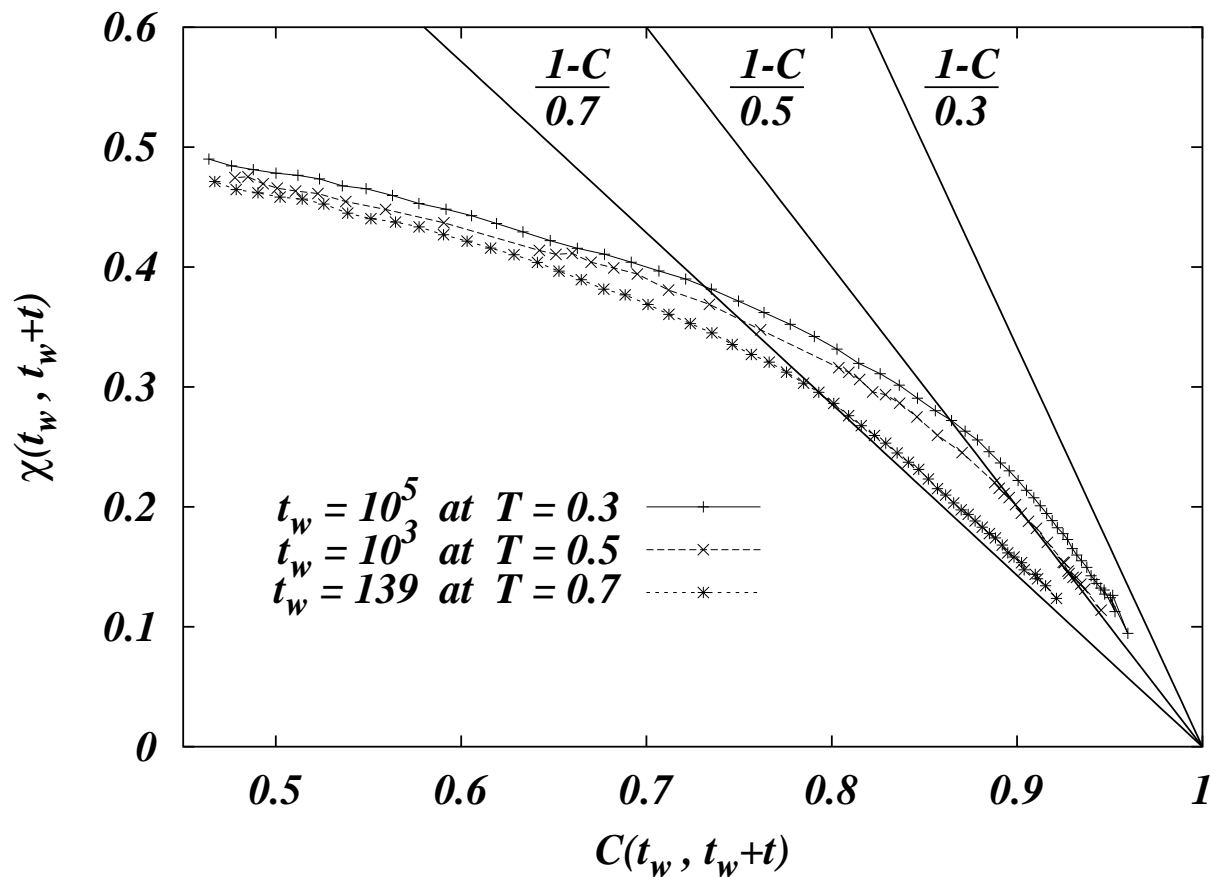
## T-shift



We choose the  $\{t_w, T_2\}$  pairs such that the size of correlated regions,  $\xi(t_w, T_2) = B t_w^A T_2$  with  $B = \mathcal{O}(1)$  and  $A \simeq 0.16$ , is the same in every experiment. We always take measurements at  $T_1 = 0.5$ . We find that similar correlation lengths give similar effective waiting time  $t_w^{eff}$  in the correlation decay.



However, temperatures in the quasi-equilibrium regime seem to be different. Remind that measurements are always taken at  $T_1 = 0.5$ .

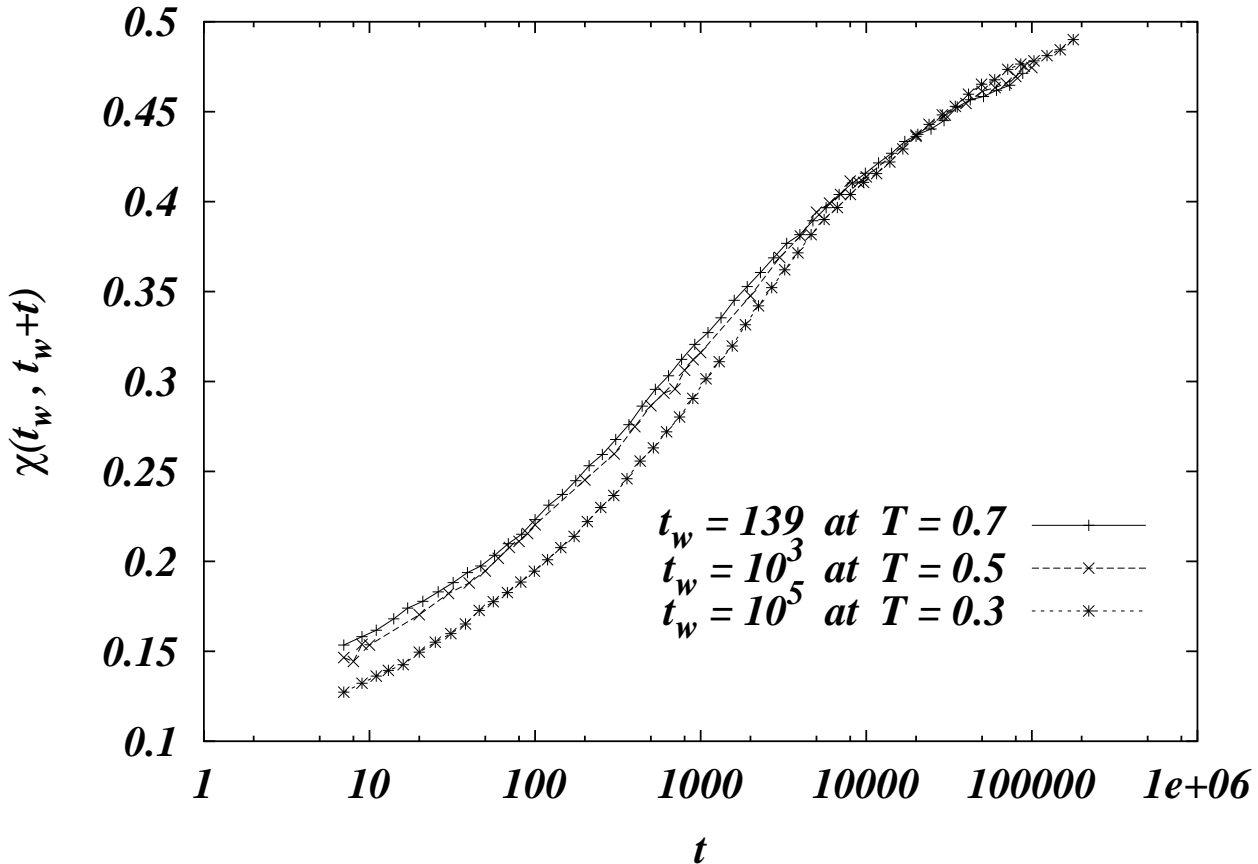


This effect is asymmetric with respect to  $\Delta T$ .

Very recently Bernardi et al. (cond-mat/0006059) have proposed a scaling for the susceptibility

$$\chi(t_w + t, t_w) = \tilde{\chi}(R(t_w), L(t))$$

which is not satisfy by our data.



In our 3 experiments  $R(t_w)$  is the same by construction and  $L(t)$  is related to  $t$  by the same law, because we measure the response always at the same temperature.

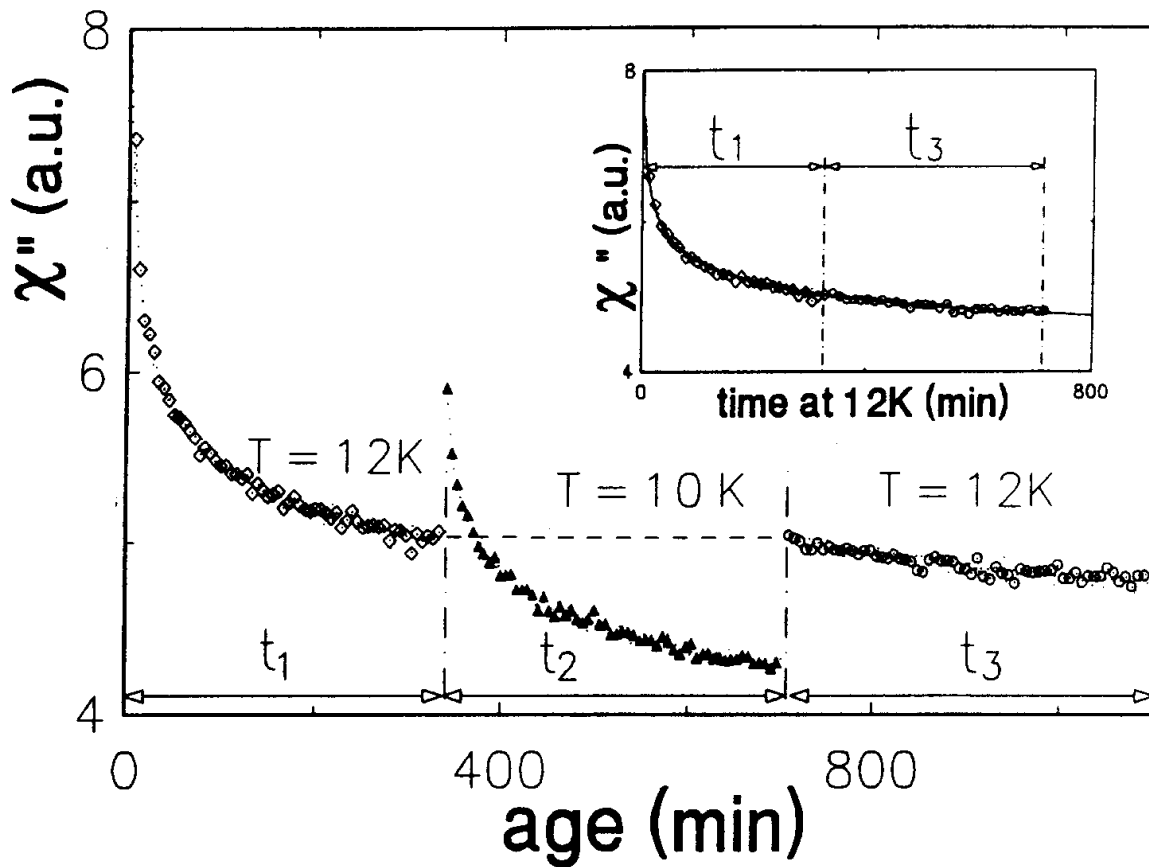
If the scaling would be correct data in the figure should collapse.

## Chaos in AC relaxation

We look for rejuvenation effects under a change in

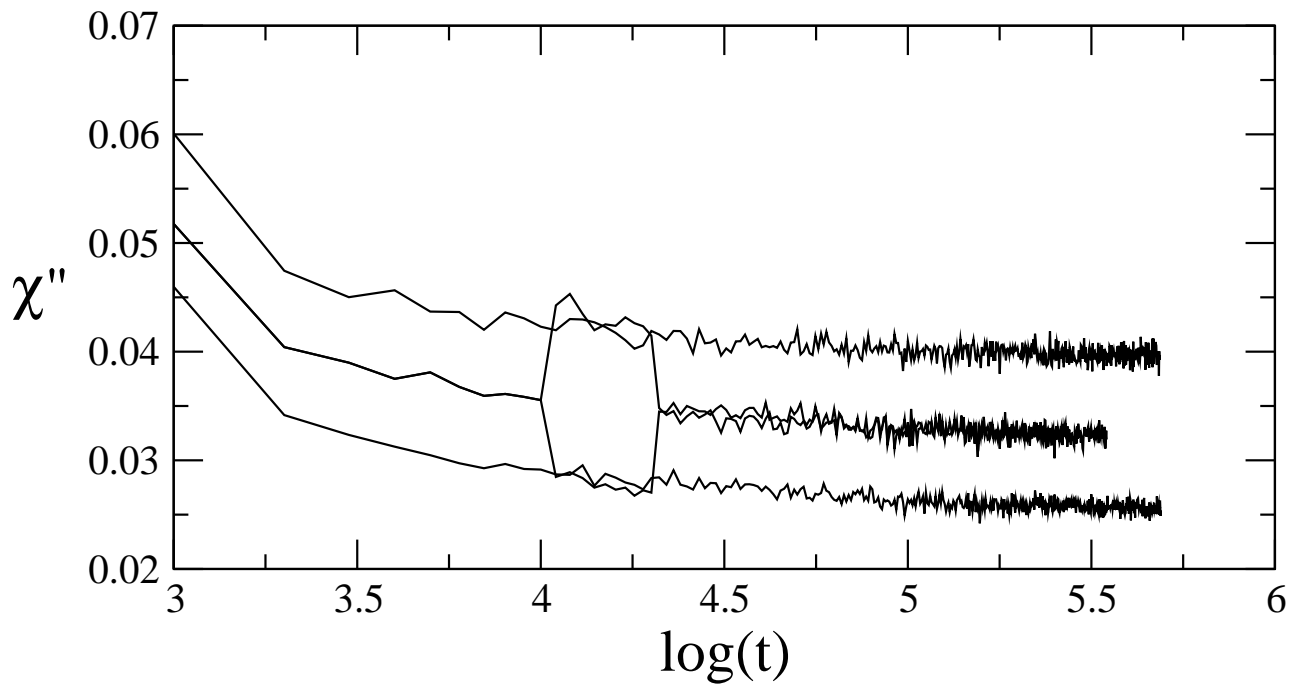
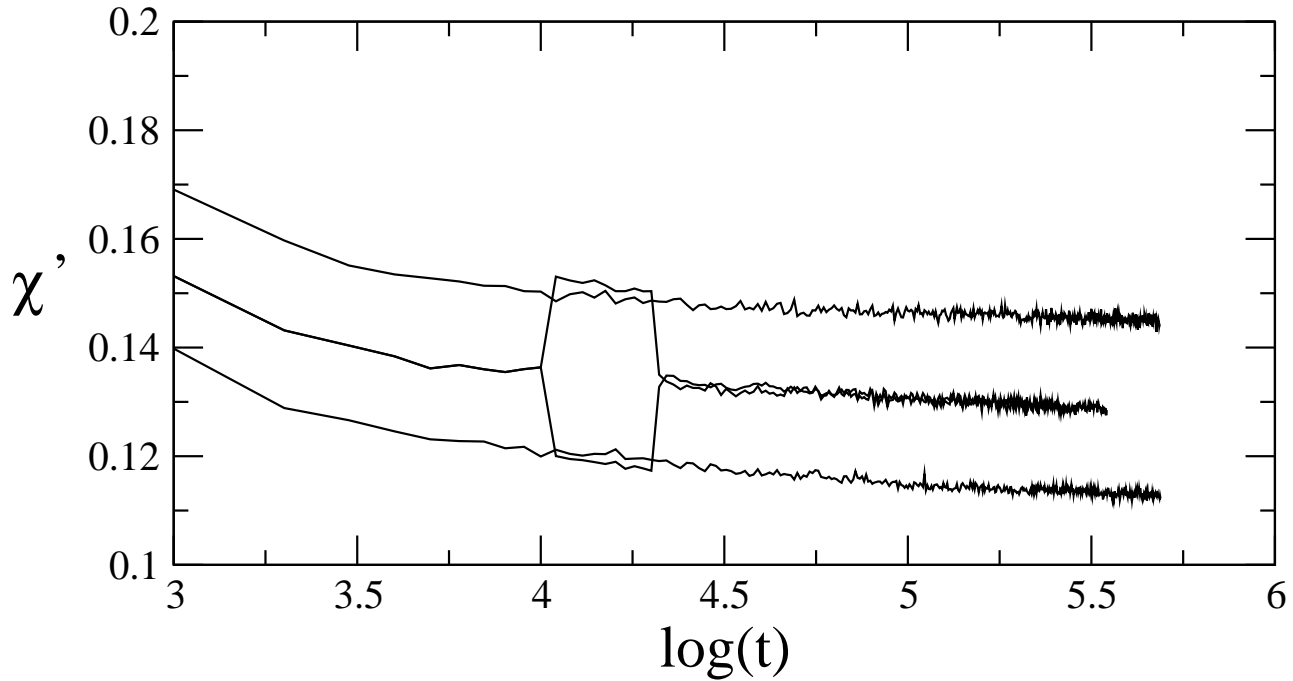
- temperature
- magnetic field
- coupling

Rejuvenation in real spin glasses under a temperature cycle.



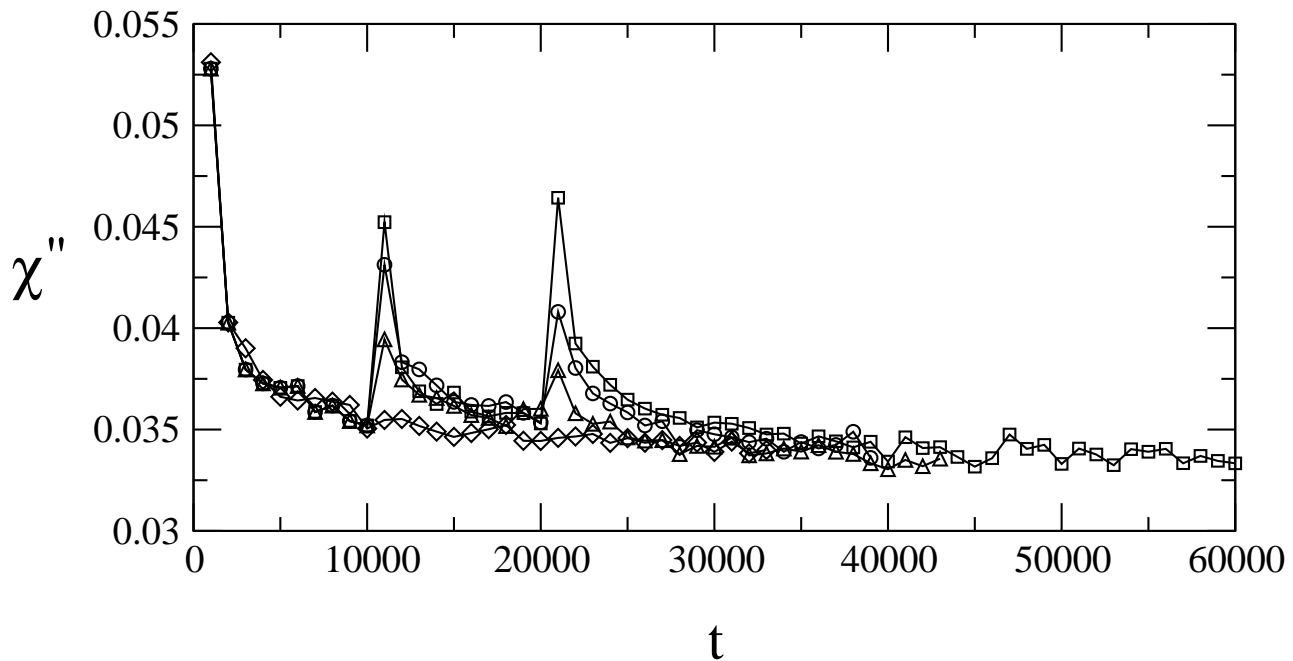
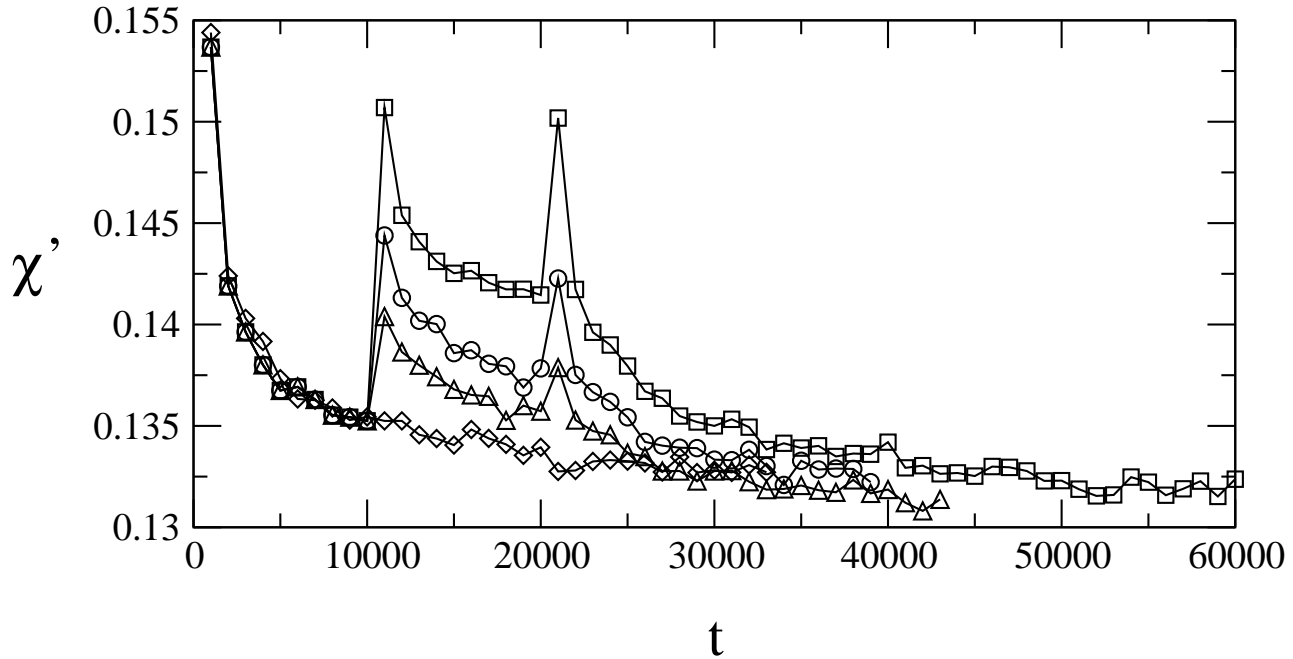
# Temperature cycle in the EA model.

Temperatures:  $T_1 = 0.6$  and  $\Delta T = \pm 0.1$



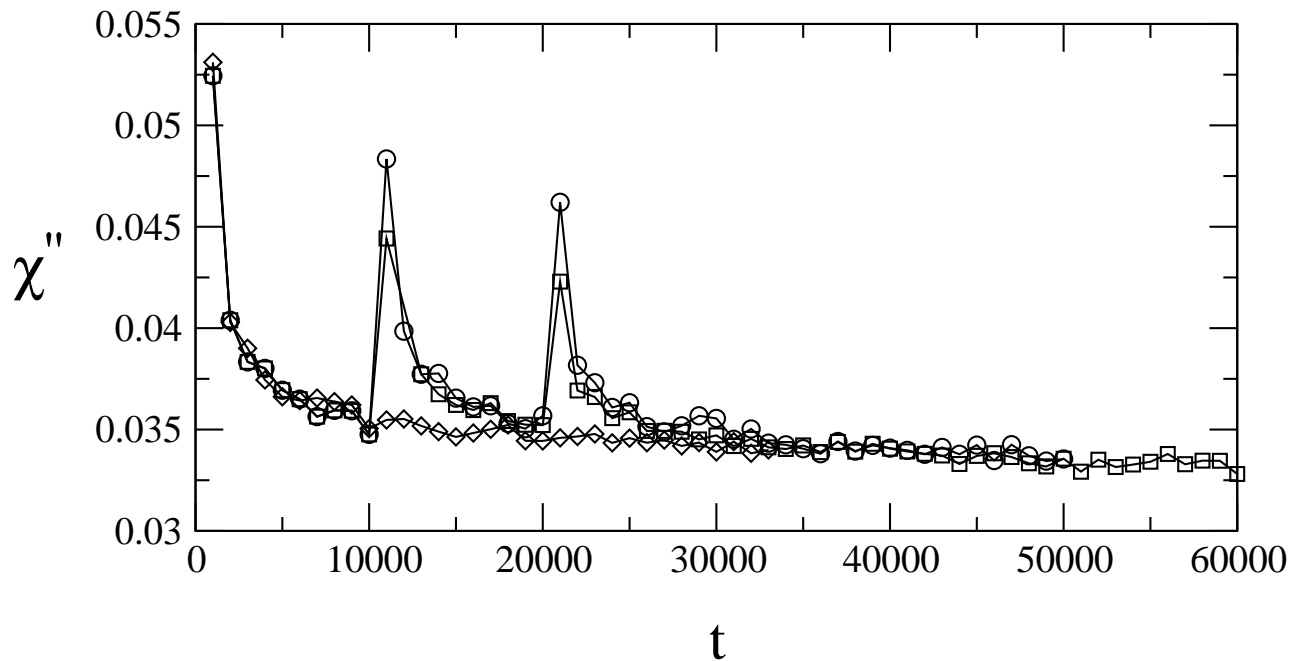
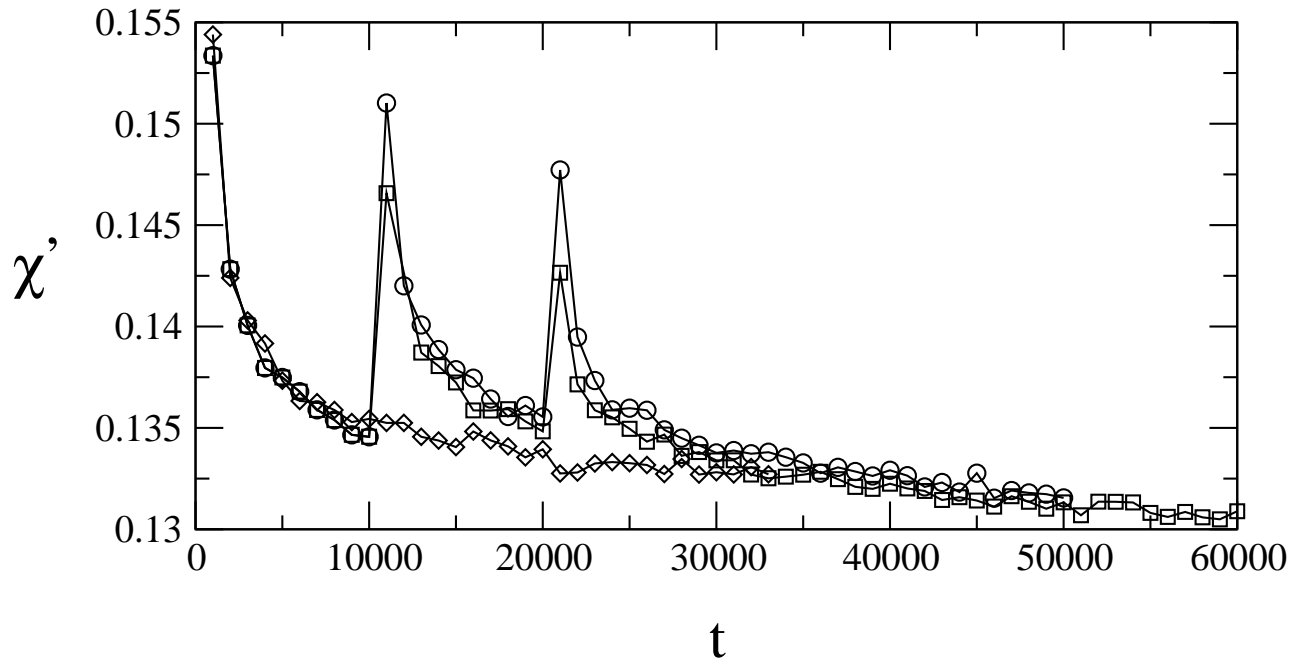
# Application of the external **DC field**.

$T = 0.6$  and perturbing field = 0.4, 0.6, 1.0



Sign change of a fraction  $r$  of **couplings**.

$T = 0.6$  and  $r = 0.05, 0.1$



## Conclusions and Discussion

On time and length scales that can be reached with present computers, we do not find any chaotic effect under temperature changes, while chaos clearly appears under changes in the external field and in the random couplings.

The **absence of temperature chaos** is confirmed also by the presence of very strong cooling rates effects.

On the contrary, in real spin glasses chaotic effects are induced by any one of the above perturbations and there are no cooling rate effects.

How can this discrepancy be explained ?

**Note** that our results seem to be in contrast with a naive scaling, which would predict stronger  $\Delta T$ -chaos with respect to  $\Delta h$ -chaos:  $L(\Delta T) \sim \Delta T^{-1/\zeta}$  with  $\zeta = d_s/2 - \theta < 3/2$  against  $L(\Delta h) \sim \Delta h^{-2/3}$ .

But, prefactors count !



Two possible explanations:

\* Everything is due to **computing limitations**:

- too small system sizes;
- too large probing fields;
- too small time and length scales.

Consider that  $\xi$  is few lattice spacing in numerical simulations, while  $\xi \sim 100$  in experiments.

\* The **EA model fails** in describing some aspects of real spin glasses.

### Some perspectives

- Pushing forward simulations of the EA model (with the next computer generation).
- Looking for different spin glass models.
- Explaining why in the EA model  $\Delta T$ -chaos is so weak.
- Experiments at high frequencies.