# UNFRUSTRATED MODELS WITH A GLASS TRANSITION

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# Two motivations for this work

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1) To find a spin model with many of the properties of a structural glass:

- no disorder, no frustration;
- at least 3 states: liquid, crystal and glass.

Hopefully, a non-disordered model of spins is much more easy to be treated (and solved!) analytically.

2) To give a more formal answer to the question:

*Why the use of models with quenched disorder and frustration usually gives very good results in the study of models which are neither disordered nor frustrated?*

# Definition of the model

Ising spins → $S_i = \pm 1$

Multi-spins interactions → $S_{i_1} S_{i_2} \dots S_{i_p}$ $(p = 3)$

Ferromagnetic couplings → $J = 1$

Fixed connectivity → every spin participate to exactly $c$ interactions.

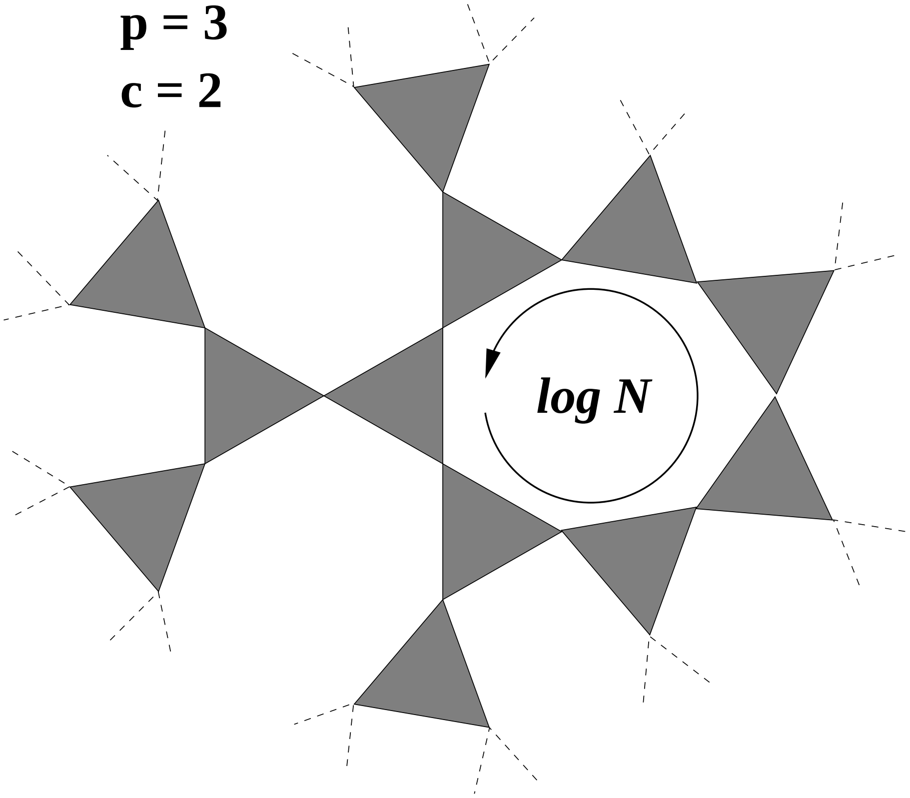
$$\mathcal{H} = - \sum_{(i,j,k) \in \mathcal{G}} S_i S_j S_k$$

$\mathcal{G}$ is a set of $\frac{cN}{3}$ triples, with the topology of a Bethe lattice.

# Bethe lattice

All sites are equivalent.

No disorder on all length scales $\ell < \log(N)$.



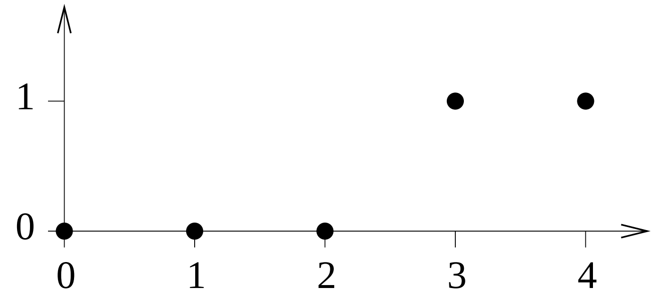
Neighbours are chosen at random $\Rightarrow$

$\Rightarrow$ long-range interactions.

# Physical relevance of multi-spins interactions

1) Geometrical constraints. E.g. $\theta(\sum_j n_j - \ell)$

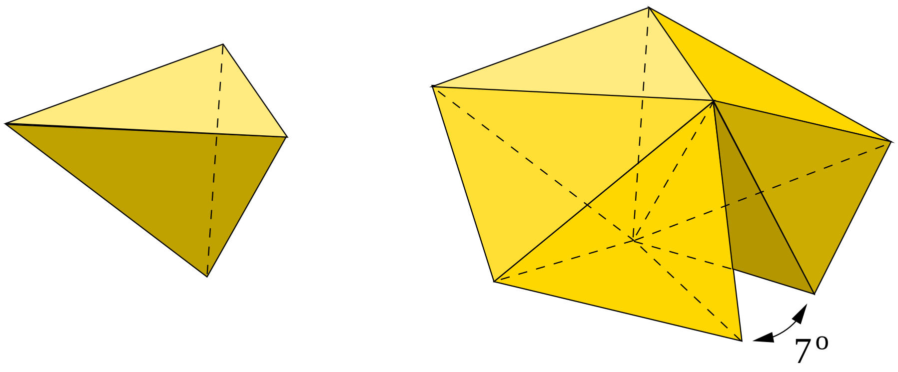
$\theta(x-2)$



$x = \sum_j n_j$

$$x\,(x-1)\,(x-2)\ldots \;\Rightarrow\; n_{j_1} n_{j_2} n_{j_3} + \ldots$$

2) Local order incompatible with global order.



Global order ↔ ground state, $S_i = 1\ \forall i$

$$S_i\,S_j\,S_k = \left\{ \begin{array}{l} +\;+\;+ \quad \to \text{ground state} \\ \left.\begin{array}{ccc} + & - & - \\ - & + & - \\ - & - & + \end{array}\right\} \begin{array}{c}\text{incompatible with} \\ \text{global order}\end{array} \end{array}\right.$$

# Comparison with different topologies

- Finite-dimensional lattices

  More realistic, but limited to numerical studies.

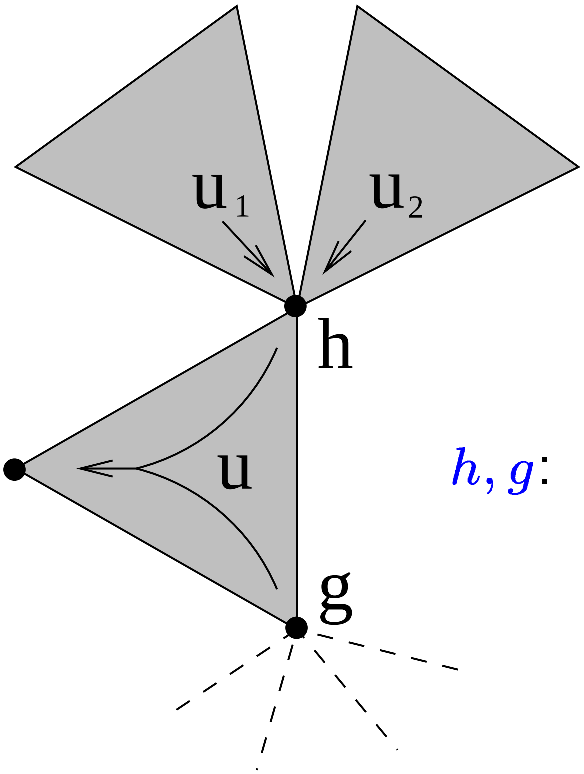
  Symmetries ⇒ Large ground state degeneracy

- Mean-field fully connected ($c = N - 1 \to \infty$)

  Each spin feels the rest of the system.

  Lack of frustration ⇒ Trivial dynamics towards the lowest energy configuration

# Recursion relations



$h, g$: cavity fields

$$P(h) = \int \prod_{i=1}^{c-1} du_i \; Q(u_i) \; \delta\left[h - \sum_{i=0}^{c-1} u_i\right]$$

$$Q(u) = \int dh \; dg \; P(h) \; P(g) \; \delta[u - u(h,g)]$$

$$\tanh[\beta u(h,g)] = \tanh(\beta) \; \tanh(\beta h) \; \tanh(\beta g)$$

$$m = \langle \tanh(\beta c u) \rangle_Q \qquad q = \langle \tanh(\beta c u)^2 \rangle_Q$$

# Thermodynamic solution

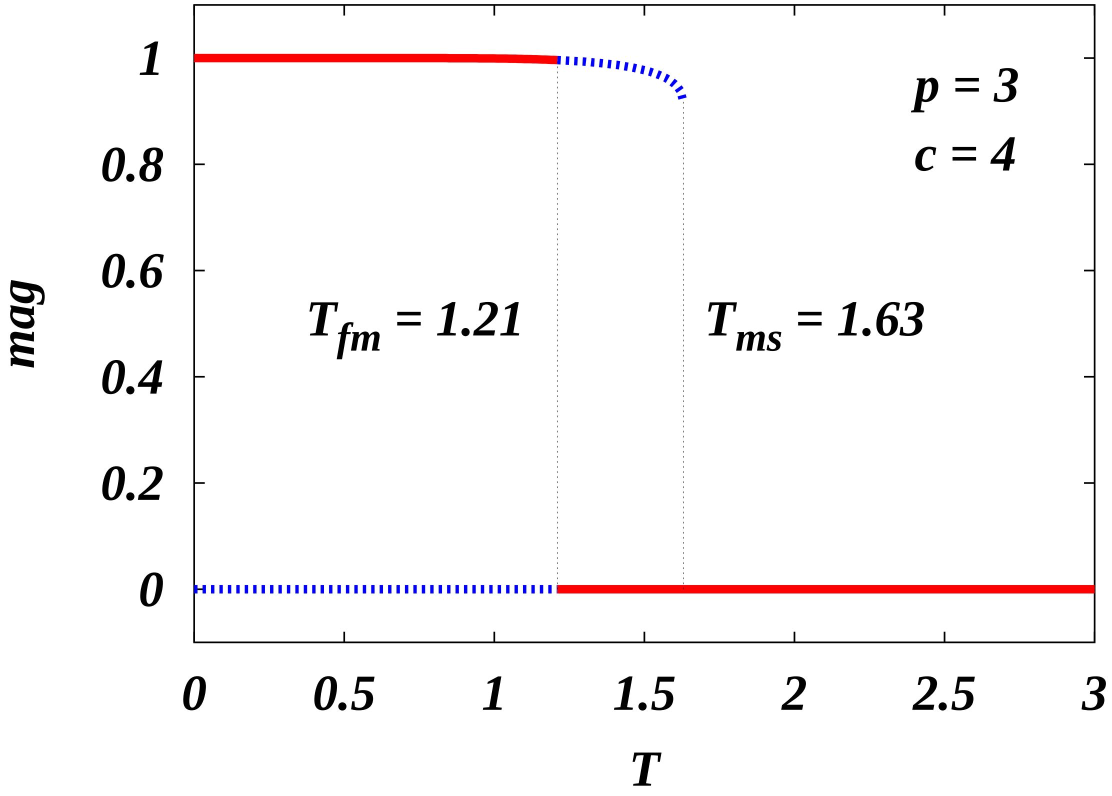
$$\left.\begin{array}{l}\text{Sites equivalence}\\ \text{Replica Symmetry}\end{array}\right\} \Rightarrow \begin{array}{l} Q(u) = \delta(u - u_0) \\ P(h) = \delta[h - (c-1)u_0] \end{array}$$

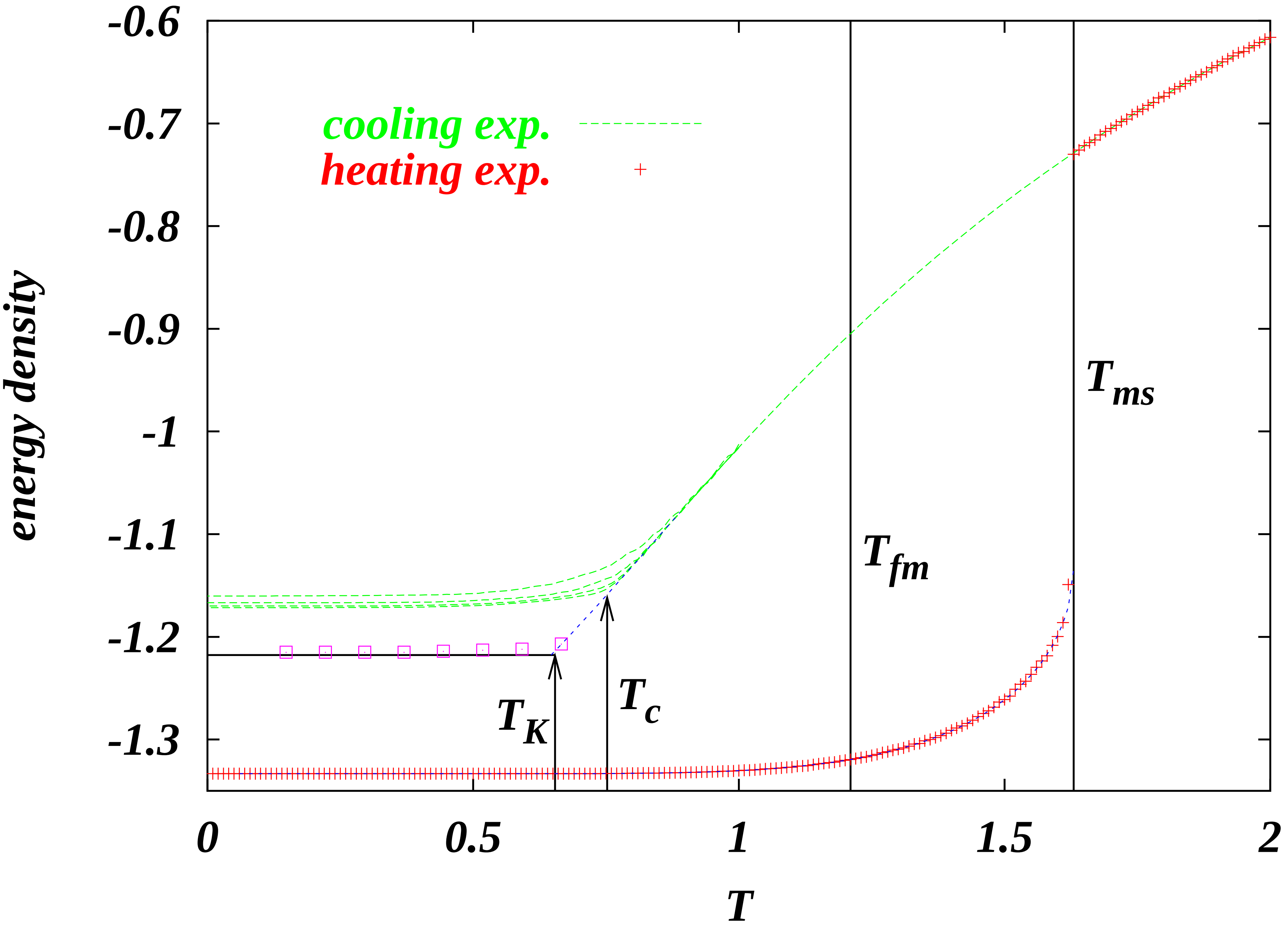
- Paramagnetic solution: $u_0 = 0$

- Ferromagnetic solution:

$$\tanh(\beta u_0) = \tanh(\beta) \tanh(\beta k u_0)$$

exist for $T \leq T_{ms}$, stable for $T \leq T_{fm}$




cooling exp.
heating exp.
energy density
-0.6
-0.7
-0.8
-0.9
-1
-1.1
-1.2
-1.3
0
0.5
1
1.5
2
T
$T_K$
$T_c$
$T_{fm}$
$T_{ms}$

# Following the out-of-equilibrium dynamics

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$m = 0$ sector always locally stable
because of finite connectivity

$m = 0 \Rightarrow P(h)$ symmetric under $h \leftrightarrow -h$

Equivalent local arrangements

$\Downarrow$

wrong choices

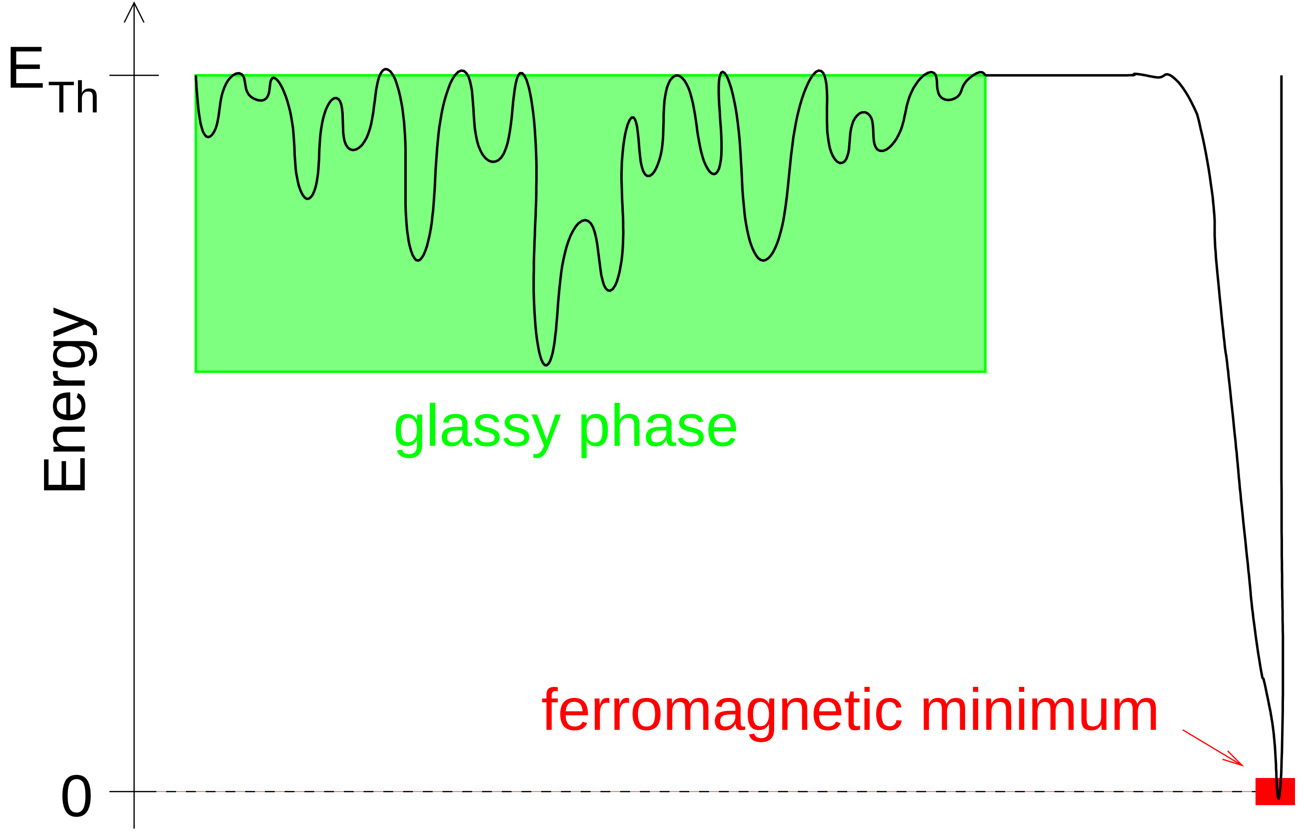
$\Downarrow$

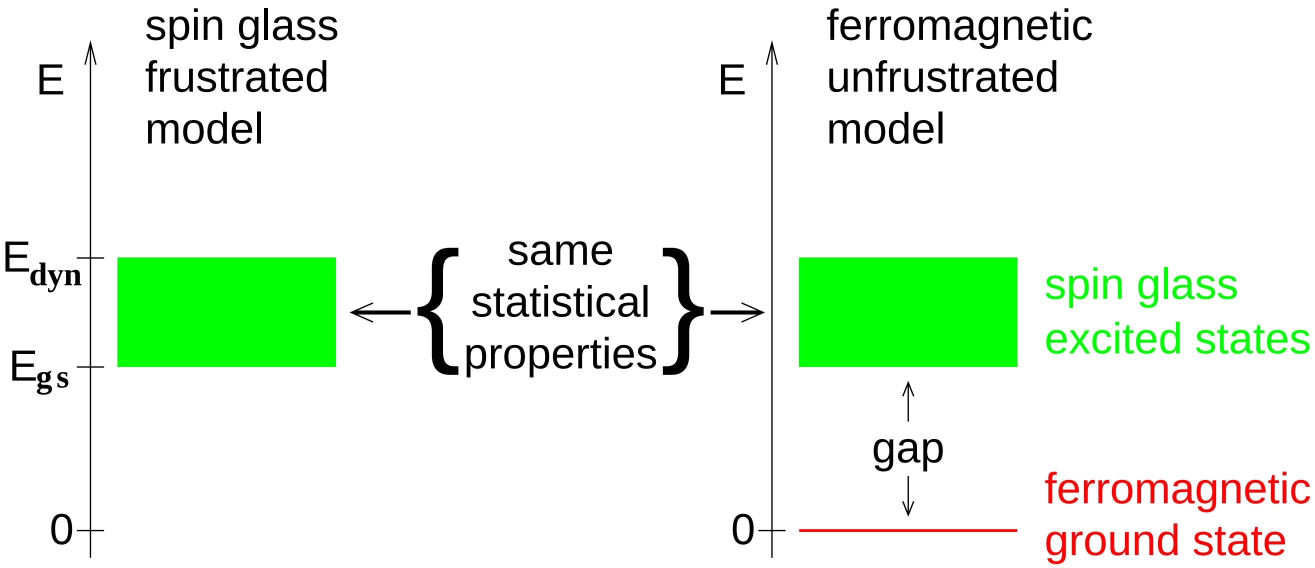
heterogeneous frustrated configuration

$\Downarrow$

$P(h)$ broad, i.e. glassy solutions $q \neq 0$

Self-induced heterogeneities and frustration!


$E_{Th}$
Energy
glassy phase
ferromagnetic minimum
0

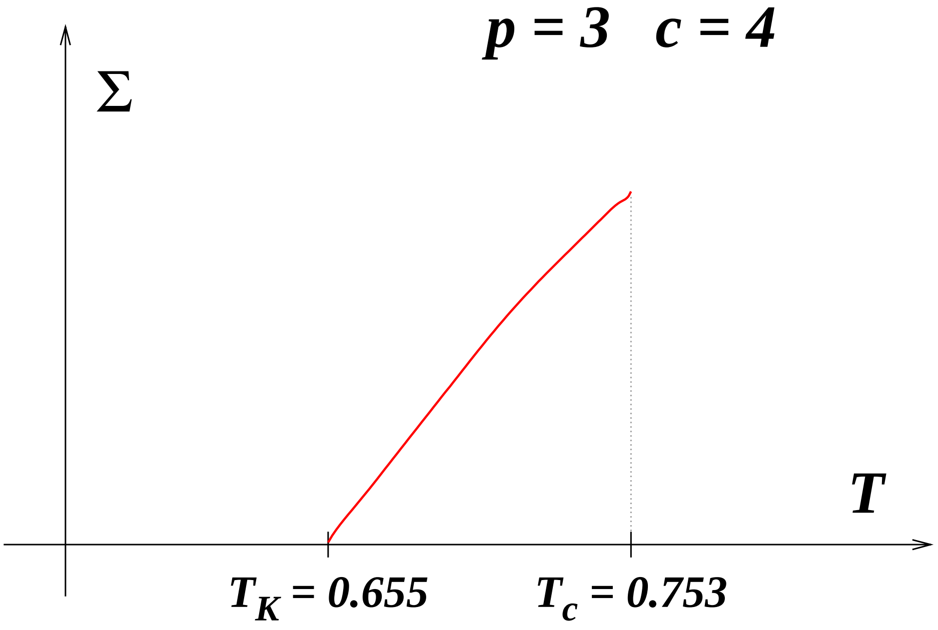

E
spin glass
frustrated
model
$E_{dyn}$
$E_{gs}$
0
same
statistical
properties
E
ferromagnetic
unfrustrated
model
spin glass
excited states
gap
ferromagnetic
ground state
0

# Replica Symmetry Breaking solutions

Dynamical RSB !

1-step ‘factorized’ Ansatz ⇒ $q_0 = 0$

$$\frac{P(h)}{\cosh(\beta h)^m} = \frac{1}{\mathcal{N}} \int \prod_{i=1}^{c-1} \frac{du_i \; Q(u_i)}{\cosh(\beta u_i)^m} \; \delta\left[h - \sum_{i=0}^{c-1} u_i\right]$$

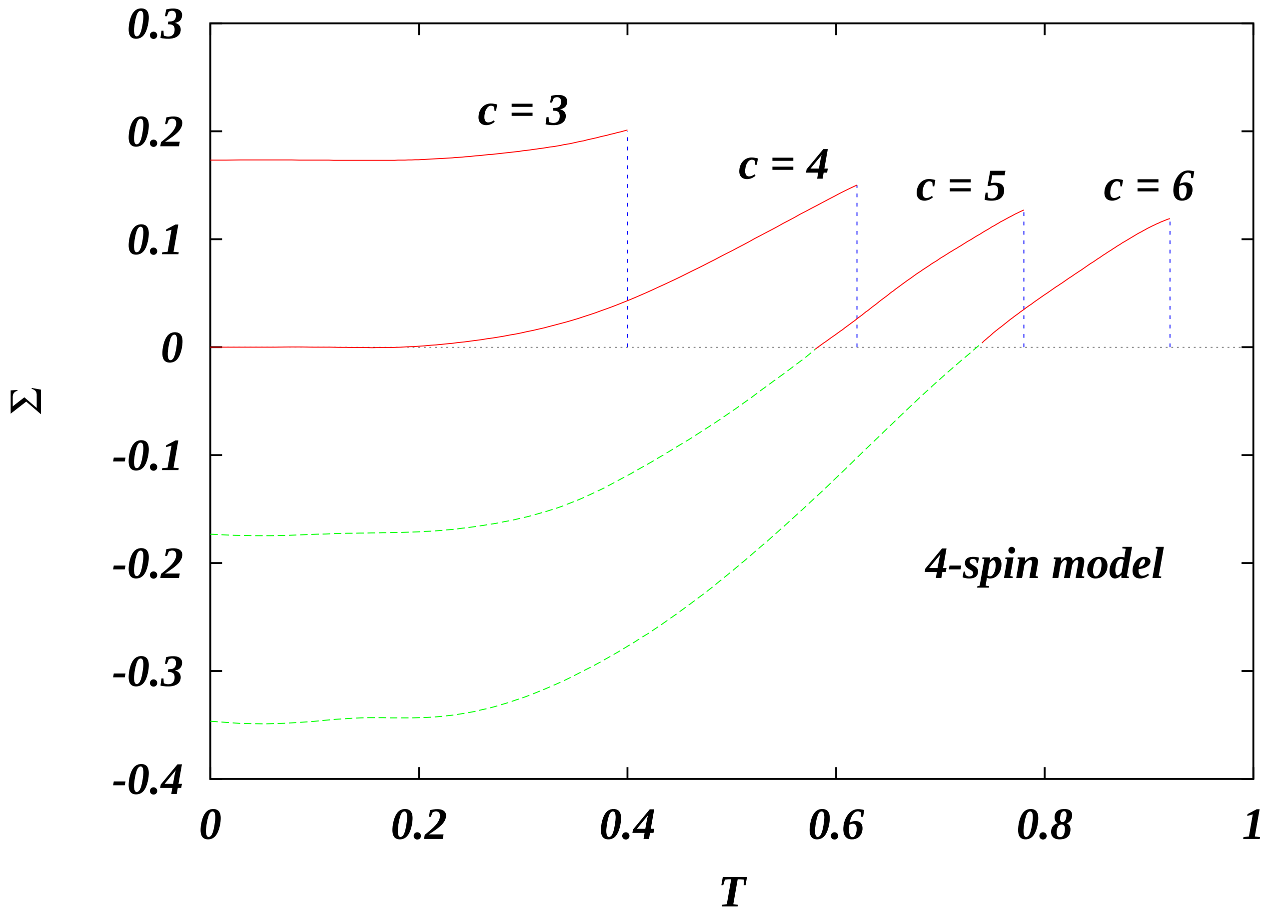


# Spin Glass version ($J = \pm 1$)

$$\tanh[\beta u(h,g)] = \tanh(\beta \underset{\substack{\uparrow \\ \pm 1}}{J}) \tanh(\beta h) \tanh(\beta g)$$

$Q(u) \rightarrow \overline{Q(u)}$ symmetric

$\boxed{\text{Spin Glass} = \text{Ferromagnet with } m = 0}$



$T_K$ may disappear, $T_c$ always exist for $c, p > 2$

# Summary

- Simple unfrustrated spin model with the phases of a structural glass: liquid, crystal and glass.
- Frustration is self-induced during relaxation → → 'dynamical' RSB.
- Exact solutions for the thermodynamics (RS) and for the asymptotic dynamical state (RSB).
- Asymptotic dynamical state ⟷ Constrained thermodynamics ($m = 0$).
- A formal justification for the use of disordered and frustrated models in the study of models with neither disorder nor frustration.

## A couple of references

- S. Franz, M. Mézard, F. Ricci-Tersenghi, M. Weigt and R. Zecchina, Europhys. Lett. **55**, 465 (2001).
- S. Franz, M. Leone, F. Ricci-Tersenghi and R. Zecchina, Phys. Rev. Lett. **87**, 127209 (2001).