ON THE NATURE OF THE LOW-TEMPERATURE PHASE IN DISCONTINUOUS MEAN-FIELD SPIN GLASSES

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Results partially appeared in cond-mat/0301591

Main results

For the generic discontinuous mean-field spin glass

- FRSB is necessary at any $T < T_d$
  threshold states are always FRSB
  off-equilibrium dynamics has ∞ time scales

- FRSB calculation for the complexity Σ
  $f_G \leq f_d \leq f_d^{\text{1RSB}}$

- Stability of 1RSB vs 2RSB in finite connectivity models (diluted p-spin and K-SAT).

- Asymptotic states for fixed-T and slow cooling off-equilibrium dynamics are different!

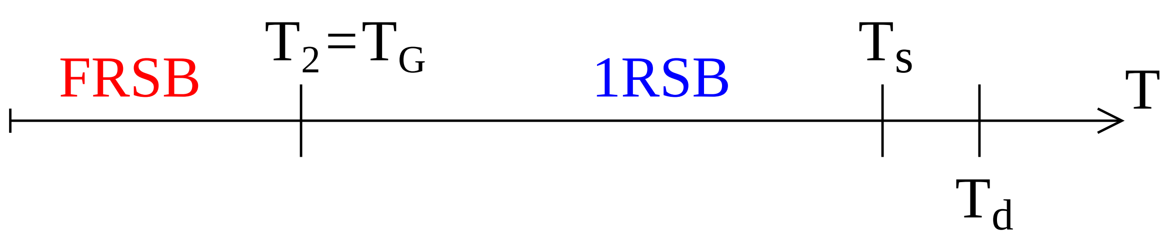
p-spin fully connected model

$$\mathcal{H}(\sigma) = -\sum_{(i_1 \ldots i_p)} J_{i_1 i_2 \ldots i_p} \sigma_{i_1} \sigma_{i_2} \cdots \sigma_{i_p} \quad ,$$

where $\sigma_i = \pm 1$, $\overline{J} = 0$ and $\overline{J^2} = p!/(2N^{p-1})$.

$p = 3$ fixed in this presentation.

Thermodynamic solution by Gardner (1984)



At T_d metastable states arise $\Longrightarrow$ off-equilibrium dynamics relaxes towards threshold states

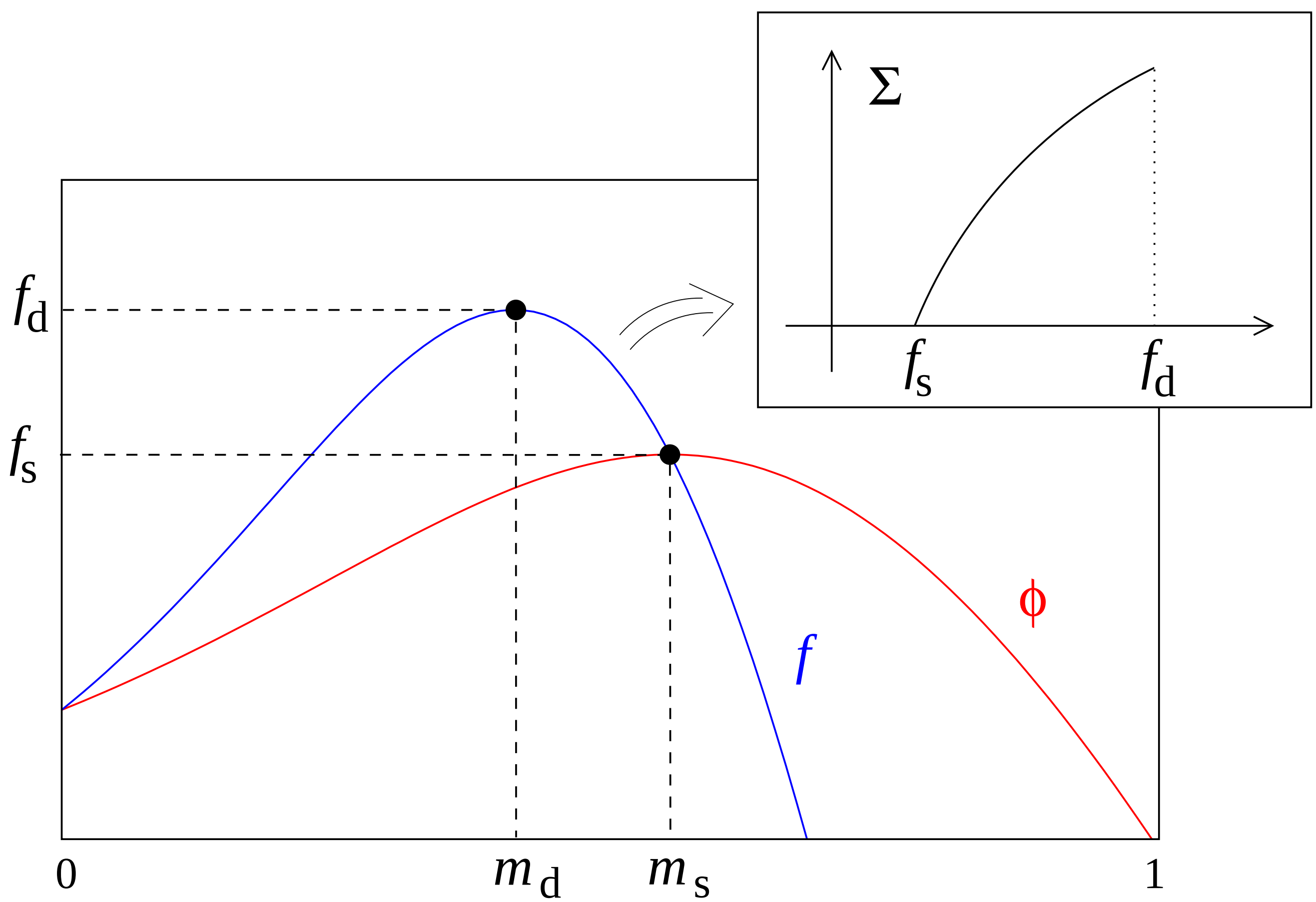
1RSB solution

For any $T < T_d$, the complexity $\Sigma_T(f)$ is given by

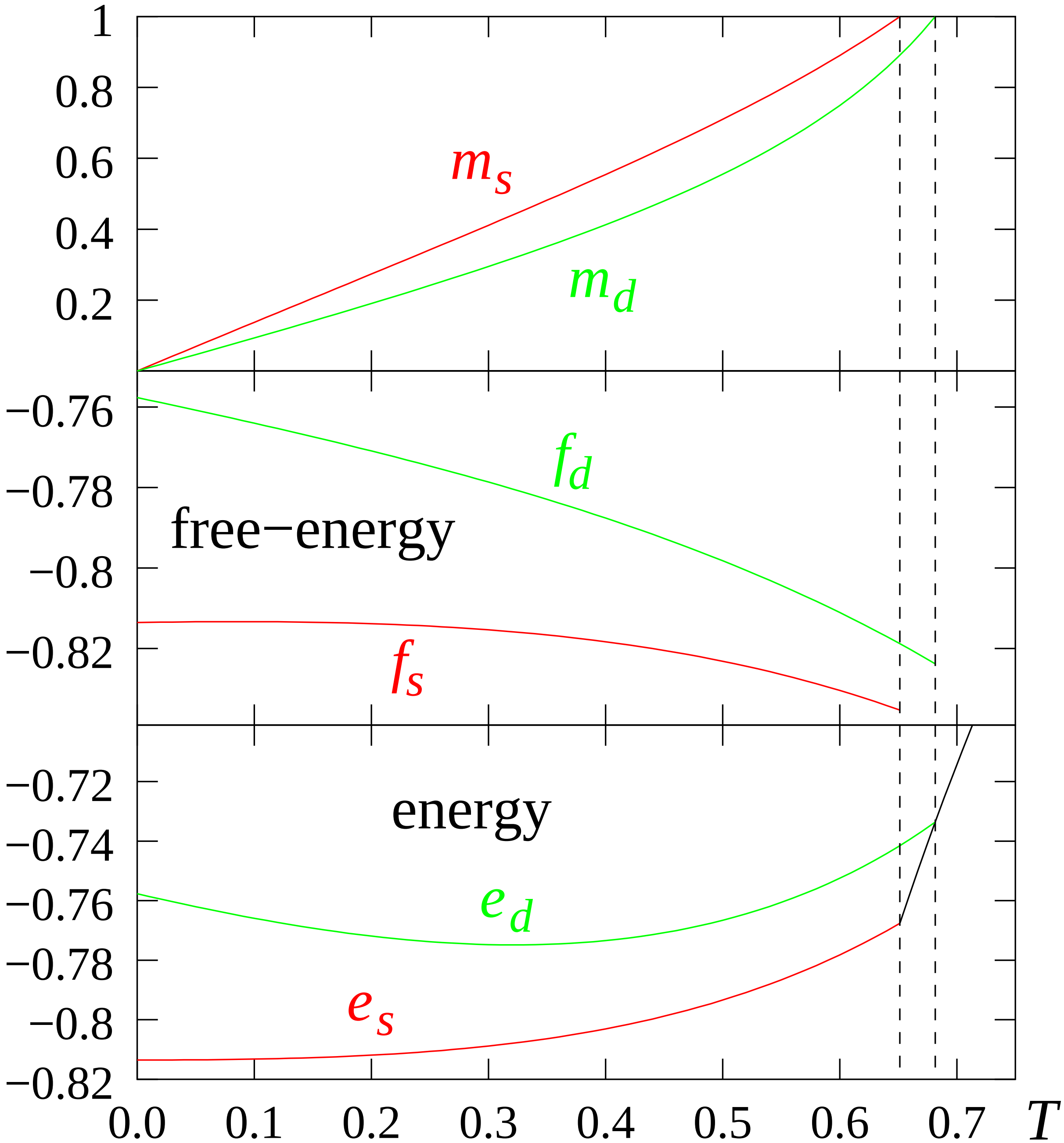
$$f = \partial_m[m\,\phi] \qquad \Sigma = \beta\, m^2 \partial_m \phi = \beta\, m\,(f - \phi)$$

where $\phi(\beta, m)$ is the replicated free energy.

For the 3-spin at $T = 0.4$



1RSB solution

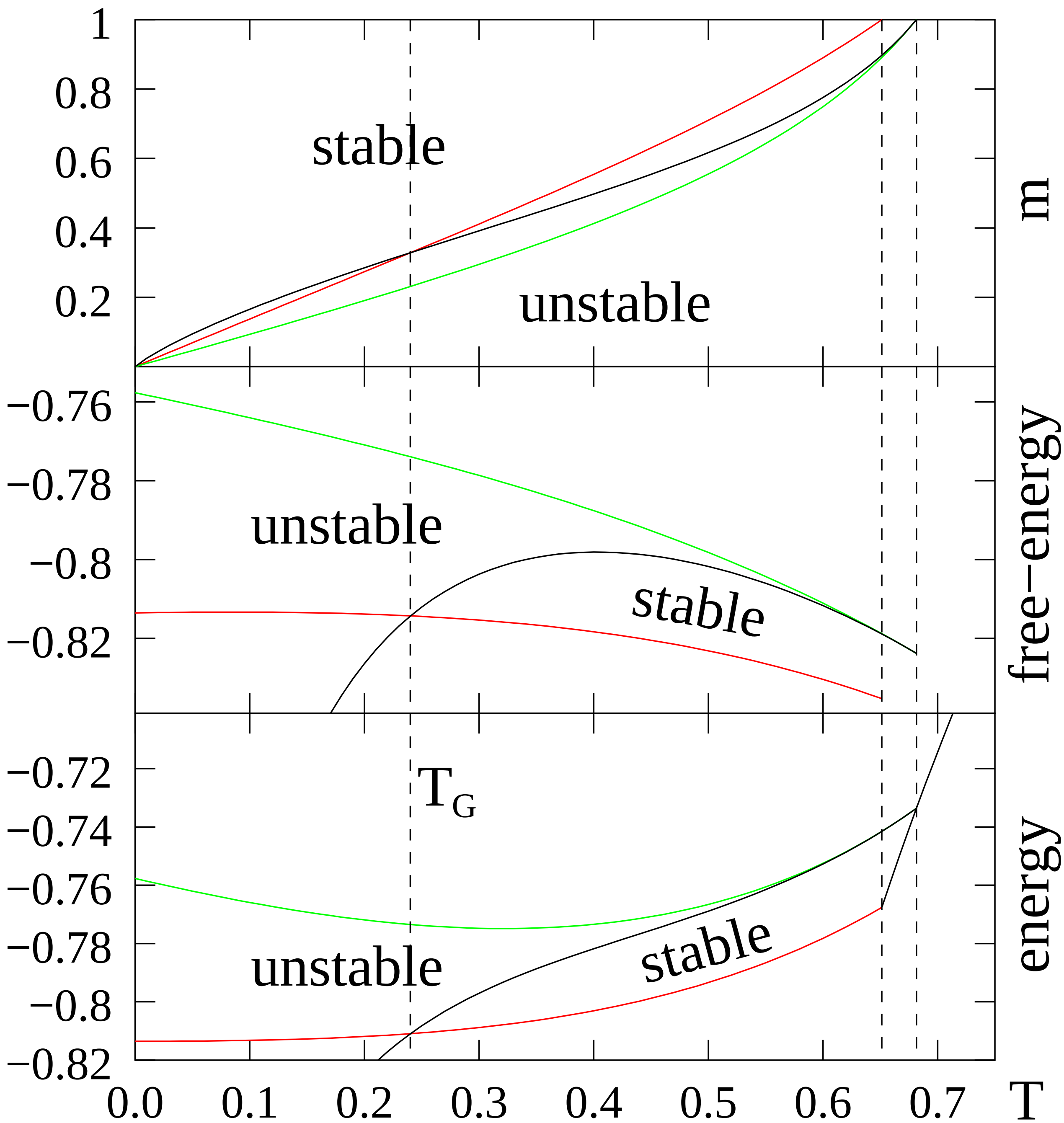


$$e(\beta, m) = -\frac{\beta}{2}\Big[1 - (1-m)\, q^p\Big]$$

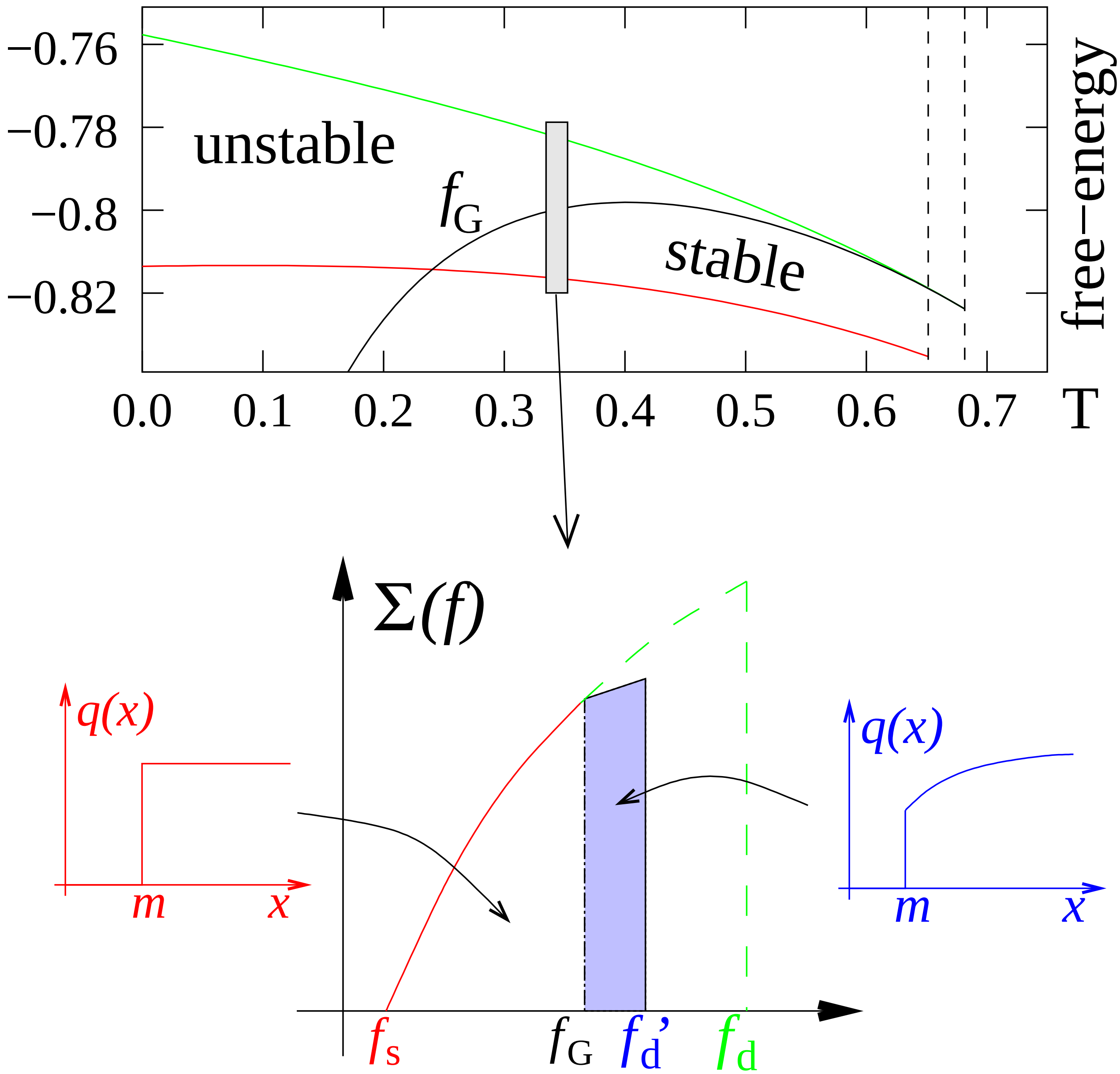
1RSB solution stability

The 1RSB solution is stable if

$$\frac{2}{p(p-1)\beta^2 q^{p-2}} > \frac{\int \mathcal{D}z \cosh^{m-4}\left(\beta\sqrt{\frac{p}{2}\,q^{p-1}}\,z\right)}{\int \mathcal{D}z \cosh^{m}\left(\beta\sqrt{\frac{p}{2}\,q^{p-1}}\,z\right)}$$



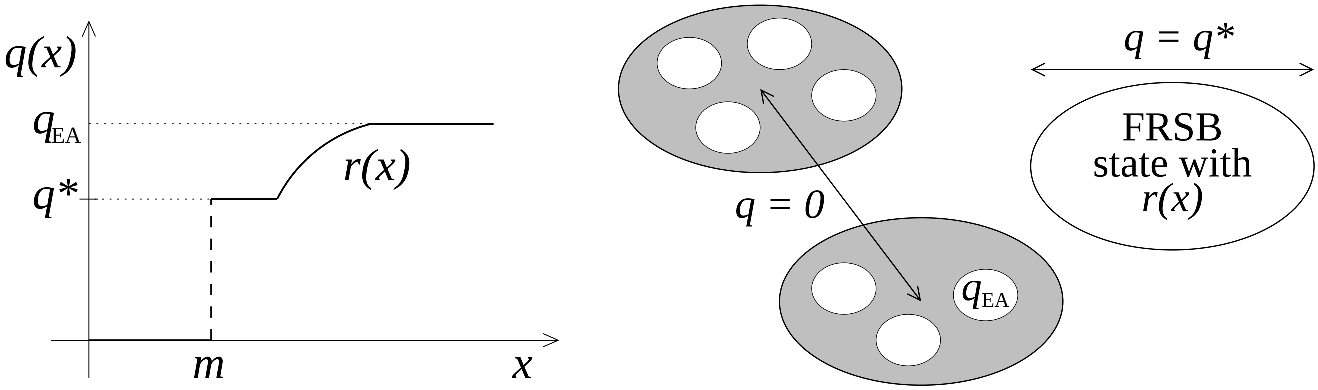
The general picture



How can we calculate $\Sigma(f)$ in the FRSB region?

FRSB complexity

General $q(x)$ for a discontinuous model with $h = 0$



Clean clustering only with an overlap $0 < q < q^*$.

Varying m we can count the number of clusters.

We propose the relevant complexity $\Sigma_T(f)$ to be the Legendre transform of

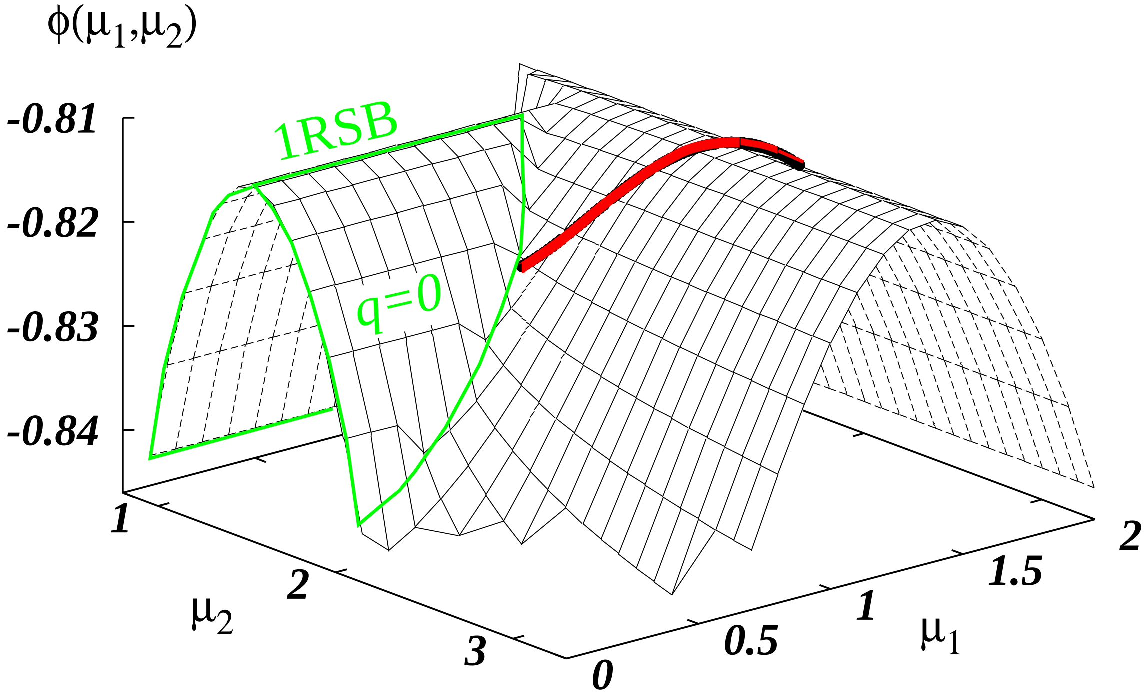
$$\phi(T,m) = \max_{r(x)} \Phi_{\text{FRSB}}[T, q(x)]$$

with respect to m

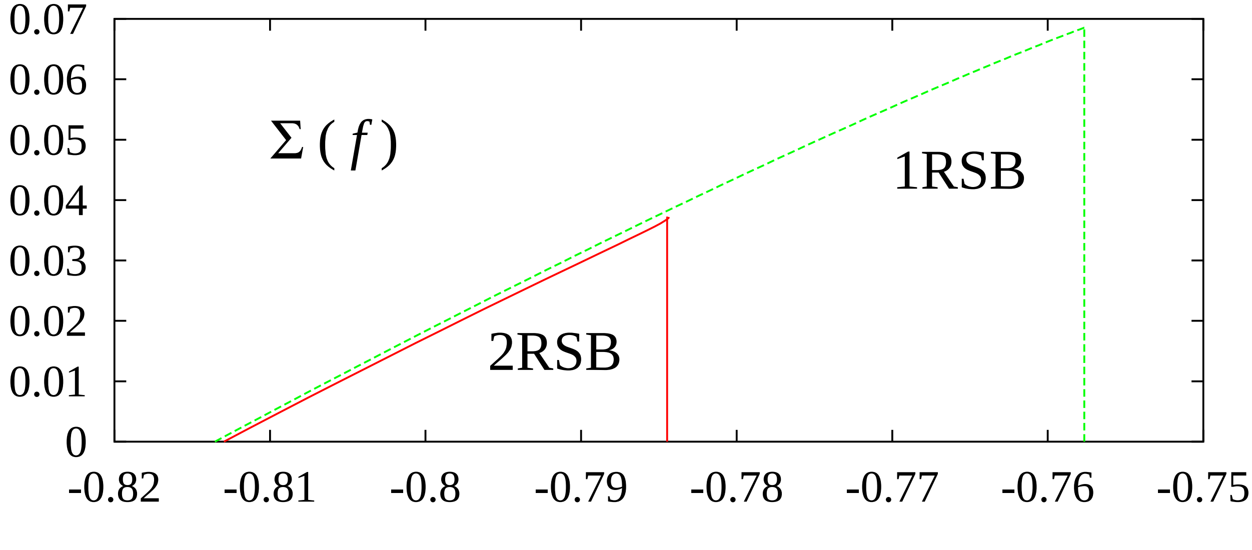
2RSB solution at $T = 0$

Three overlaps: $q_0 = 0 \quad q_1 = q \quad q_2 = 1$

Two parameters: $\beta m_1 \to \mu_1 \quad \beta m_2 \to \mu_2$



red line: local maxima in μ_2 at fixed μ_1

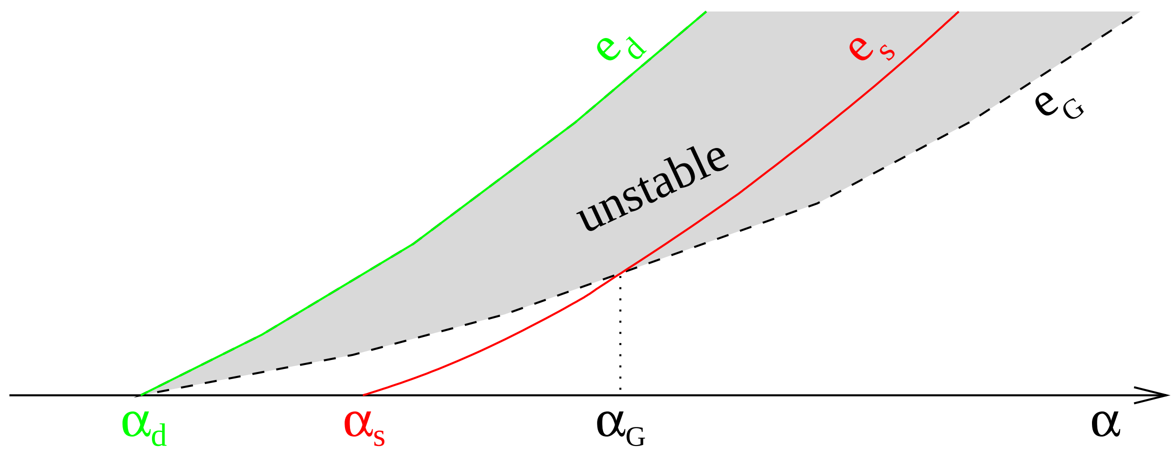


Finite connectivity models

We have studied three models at $T = 0$:

- diluted p-spin with fixed connectivity
- diluted p-spin with fluctuating connectivity
- random K-SAT

All have the same behavior at zero temperature



Some values for α_G

3-spin with fixed conn. $\longrightarrow 10 < c_G < 11$

3-spin with fluct. conn. $\longrightarrow c_G = 9.216(6)$

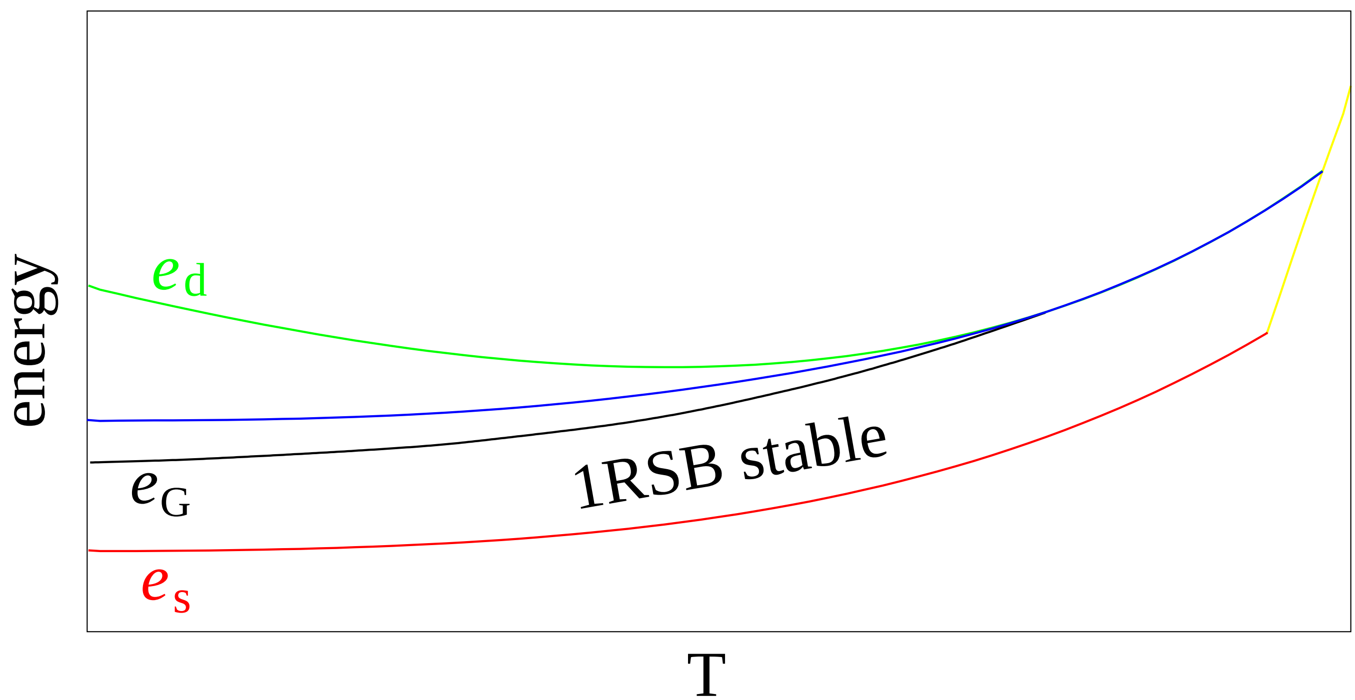
random 3-SAT $\longrightarrow \alpha_G \simeq 9$

note: at finite temperatures K-SAT is different from diluted p-spin because it has fields $\propto T$

Finite connectivity 3-spin at $T > 0$

For connectivities such that $e_G < e_s \rightarrow$
$\rightarrow$ similar to fully-connected version

For connectivities such that $e_s < e_G$



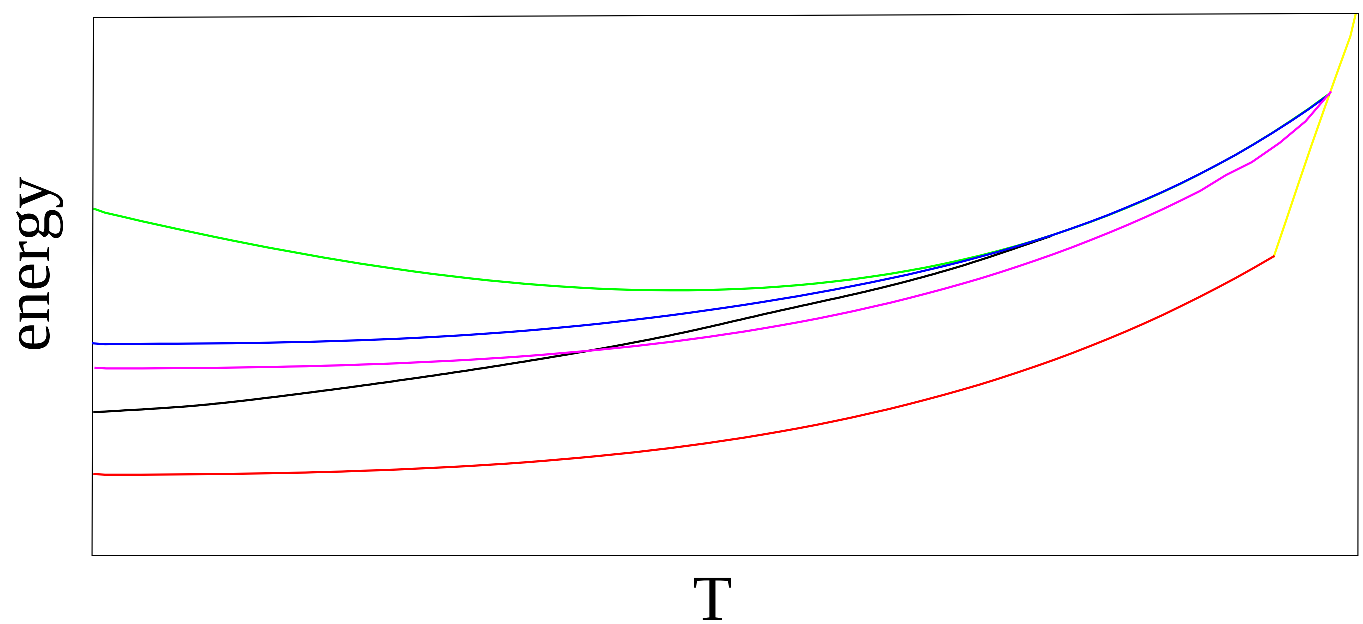
- ground state inapproximability

- does relaxation dynamics stop at the threshold states (blue line)?

Off-equilibrium dynamics

A quench from ∞ to temperature T relaxes towards threshold states (blue line)

A very slow cooling follows the magenta line, so it goes below threshold states



Along the magenta line $\rightarrow$ Σ =costant

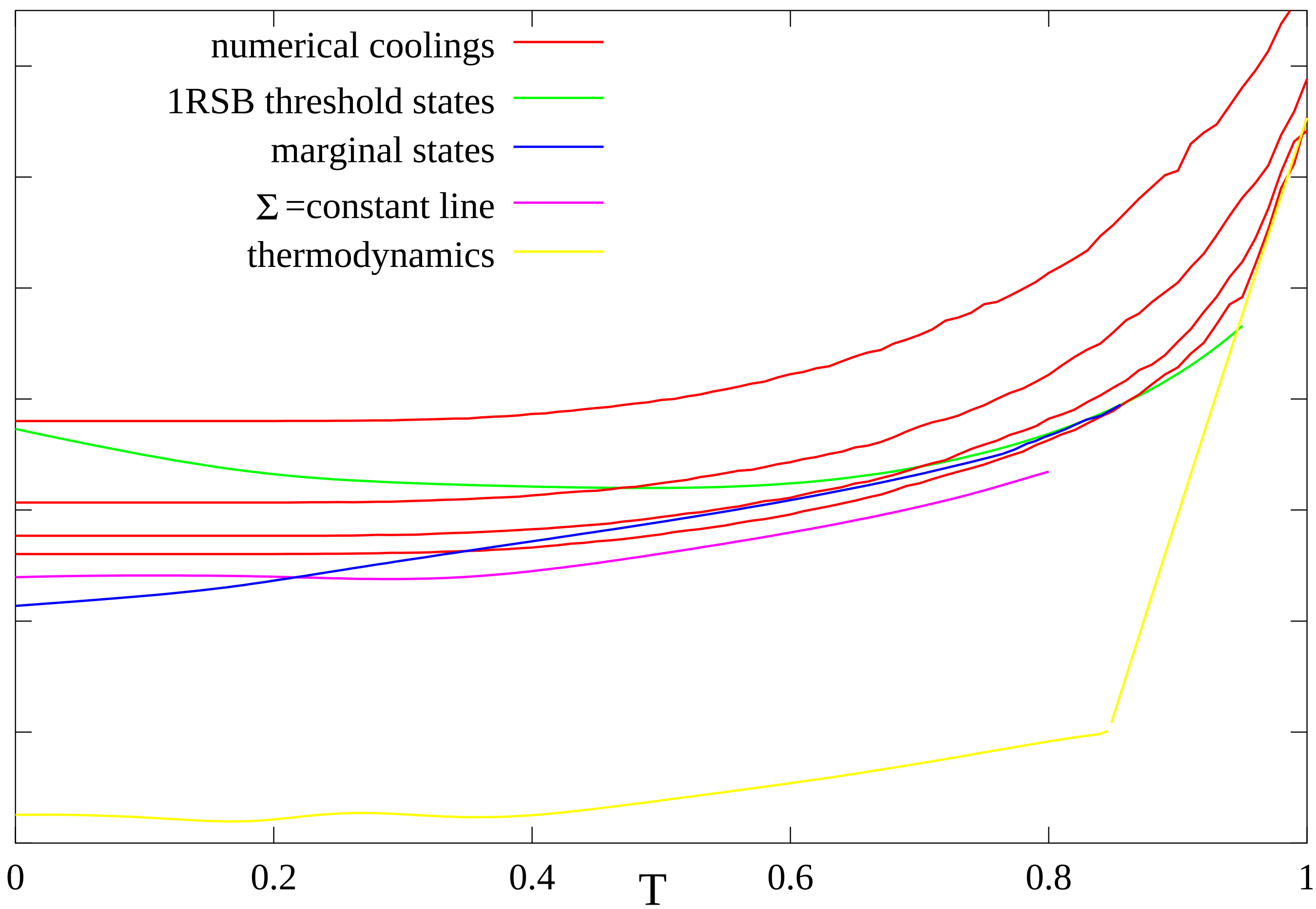
During the cooling the system remains trapped within a state. Inside a state we have that

$$e = \partial_\beta(\beta f) = \partial_\beta(\beta\phi) + \frac{1}{m}\partial_\beta\Sigma \quad \Rightarrow \quad \partial_\beta\Sigma = 0$$

Lopatin and Ioffe, PRB 66, 174202 (2002)

Numerics vs analytics

3-spin with fixed connectivity 5 ($N \sim 10^5$)



numerical coolings are below both 1RSB threshold states and marginal states

For infinitely slow rate, numerical coolings extrapolate to the iso-complexity line

Extrapolations